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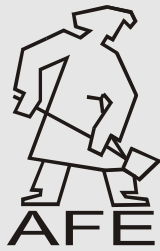
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# Inhomogeneity of the grain size of aircraft engine turbine polycrystalline blades

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## Abstract

The determination of the behaviour of inhomogeneous materials with a complex microstructure requires taking into account the inhomogeneity of the grain size, as it is the basis for the process of designing and modelling effective behaviours. Therefore, the functional description of the inhomogeneity is becoming an important issue. The paper presents an analytical approach to the grain size inhomogeneity, based on the derivative of a logarithmic-logistic function. The solution applied enabled an effective evaluation of the inhomogeneity of two macrostructures of aircraft engine turbine blades, characterized by a high degree of diversity in the grain size. For the investigated single-modal and bimodal grain size distributions on a perpendicular projection and for grains with a non-planar surface, we identified the parameters that describe the degree of inhomogeneity of the constituents of weight distributions and we also derived a formula describing the overall degree of inhomogeneity of bimodal distributions. The solution presented in the paper is of a general nature and it can be used to describe the degree of inhomogeneity of multi-modal distributions. All the calculations were performed using the Mathematica<sup>®</sup> package.

**Keywords:** Turbine blade, Inhomogeneity of the grain size, Logarithmic-logistic function

## 1. Introduction

Modern material engineering requires the application of a precise quantitative description of the material microstructure. In the case of polycrystalline materials, a description of grain geometry is required, taking into account the shape, size, and degree of the grain size diversity. The parameter that is determined most often to describe the microstructure of polycrystalline materials is the grain size [1]. Since grains in the polycrystal are three-dimensional elements of complex shapes, the natural measure of their size is their volume [2-3]. However, the measurement of the grain volume poses considerable technical

difficulties. For this reason, the grain size is most frequently described by means of parameters measured on cross-sections of polycrystals. It should be emphasized that the grain size expressed by means of the cross-section area is the parameter that is most often used when describing the microstructure because it depends on the shape of the grain to the least extent. Taking into consideration the grain size diversity in polycrystalline materials, in the description of the microstructure and its relations to mechanical properties, the issue of inhomogeneity of the grain size is often reduced to the average size of the grain. In rare cases, the quantitative description of the mechanical properties takes into account the parameters of the grain size distribution. In the field

of quantitative metallography, there is no functional description of the inhomogeneity of the grain size.

## 2. Quantitative approach to the grain size inhomogeneity

The determination of the behaviour of inhomogeneous materials with a complex microstructure is a huge challenge, both in the process of designing and modelling their effective behaviours. The volume fraction, crystallographic and morphological orientation, as well as the size and shape of grains are the constituents of the microstructure of inhomogeneous materials (metals, intermetallics, ceramics) and they have to be properly selected. The parameter that is determined most often to describe the microstructure is the grain size. This issue does not only refer to average sized grain, which size is the basis for the Hall-Petch equation describing the correlation between this parameter and the yield point in quantitative terms, but also to the single-modal log-normal distributions of the grain size [4]. Other studies [5] describe the correlation between grain size distributions and the Hall-Petch strengthening mechanism for "large" grains, as well as other mechanisms, such as the transport of vacancies in the areas of grain boundaries for grain of the size of nanometers. The engineering of material grain size distributions is one of the methods of modifying the response of the material to load, with a particular emphasis on increasing the allowed elongation [6]. There are few theoretical models which describe the role of the grain size distribution in modifying the response of the material. A majority of such studies are concentrated on the yield point and elastic-plastic transition [7]. There have also been attempts to describe in quantitative terms the relations between the grain size distribution and the rate of strengthening [8]. Kurzydowski [4] was the first to present a simple analytical expression explaining the fall in the slope of the Hall-Petch curve with a growing width of the grain size distribution. A similar effect of transition of mechanical properties, close to the behaviour of grain larger than average in size with a broadened distribution, is presented in paper [8]. The above presented effect of the grain size distribution shows that the average grain size is an insufficient parameter to link the grain size distribution with mechanical properties. The authors of paper [8] suggested a geometrical parameter connected with the log-normal grain size distribution, called "representative grain size", which is a natural link connecting the yield point with the grain size distribution. Replacement of the average grain size with a "representative" size facilitated obtaining the Hall-Petch curve, whose slope is independent of the size distribution. The analysis of the grain size inhomogeneity presented above, which includes the relations between the mechanical properties, the average grain size and the distribution parameters, as well as the "representative" size, is limited mainly to an analysis of log-normal distributions of a single-modal nature. In many cases, the real structures of polycrystalline materials are more complex and are characterized by bimodal or multi-modal grain size distributions. An analysis of such distributions is presented in papers [8,9].

