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**UNITY AND DISUNITY BETWEEN  
GOTTLOB FREGE AND GIUSEPPE PEANO  
ON THE BASIS OF THEIR CORRESPONDENCE  
IN THE YEARS 1891-1903**

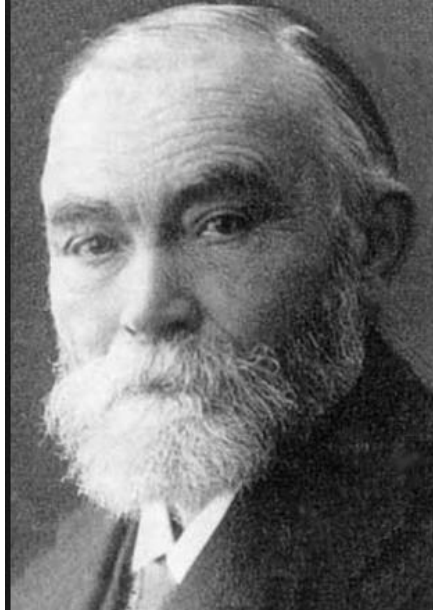
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Gabriela Besler, University of Silesia,  
8th International Conference of the European Society  
for the History of Science,  
London 2018 September 14-17

# Contents:

1. Preserved letters.
2. Introductory remarks.
3. Unity and disunity of aims of Frege's logic and Peano's mathematical logic.
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## Frege (1848-1925)



- University in Jena;
- Concept script [*Begriffsschrift*];

## Peano (1858–1932)



- University in Turin;
- Mathematical logic;
- Peano founded the journal „Rivista di Matematica” (1891).

# 1. Preserved letters

- Frege an Peano, without date, after 1891 (draft version found in Frege's legacy);
- Peano an Frege, 30. 1. 1894;
- Peano an Frege, 10. 2. 1894;
- Peano an Frege, 14. 8. 1895;
- Peano an Frege, 4. 10. 1895
- Peano an Frege, 5. 4. 1896
- Frege an Peano, 29. 9. 1896, published in „Rivista...”, 1899
- Peano an Frege, without date, published in „Rivista...”, 1899
- Peano an Frege, 3. 10. 1896
- Peano an Frege, 14. 10. 1896
- Frege an Peano, without date (draft version found in Frege's legacy);
- Peano an Frege, 7. 1. 1903.

## 2. Introductory remarks.

## „Golden period” of their academic activity

1. The peak of **Frege’s logicism**, a standpoint in which arithmetical notions are said to have been reduced to pure logical notions.

2. **Peano**, before starting the correspondence, published

*Arithmetices principia, nova methodo exposita* (1889)

and following papers during the correspondence:

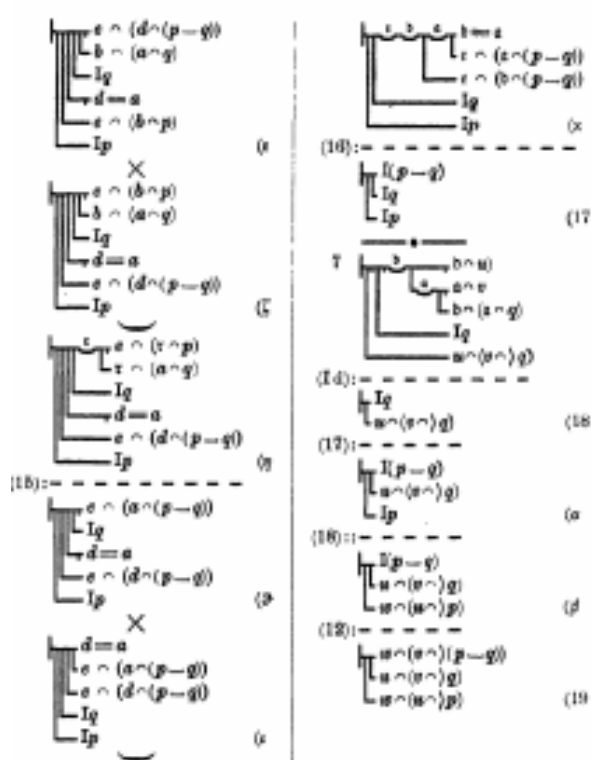
*Notations de logique mathématique* (1894).

