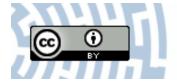


## You have downloaded a document from RE-BUŚ repository of the University of Silesia in Katowice

Title: A theorem supplementing a result of James A. Yorke

Author: Józef Kalinowski

**Citation style:** Kalinowski Józef. (1978). A theorem supplementing a result of James A. Yorke. "Demonstratio Mathematica" (Vol. 11, nr 4 (1978) s. 899-900).



Uznanie autorstwa - Licencja ta pozwala na kopiowanie, zmienianie, rozprowadzanie, przedstawianie i wykonywanie utworu jedynie pod warunkiem oznaczenia autorstwa.







## Józef Kalinowski

## A THEOREM SUPPLEMENTING A RESULT OF JAMES A. YORKE

In this paper a theorem will be proved showing that the assumptions of one of Yorke's theorems are never fulfilled in case n=1.

Consider the system of differential equations of the first order

$$x' = F(x).$$

where  $x(t) = (x^1(t), x^2(t), ..., x^n(x))$  is a sought vector-function for  $t \in \mathbb{R}$ ,  $\mathbb{R}$  - the set of real numbers and F(x) is a function fulfilling the Lipschitz condition

(2) 
$$\|F(x) - F(\bar{x})\| \le L\|x - \bar{x}\|$$
.

denotes Euclidean norm in Rn.

The author of notes [1] and [2] considered periodic solutions of the system (1), i.e. solutions satisfying the equation

$$x(t + p) = x(t)$$
 for  $t \in \mathbb{R}$ .

In the note [1] the following theorem has been proved.

Theorem 1. If x(t) is a periodic solution of the system (1) fulfilling condition (2), then  $p \le 2\pi/L$ .

Under weaker assumptions about F one may prove for n=1 the following

The orem 2. Let F be a continuous function. If the equation (1) possesses a solution  $x(t) \in C^1$ ,  $t \in R$  for n = 1, then it is not periodic.

Proof. Suppose that the solution x(t) is periodic and non-constant. The function x(t) assumes minimum at the point  $t_{min}$  and maximum at the point  $t_{max}$  ( $t_{min} < t_{max} < t_{min} + p$ ). Consider the intervals  $A = \begin{bmatrix} t_{min}, t_{max} \end{bmatrix}$ ,  $B = \begin{bmatrix} t_{mex}, t_{min} + p \end{bmatrix}$ . In A there exists  $\xi_1$  such that  $x'(\xi_1) > 0$  and in B there exists  $\xi_2$  such that  $x(\xi_1) = x(\xi_2)$  (Darboux's property). For one of these numbers there must be  $x'(\xi_2) \ge 0$ . Hence  $x'(\xi_1) > x'(\xi_2)$  and this together with  $F(x(\xi_1)) = F(x(\xi_2))$  leads to a contradiction with (1).

From Theorem 2 it follows that in case n = 1 the assumptions of Theorem 1 are never fulfilled.

Theorem 2 is no longer true for  $n \ge 2$  (see [1]).

## REFERENCES

- [1] James A. Yorke: The Lipschitz constant and the period of periodic solutions, Proc. Amer. Math. Soc. 22 (1969) 509-512.
- [2] James A. Yorke: The period of periodic solutions and charged particles in magnetic fields, Lecture Notes in Mathematics 144 (1970) 267-268.

INSTITUTE OF MATHEMATICS, SILESIAN UNIVERSITY, KATOWICE Received July 29, 1976.