The issue of inhomogeneity is the part of quantitative metallography that has been developed to the least extent. In the subject literature, the quantitative description of this type of inhomogeneity, i.e. the inhomogeneity of the size, is reduced to the variability index. The size diversity of elements of a microstructure characterized by bimodal and multi-modal distribution, described quantitatively by means of a variability index, is a huge simplification. A proper analytical description of the size inhomogeneity should take into account the nature of the distribution, since such approach to the issue has been justified in the studies on deformation mechanisms, based on methods using electron microscopy (TEM) and the EBSD technique. The essence of the analytical description of size inhomogeneity is the application of such a function that would enable a comprehensive distribution characterization taking into account not only the nature of the distribution but also the quantitative description of the degree of grain size diversity in polycrystalline materials. For the purpose of a quantitative evaluation of the size inhomogeneity, we will use a logarithmic-logistic function, known in the literature [9], in the form (1):

$$y = y_{\max} \left( 1 - \frac{1 - y_{\min} \cdot y_{\max}^{-1}}{1 + \exp[b(X - Q)]} \right) \quad (1)$$

where:  $X$  – logarithmic value of the independent variable;  $y_{\min}$ ,  $y_{\max}$  – minimum and maximum value of the dependent variable;  $b$  – inhomogeneity measure of the independent variable;  $Q$  – mean value of the logarithmic independent variable corresponding to the value of  $y = 0,5(y_{\min} + y_{\max})$ .

The presented function is applied in the analytical description of cumulative distributions and in fractal description scaling procedures [10]. The derivative of this function described with expression (2) can be used for a quantitative description of single-modal and multi-modal distributions, and in the fractal analysis for quantifying fractal properties [11]:

$$f(X) = \frac{dy}{dX} = \frac{b(y_{\max} - y_{\min}) \exp[b(X - Q)]}{\{1 + \exp[b(X - Q)]\}^2} \quad (2)$$

## 3. Material and methodology

The research material consisted of 2 polycrystalline blades made of IN-100 nickel-based superalloy through investment casting at WSK "PZL Rzeszów" S.A.

To detect the presence of primary grains in the investigated superalloys, the airfoil surfaces were etched in a reagent of the following chemical composition: 18 g/dm<sup>3</sup> HNO<sub>3</sub>, 280-320 g/dm<sup>3</sup> HCl, 151-173 g/dm<sup>3</sup> FeCl<sub>3</sub> (anhydrous) and 110 ml/dm<sup>3</sup> H<sub>2</sub>O. Before etching, the blade must be degreased; the etching time is 5 seconds.

On the surface of blade made of IN-100 – test 1, there are the following zones: large equiaxed grains on the airfoil; on the airfoil edge, there is a very wide zone of chilled crystals and a wide zone of columnar crystals. Grains of a very large size prevail on the blade root. Blade made of IN-100 - test 2 are uniform throughout

their surface in terms of the macrostructure. On both, two roots and the blade airfoil, only equiaxed grains are present (Fig. 2). To obtain an image of the airfoil projection, a stereoscopic microscope, Olympus SZX-9, was used. Images were recorded in polarized light at magnification of 10x – 25x. The initial images were then transformed into binary images revealing using the grain boundary lattice using the MetIlo® program (Fig. 3).

