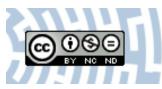


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Author: Aleksander Błaszczyk

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Ministerstwo Nauki i Szkolnictwa Wyższego

ALEKSANDER BŁASZCZYK*

A NOTE ON REMAINDERS OF COMPACT EXTENSIONS

Abstract. The paper contains a construction of a Tychonoff space X such that for every compact extension bX the subset bX-X contains a non-empty \mathscr{G}_{δ} -set G such that Int $G = \emptyset$.

In 1960 Fine and Gillman [4] proved that if X is a locally compact real compact space, then the remainder of the Čech-Stone compactification of X (abbreviated $\beta X - X$) has the following property: every non-empty \mathscr{G}_{δ} -set contains a nonempty open set. Hausdorff spaces satisfying this property are called *P'*-spaces, whereas *P*-spaces are the spaces in which all \mathscr{G}_{δ} -sets are open; see e.g. Gillman and Jerison [5], Plank [6] or Veksler [7]. Although every compact (Hausdorff) *P*-space is finite, there exist non-trivial compact *P'*-spaces. As an example of a non-trivial compact *P'*-space one can state $\beta N - N$, the remainder of the Čech-Stone compactification of the integers.

Recently Aniskovič [1] has shown that the result of Fine and Gillman can be improved by replacing the Čech-Stone compactification by a wide class of compactifications. He has also pointed out that by an additional set-theoretical assumption one can construct a Tychonoff space no compactification of which has the remainder being a P'-space. The aim of this note is to construct such spaces without any additional set-theoretical assumptions.

LEMMA 1. Every countable P'-space is discrete.

Proof. Indeed, in countable Hausdorff spaces every point is a \mathscr{G}_{δ} -set.

LEMMA 2. Every uncountable P'-space is non-separable.

Proof. Let D be a countable subset of an uncountable P'-space X. Choose a point x of X-D. There exists a \mathscr{G}_{δ} -set G such that $x \in G \subset X-D$. Since $\operatorname{Int} G \neq \emptyset$, D is not dense in X.

A topological space E is extremally disconnected (abbreviated e.d.) if the closure of every open subset of E is open. Clearly, dense subspaces of e.d. spaces are e.d. For every topological space X there exists so called *absolute* (or *Gleason space*) of X, that is an e.d. space G(X) which can be mapped onto X by a continuous

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^{*} Instytut Matematyki Uniwersytetu Śląskiego, Katowice, ul. Bankowa 14, Poland.

irreducible perfect mapping; see e.g. Comfort and Negrepontis [2] for the compact case.

THEOREM 1. There exists an e.d. locally compact space X such that for every compactification bX of the space X, the remainder bX - X is either finite or is not a P'-space.

Proof. Let *E* be an e.d. compact space without isolated points (e.g. the absolute of the Cantor set). Choose a countable discrete subset *N* of *E*. Clearly, cl*N* is a nowhere dense subset of *E*. Thus, *E* is a compactification of the e.d. locally compact space X = E - clN. By a theorem of Taimanov (see e.g. Engelking [3, p. 182]), every continuous mapping of X into a compact space has a continuous extension over *E*. Thus, *E* is equivalent to βX . Now, let bX be an arbitrary compactification of X. Then, there exists a continuous mapping *f* from *E* onto bX such that f(clN) == bX - X. Hence bX - X is a separable compact space. Assume that it is a *P'*-space. Then, by Lemma 2 and Lemma 1, it must be finite.

In particular, Theorem 1 says that in the Theorem of Fine and Gillman mentioned above the assumption of realcompactness cannot be removed.

A Tychonoff space is called *nowhere locally compact* whenever every compact subset of this space is nowhere dense.

THEOREM 2. There exists an e.d. nowhere locally compact space no compactification of which has the remainder being a P'-space.

Proof. Let E be the absolute of the Cantor set and let D be a countable dense subset of E. We set X = E - D. By the Taimanov's Theorem (see the proof of Theorem 1), E is a compactification of X equivalent to βX . Since D is countable, the remainder of any compactification of X is countable. Furthermore, X is nowhere locally compact because D is dense in E. Suppose bX is a compactification of X with bX - X being a P'-space. By Lemma 1, bX - X is discrete. Thus, X is locally compact in some points; we get a contradiction.

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