*Formulaire mathématique* (1895).

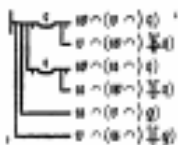


# Frege: concept script [*Begriffsschrift*]

# Peano: mathematical logic



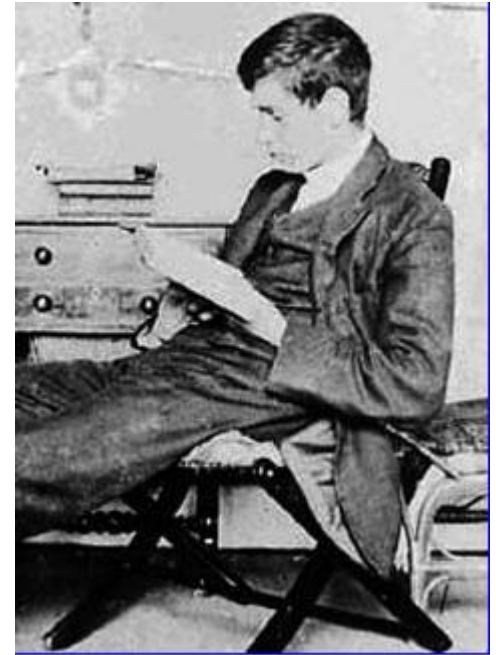
b) Beweis des Satzes



und Ende des Abschnittes A.

- \* 8.  $ab = ba$  {P7,  $\begin{smallmatrix} b, a \\ a, b \end{smallmatrix}$ } P7 . 0 . P8]
- 9.  $abc \supset acb$  [Pp.]
- \*10.  $abc = acb$  [P9 . 0 . P10]
- \*11.  $a \supset b . 0 . a \supset b e$  [Pp.]
- \*12.  $a . a \supset b . 0 . b$  [Pp.]
- \*13.  $a \supset b . b \supset c . 0 . a \supset c$  [Pp.]
- 14.  $abc \supset bac$  [P7,  $\begin{pmatrix} ab, ba \\ a, b \end{pmatrix}$ ] P11 . P12 . 0 . P14]
- 15.  $ab \supset b$  [P7,  $\begin{pmatrix} b, a \\ a, b \end{pmatrix}$ ] P5 . P13 . 0 . P15]
- 16.  $a = b . 0 . a \supset b$  [P3 . P5 . 0 . P14]
- 17.  $a = b . 0 . b \supset a$  [P3 . P15 . 0 . P17]
- 18.  $a \supset b . b \supset a . 0 . a = b$  [P3 . P1 . 0 . P18]
- 19.  $a = b . 0 . b = a$  [P3 . P7 . 0 . P19]
- \*20.  $a = b . = . b = a$  [P19,  $\begin{pmatrix} b, a \\ a, b \end{pmatrix}$ ] P19 . = . P20]
- 21.  $a = b . b \supset c . 0 . a \supset c$  [Pp, 0 . a \supset b . b \supset c . 0 . a \supset c]
- 22.  $a \supset b . b = c . 0 . a \supset c$  [Idem]
- \*23.  $a = b . b = c . 0 . a = c$  [Pp, 0 . a \supset b . b \supset c . 0 . a \supset c . 0 . a \supset c . 0 . a \supset c . 0 . T8.]
- ( $\alpha$ )  $a \supset b . b \supset c . 0 . a \supset c . 0 . a \supset c$  [P13 . P11 . 0 . ( $\alpha$ )]
- \*24.  $a \supset b . b \supset c . 0 . a \supset d$  [( $\alpha$ ), P13 . 0 . P24]
- 25.  $a = b . b = c . 0 . a = d$  [Pp, P23 . 0 . a = c . c = d . 0 . T8.]
- 26.  $b \supset a \supset ab$  [Pp.]
- ( $\alpha$ )  $c . 0 . a \supset ac$  [ $\begin{pmatrix} c \\ b \end{pmatrix}$ ] P26 . 0 . ( $\alpha$ )]
- ( $\beta$ )  $c . a \supset b \supset c . 0 . a \supset ac . a \supset bc$  [( $\alpha$ ), P11 . 0 . ( $\beta$ )]