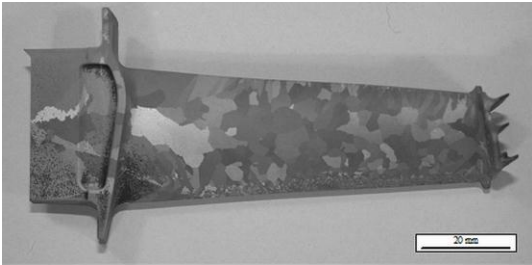


Fig. 1. Macrostructure of blade made of IN-100 – test 1

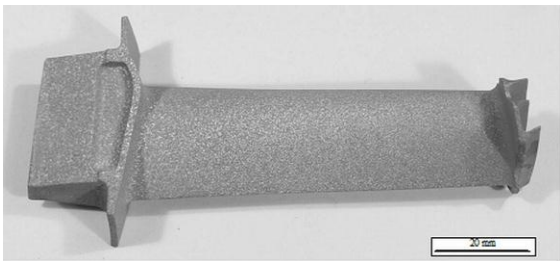


Fig. 2. Macrostructure of blade made of IN-100 – test 2

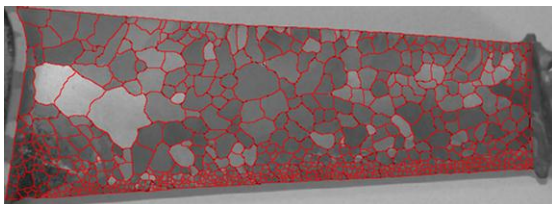


Fig. 3. Grain boundaries of blade made of IN-100 – test 1

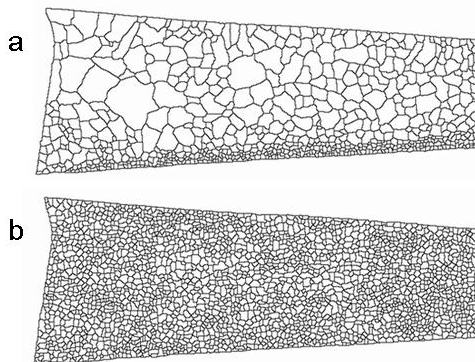


Fig. 4. Grain boundaries on the perpendicular projection in the input files: (a) IN-100 test 1, (b) IN-100 test 2

Grain boundaries in a perpendicular projection and their representation on the non-planar surfaces of blades are presented in Fig. 4 and Fig. 5.

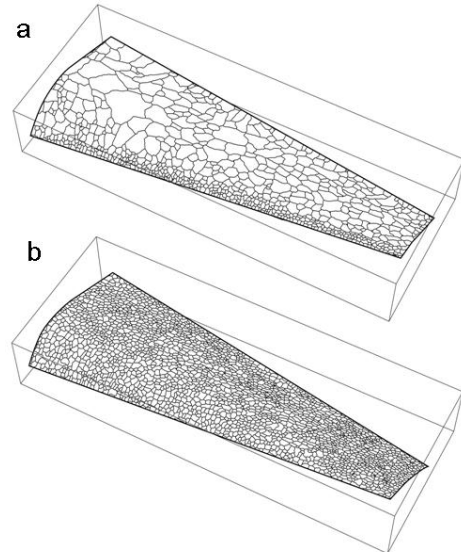


Fig. 5. Grain boundaries on the non-planar surfaces in the input files: (a) IN-100 test 1, (b) IN-100 test 2

Detail description of geometric measurements, methodology of imaging of grain boundaries on non-planar surfaces and numerical calculations of stereological parameters is presented in papers [12,13].

## 4. Results

The macrostructure of blade IN-100 test 1 is characterized by very significant differences in the grain size, which is reflected in the quantitative analysis. The distributions of the grain areas in the projection plane and of the grain with a non-planar surface have the nature of bimodal distributions (Fig. 6).

For the purpose of a quantitative description of the grain area size inhomogeneity, an expression was used in the form of a sum of weighted distributions (3):

$$F_C(X) = \sum_{i=1}^{n \geq 2} w_i F_i(X) \quad (3)$$

where:  $w_i$  – weight coefficient for each modal value  $i$  ( $\forall i, w_i < 1$ );  $F_i(X)$  – constituent weight function described with expression (2).

By substituting expression (3) for (2), we obtain an analytical description of the bimodal distribution (4):

$$F_C(X) = \frac{w_1 b_1 (y_{1\max} - y_{1\min}) \exp[b_1(X_1 - Q_1)]}{\{1 + \exp[b_1(X_1 - Q_1)]\}^2} + \frac{w_2 b_2 (y_{2\max} - y_{2\min}) \exp[b_2(X_2 - Q_2)]}{\{1 + \exp[b_2(X_2 - Q_2)]\}^2} \quad (4)$$

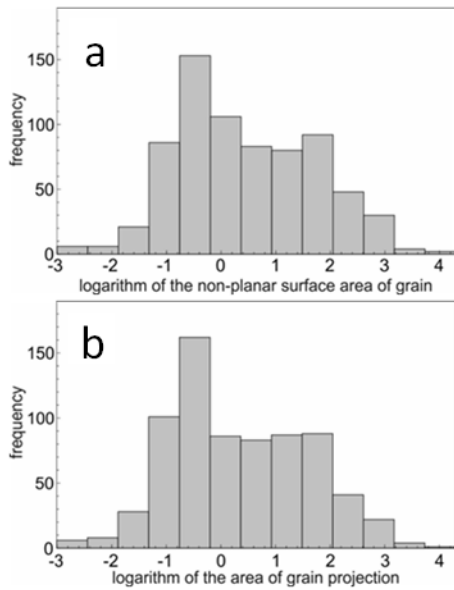


Fig. 6. Grain size distribution of macrostructure IN-100 test 1: (a) – on the non-planar surface 3D ; (b) – on the perpendicular projection

Parameters  $b_i$ ,  $Q_i$  and the ranges of variation ( $y_{1max} - y_{1min}$ ;  $y_{2max} - y_{2min}$ ) for weight functions were determined in accordance with the procedures described in papers [10,11]. The calculation results in the form of weight function graphs are presented in Fig. 7.

