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Studying Political Violence Using Game Theory Models: Research Approaches and Assumptions

Abstract: The purpose of the paper is to concisely present basic applications of game theory models for a scientific description of political violence. The paper is divided into four parts. The first part discusses the key theoretical issues including: the assumption of the players' rationality, the assumption of the players' common knowledge of their rationality, the Nash equilibrium concept, Pareto optimality, the Nash arbitration scheme and the concept of evolutionarily stable strategies. The second and third parts contain examples of uses of selected models of classical and evolutionary games in the studies on political violence. The following two interaction schemes were used to that end: the Prisoner's Dilemma and Chicken. The paper ends with a summary and discussion. The key feature of the discussed models is their methodological simplicity, as demonstrated by the lack of need to use complicated mathematical methods. This is why the paper is mainly addressed to individuals who had not studied game theory before or who have insufficient knowledge in the field to conduct own studies.

Keywords: *political violence, game theory, classical models, evolutionary models, Prisoner's Dilemma game, Chicken game*

Violence is an integral part of human history. Due to its ubiquity, violence is a social problem of interest to many scientific disciplines, such as medicine (Gin et al., 1991; Denninghoff et al., 2002; Farmer et al., 2006), geography (Le Billon, 2001; Toft, 2003; Springer & Le Billon, 2016), anthropology (Fry, 2006; Schmidt & Schröder, 2001), psychology (Walker, 1997; Bonta, Law, & Hanson, 1998; Michaels, 2017), law (Zimring, 1998; Schabas, 2000), history

(Gordon, 1988; Raaflaub, 2007), sociology (Sampson, 1987; Malešević, 2010; Walby, 2013) and political science (Nieburg, 1969; Apter, 1997; Kalyvas, 2006; Mider, 2013; Rak, 2017) to name but a few. The latter considers a particular type of violence, namely political violence, which is the subject of this article.

The following definition of political violence has been adopted for the purposes of this article: 'Political violence is the deliberate use of power and force to achieve political goals' (Sousa, 2013, p. 169). Political violence can manifest itself both in the use of physical force, e.g. murder, beatings, destruction of property, as well as verbal and non-verbal means of interpersonal communication, e.g. defamation, intimidation or blackmail of political opponents (Euchner, 1996, p. 168). Political conflicts leading to violence usually stem from three types of contradictions: (1) structural contradictions – resulting from disproportions in the access of individuals and social groups to goods; (2) axiological contradictions – arising from discrepancies in the value systems of key individuals and social groups; (3) political contradictions – resulting from unequal access of individuals and groups to state decision-making authorities (Seidler, Groszyk, & Pieniążek, 2010, pp. 50–51).

Different theoretical and methodological approaches can be applied in surveys of political violence. The constitutive feature of some of them is methodological individualism (e.g. rational choice theory), while some refer to methodological holism (e.g. functionalism). Methodological individualism was adopted for the analyses presented herein. The article presents selected possibilities of the application of simple game theory models in studies on political violence. The purpose of the article is not to explain political violence but to focus on key technical issues related to the description of violent interactions in politics using game theory models. Precision and intersubjective communicability, as well as the intersubjective verifiability of results expressed in language mathematics, are features that make the theory of games a tool often applied in the analyses of politically motivated violence. Examples include analyses of such phenomena as tactical aerial warfare (Berkovitz & Dresher, 1959), strategy and arms control (Schelling, 1966), detection of submarines (Danskin, 1968), bargaining during military conflicts (Powell, 2002), and terrorism and counterterrorism (Lapan & Sandler, 1993; Sandler & Arce, 2003; Bueno de Mesquita, 2005).

The article consists of four parts. In the first part, we discuss key theoretical issues, distinguishing between classical game theory and evolutionary game theory. In parts two and three, we describe and interpret the results of the analyses. We have examined selected models of classical and evolutionary games in detail. The final part contains a summary and discussion.

