Title: A theorem supplementing a result of James A. Yorke

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A THEOREM SUPPLEMENTING A RESULT OF JAMES A. YORKE

In this paper a theorem will be proved showing that the assumptions of one of Yorke's theorems are never fulfilled in case $n = 1$.

Consider the system of differential equations of the first order

$$x' = F(x),$$

where $x(t) = (x^1(t), x^2(t), ..., x^n(t))$ is a sought vector-function for $t \in \mathbb{R}$, $\mathbb{R}$ - the set of real numbers and $F(x)$ is a function fulfilling the Lipschitz condition

$$\|F(x) - F(\bar{x})\| \leq L\|x - \bar{x}\|.$$

$\| \cdot \|$ denotes Euclidean norm in $\mathbb{R}^n$.

The author of notes [1] and [2] considered periodic solutions of the system (1), i.e. solutions satisfying the equation

$$x(t + p) = x(t) \quad \text{for} \quad t \in \mathbb{R}.$$

In the note [1] the following theorem has been proved.

Theorem 1. If $x(t)$ is a periodic solution of the system (1) fulfilling condition (2), then $p \leq 2\pi/L$.

Under weaker assumptions about $F$ one may prove for $n = 1$ the following
Theorem 2. Let $F$ be a continuous function. If the equation (1) possesses a solution $x(t) \in C^1$, $t \in \mathbb{R}$ for $n = 1$, then it is not periodic.

Proof. Suppose that the solution $x(t)$ is periodic and non-constant. The function $x(t)$ assumes minimum at the point $t_{\min}$ and maximum at the point $t_{\max}$ ($t_{\min} < t_{\max} < t_{\min} + p$). Consider the intervals $A = [t_{\min}, t_{\max}]$, $B = [t_{\max}, t_{\min} + p]$. In $A$ there exists $\xi_1$ such that $x'(\xi_1) > 0$ and in $B$ there exists $\xi_2$ such that $x(\xi_1) = x(\xi_2)$ (Darboux's property). For one of these numbers there must be $x'(\xi_2) \geqslant 0$. Hence $x'(\xi_1) > x'(\xi_2)$ and this together with $F(x(\xi_1)) = F(x(\xi_2))$ leads to a contradiction with (1).

From Theorem 2 it follows that in case $n = 1$ the assumptions of Theorem 1 are never fulfilled.

Theorem 2 is no longer true for $n \geqslant 2$ (see [1]).

REFERENCES


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