# Frege to Philip Jourdain (1879-1919)



I believe I have perfected my conceptual notation somewhat in this work [ *Grundgesetze der Arithmetik*] (...) I regarded it in many respects as better than Peano's, even though it may appear less simple at the first glance (Frege to Jourdain, 23. 9. 1902, s. 73).

# Peano wrote:

„I have some difficulty in reading your symbols; but I shall get better at it, and if I still find difficulties, I shall take the liberty of writing to you

(Peano to Frege 30.01.1894).

# 3. Aims of Frege's concept script and Peano's mathematical logic.

# Unity of their aims

Their common aim was **to improve mathematics by logic** with the small number of primitive undefinable signs. As it was written by Peano they

„[...] are taking the same route in science (Peano an Frege, 30.01.1894, p. 178)

and they have much to gain from the parallel between their logical systems (Peano to Frege, 7.01.1903).

The scientific activity of both Frege and Peano resulted from the intention to implement Gottfried Wilhelm Leibniz's (1646-1716) postulate of creating **universal language**.



**"A Universal Symbolism, very popular, very agreeable to the people...might be introduced if small figures were employed in the place of words, which would represent visible things by their lines, and the invisible, by the visible, which accompany them."**

**— Leibniz, 1679**

## Disunity of their aims:

**Peano's** primary interest was in **axiomatics**, that he never used the mathematical logic developed by him for the reduction of mathematical concepts to logical concepts, and that, instead, he denied the validity of such a reduction. (Kennedy 2002, p. 11)

That Peano considered his work no more than axiomatization, and not an answer to the basic question—What is a number?, may be seen in “Sul concetto di numero.” (Kennedy 2002, p. 8)

- **Peano** created logic as a useful tool to **axiomatize** mathematics (arithmetic and geometry as well) and as a tool to examine the principles of arithmetic and geometry.
- **Frege** created his logical system to realize **logistic program**.
- **Peano** did not remark the differences between his aim of creating logical system and Frege's. He maintained that Frege was his predecessor in the area of mathematical logic

(G. Peano: *Recensione: G. Frege, Grundgesetze der Arithmetik, begriffsschriftlich abgeleitet*. „Rivista di matematica “ 1895, vol. 5, p. 122).



## 4. Topics raised in the correspondence

# List of topics:

- 4.1. The comparison of logical **symbolisms**.
- 4.2. The required number of **primitive, undefinable signs**.
- 4.3. Symbols of **quantification** and its understanding;
- 4.4 Translation of some of **Peano's formulas into Frege's logic**.
- 4.5 Two kinds of **propositions**: general and singular.
- 4.6 The conditions that should be met by a **correct definition**.
- 4.7 Critique of definitions introduced by Peano, in particular definitions of **addition** and **equality**.
- 4.8 Other topics: **class, identity**, what is **function**, **semiotic-philosophical notions**.

## 4.1. Comparison of logical symbolisms

They claimed that „[...] the parallel between the two systems of writing,

Mathematical Logic = Conceptual Notation  
will have much to gain”

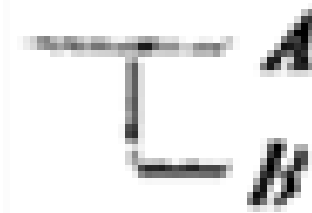
(Peano to Frege 7.01.1903).

# They explained how to read

- the Peano's sign of deduction "⊃"

The image shows the Peano's sign of deduction symbol, which is a stylized 'C' shape with a vertical line on the right side, resembling a '3' or a 'C' with a tail.

- and the Frege's conditional stroke

The image shows Frege's conditional stroke symbol, which consists of a horizontal line on the left, a vertical line on the right, and a horizontal line at the bottom, forming a U-shape. To the right of the vertical line, there are two small, stylized symbols that look like 'A' and 'B' stacked vertically.