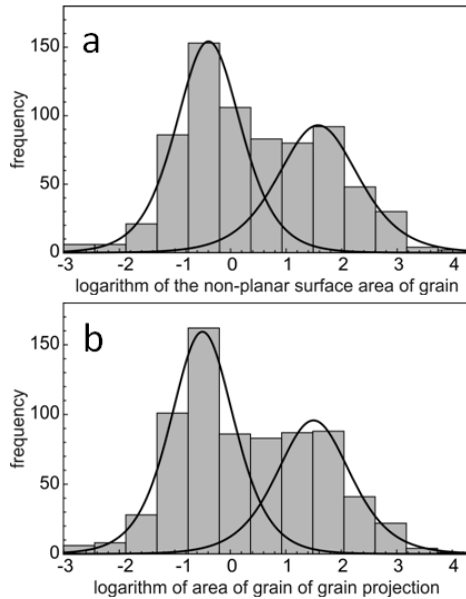


Fig. 7. Bimodal distribution weight functions: (a) – for grain size distribution on a non-planar surface; (b) – for grain size distribution in a perpendicular projection

Quantitative description of bimodal distributions based on the sum of weighted distributions makes it possible to express numerically, by means of parameters  $b_1$  and  $b_2$ , the

inhomogeneity of the grain size in subsets. Weight coefficients  $w_i$ , selected at an earlier stage of calculations, enable expressing numerically the inhomogeneity of the grain size of a complex macrostructure characterized by a bimodal distribution, based on a simple expression (5):

$$b_{3D/2D} = \sum_{i=1}^{n=2} w_i b_i \quad (5)$$

Therefore, the size inhomogeneity of grains with a non-planar surface of the whole set is depicted by the dependence (6)

$$b_{3D} = w_1(3D)b_1(3D) + w_2(3D)b_2(3D) = 2.36495 \quad (6)$$

The inhomogeneity of grain size of the whole set in the projection plane is described by dependence (7):

$$b_{2D} = w_1(2D)b_1(2D) + w_2(2D)b_2(2D) = 2.48107 \quad (7)$$

The comparison of parameters  $b_{2D} > b_{3D}$  shows a higher degree of homogeneity of the grain set in the projection plane. The weight functions that occur in expression (4) reach their maximum in point  $X_i = Q_i$ , which can be easily shown by differentiating these functions and finding the zero of the derivative. By calculating the second derivative in point  $X_i = Q_i$ , we obtain a value lower than zero, which proves the fulfilment of the condition necessary for the existence of a maximum in the investigated point. Therefore, the size of the grain area in points where the weight functions reach their maxima is determined in a simple way:  $P_i = \exp(Q_i)$ . Knowing the diversity of grain area distributions and the numerical description of their size inhomogeneity, we can determine the "representative" grain size for the bimodal distribution:

$$P(Q_{3D/2D}) = \exp(Q_{3D/2D}) \quad (8)$$

where:  $Q_{3D/2D} = w_i Q_i$ . After substituting the data, we obtain, for the grains with a non-planar surface:

$$Q_{3D} = w_1(3D)Q_1(3D) + w_2(3D)Q_2(3D) = 0.343539 \quad (9)$$

for the grains in the plane of projection:

$$Q_{2D} = w_1(2D)Q_1(2D) + w_2(2D)Q_2(2D) = 0.202938 \quad (10)$$

The macrostructure of the IN-100 test 2 blade is characterized by equiaxed grains with a high degree of homogeneity, which is corroborated by the single-modal nature of the distribution presented in Fig. 8. Due to the single-modal nature of the distribution, the quantitative definition of the grain size inhomogeneity was based on function (2). The calculation results and graphic image of this function are shown in Fig. 9. The comparison of parameters  $b_{2D} > b_{3D}$  shows a higher degree of homogeneity of the single-modal grain set in the projection plane.