We have given two names to the sign „ $\supset$ ”:

1. „we deduce” and
2. „is contained”,

and it can also be read in many other ways. This does not mean that the sign „ $\supset$ ” has several meanings. My is better expressed by saying that the sign „ $\supset$ ” has a single meaning, but that in ordinary language this meaning is represented by several different words, according to the circumstances

(Peano to Frege, 14.10.1896, s. 121).

4.2. The required number of primitive, undefinable signs.

- In *Formulaire* Peano wrote that all relations and operations between propositions and classes are reduced to six represented by symbols:

$\wedge, \vee, \neg, =, \emptyset, \Delta$

- The first three are primitive and undefinable, the next three are defined by the primitive terms.
- Peano maintained that there are five characters in Frege's system:

$1, \neg, \top, \perp, \cup$

# How to understand a primitive term

- Frege - all undefinable signs in a logical system.
- Peano - number of constants.
- Frege doubted that Peano used only three undefinable symbols. (Frege - Peano, 29. 9. 1896) and Peano finally agreed with him (8 Peano to Frege, without date).
- For example, Peano introduced names for particular sets of numbers like positive numbers and rational numbers while Frege defined them using primitive signs (7 Frege – Peano, 29.09.1896).



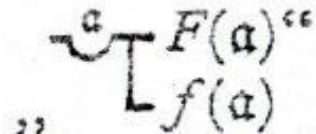
## 4.3. Symbols of quantification and its understanding.

# The comparison given by Frege

[...] your

$$"f(x) \supset_x F(x)"$$

would correspond to my


$$" \overset{a}{f} \vdash \overset{a}{F} "$$

(1 Frege – Peano, without date, p. 109).

# Peano: General quantifier

- Peano, *Aritmetices Principia. Nova methodo exposita* 1889, s. VIII

$a \Downarrow x, y, \dots b$

- It means whatever are  $x$  and  $y$ , from proposition  $a$  one deduces  $b$ .
- In *Notations de logique mathématique* (1894) Peano introduced another notation for quantification, downwards arrows bottom (someone is) and upwards arrow (everyone is).

$a \downarrow \varepsilon b$

$a \uparrow \varepsilon b$

# 5. The problem of antinomy

- In a letter to Philip Jourdain, Frege wrote that the problem of antinomy also referred to Peano's logic (Frege to Jourdain, 23.09.1902, p. 73).
- Among the above themes, there is no discussion of the problem of antinomy, although Russell informed Peano about the problem in a letter to him earlier than in the famous first letter to Frege from June 16, 1902 . At the end of this letter to Frege, Russell wrote in Peano's notation the antinomy of a class not belonging to itself, adding that he had already written about it to Peano, but he had not received any answer yet.
- The first of the nine published letters by Peano to Russell is dated March 19, 1901 and there is no reference to the problem of antinomy. Peano mentioned "the antinomy combined with the name of Russell" only in the letter of February 16, 1906.
- Therefore, the problem of antinomy was not taken up by Peano as quickly as one would expect, despite the fact that the antinomy is also constructible on the ground of his mathematical logic.

# 6. Conclusions

- It was disunity of assumed logical symbolism and in some logical solutions. They did not agree how much their systems count primitive terms. Frege criticized assumed by Peano definitions of equality and addition. They both did not agree what it means to create a good definition.
- There was much more unity than disunity between them. They did not struggle but well understand each other. However, there is a lot of constructive and serious criticism in their letters. It is why their correspondence is worth to study seriously.
- Frege wrote two papers on unity and disunity of his and Peanos logic:
  1. *Über die Begriffsschrift des Herrn Peano und meine eigene* (1897);
  2. *Begründung meiner strengeren Grundsätze des Definierens* (1897/1898).

Peano quoted Frege's papers five times. In comparison with his other quotations – it is very little.

Unity and disunity between their logics:

[...] mathematicians agree indeed on the external form of their propositions but **not on the thoughts** they attach to them, and these are surely what is essential (Frege to Peano, without date, p. 195).



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Thank you for your attention