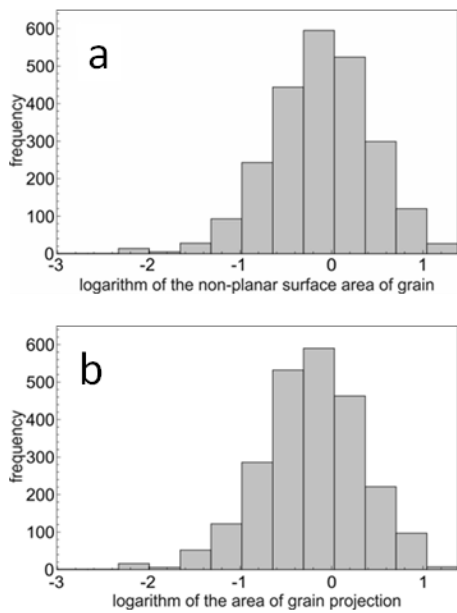


Fig. 8. Grain size distribution of macrostructure IN-100 test 2: (a) – on the non-planar surface 3D ; (b) – on the perpendicular projection

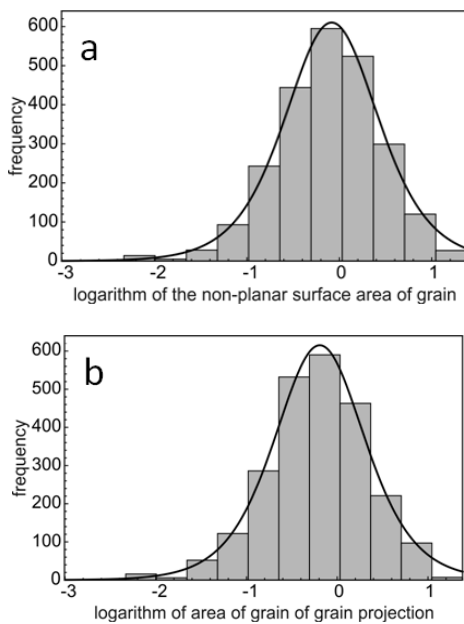


Fig. 9. Functions for approximation of grain size distribution: (a) – for the non-planar surface 3D; (b) – for grains on the perpendicular projection

## 5. Discussion of results

The quantitative description of the grain size inhomogeneity in polycrystalline materials is an important issue, both in the field

of quantitative metallography and in the analytical approach to the relations between the mechanical properties and the material microstructure. The importance of the issue of grain size inhomogeneity is emphasized in literature, and its complexity is corroborated by the few theoretical models which describe the role of the grain size distribution in modifying the response of the material to load.

A widely known parameter used for the evaluation of inhomogeneity in quantitative metallography is the variability index. This value, in the case of grain size inhomogeneity characterized by bimodal or multi-modal distributions, is substituted with a description based on distribution moments of higher orders, i.e. by the skewness factor and the dissipation factor [4]. Such a solution is troublesome and is not practicable for a functional description of the interrelations between the microstructure parameters and its properties.

The analytical description of inhomogeneity applied in the present paper, based on the derivative of the logarithmic-logistic function, precisely characterizes the grain size inhomogeneity. This refers to both the single-modal distributions (macrostructure IN-100 test 2), for which we precisely defined the inhomogeneity (parameters  $b_{2D}$  and  $b_{3D}$ ) of the grain areas on a perpendicular projection, as well as the areas of grain with a non-planar surface. The comparison of parameters  $b_{3D}$  and  $b_{2D}$  shows that the grain areas on a perpendicular projection are characterized by a higher level of homogeneity ( $b_{2D} > b_{3D}$ ) than the areas of grains with a non-planar surface whose stereological parameters were obtained using numerical calculations.

The analytical description of bimodal distributions (macrostructure IN-100 test 1) based on the sum of weight distributions (4) is fully justified. It identifies precisely both the degree of inhomogeneity of subsets characterized by weight function parameters and the inhomogeneity of the whole set. Dependencies (6) and (7) explicitly describe the degree of inhomogeneity of whole sets of grain areas on a perpendicular projection and of grain with a non-planar surface. The comparison of calculation results obtained from the above equations ( $b_{2D} > b_{3D}$ ) indicates clearly a higher degree of grain homogeneity on a perpendicular projection. The application of weight distributions for the quantitative description of complex structures characterized by a high degree of inhomogeneity (bimodal and multi-modal distributions) makes it possible to define easily also the “representative” size of the grain (10) for the whole set.

## 6. Conclusions

Analytical description of the grain size inhomogeneity based on the derivative of the logarithmic-logistic function ensures effectiveness of the solution, for both single-modal distributions and complex structures characterized by bimodal or multi-modal distributions. The function applied in the study can also be used for the purpose of scaling and analytical description of the mechanical properties and stereological parameters of the microstructure [10, 11].

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