1. Theoretical Background

1.1. Classical Game Theory

It usually is agreed that classical game theory was developed in 1944, when John von Neumann and Oskar Morgenstern published their monograph *Theory of Games and Economic Behavior*. Classical game theory is used to develop and study mathematical models of conflict situations and cooperation between intelligent and rational players (Myerson, 1997, p. 1). Basic assumptions of classical game theory include the assumption of the rationality of players and the assumption of the common knowledge of players regarding their rationality. According to the first assumption, players should strive to maximize their utility functions. According to the second, each player is required to know that the others are rational and that they know that s/he is aware of this fact, and so on *ad infinitum* (Riechmann, 2014, pp. 31–32).

The main purpose of game theory analyses is to find the optimal strategies for players, i.e. to find the strategies which constitute the best responses to other strategies. In other words, to find a situation whereby no player can obtain further benefits by changing their strategy, provided that the choices of the rest of the players remain unchanged. A pair of such strategies is called the Nash equilibrium (Nash, 1950, 1951). The Nash equilibrium is not always Pareto optimal because it is possible that the game will have different results, which will guarantee higher payoffs for both or only one of the players (Haman, 2014, pp. 28–49). The Pareto optimal equilibrium can clearly favor one player. How can we, then, proceed in order to end the game with a mutually acceptable outcome? One possibility is to allow arbitration.

The arbitration solution requires the unanimity of the players, as any decision adopted without their full consent can be challenged in the future. In addition, this solution should benefit the participants in the interaction more than any other solution adopted unilaterally. If arbitration was not to fulfil these expectations, the game would end with a less favorable, predetermined *status quo* (SQ) outcome. John F. Nash (1950) formulated four axioms, which must be fulfilled by any acceptable arbitration scheme:

- Axiom 1: Rationality. The solution should be part of the negotiation set (a set of Pareto optimal outcomes, not lower than SQ for any of the players).
- Axiom 2: Linear invariance. The solution should not depend on the linear transformation of the utility functions of players.
- Axiom 3: Symmetry. A symmetric negotiation problem should determine a result without differentiation between the entities entering into an agreement; this axiom reflects the equal negotiating potential and negotiating skills of both players.

Axiom 4: Independence of irrelevant alternatives. Assuming that within the P payoff polygon containing the points of SQ and NBS (arbitration outcome), there is a Q polygon, which also contains the SQ and NBS points – if SQ is the *status quo* for Q, then NBS should also be a solution for Q.

According to the claim proven by Nash, only one arbitration scheme fulfils the above axioms. If the payoff polygon contains the SQ point with coordinates (x_0, y_0) , the arbitration solution shall be the NBS point belonging to that polygon, with coordinates (x, y) , where $(x, y) \geq (x_0, y_0)$, which maximises the value of $(x - x_0)(y - y_0)$. In specialist literature, the discussed procedure of reaching a mutually satisfactory solution is called the Nash arbitration scheme (Straffin, 1993, pp. 103–106).

1.2. Evolutionary Game Theory

The formulation of evolutionary game theory is connected with the publication of *The Logic of Animal Conflict* (1973) by John Maynard Smith and George R. Price. The subject of evolutionary game theory research is the interactions between individual organisms within specified biological populations. The representatives of one population usually compete for the same resources (territory, food, females, etc.) using various strategies (genetically determined patterns of behavior, such as aggression or altruism). The measure of success of a given strategy is the frequency of its occurrence in the population as a result of the increased rate of reproduction of the players using this particular strategy. In other words, players displaying behaviors related to the strategy, which gains an advantage in subsequent generations, receive on average higher payoffs, interpreted in terms of adaptability, i.e. the ability to survive and raise offspring (Vincent & Brown, 2005, pp. 72–75).

None of the players in the Nash equilibrium is motivated to deviate from the adopted strategy and, as a result, this choice is fixed over time. A similar concept in evolutionary game theory is evolutionarily stable strategy (ESS) (Maynard Smith & Price, 1973; Maynard Smith, 1982). A strategy is evolutionarily stable when it is ‘used’ by the whole population, and each small group of invaders¹ ‘using’ a different strategy face a high probability of losing the competitive struggle for survival, eventually disappearing from the population for many generations (Schechter & Gintis, 2016, pp. 207–208; Easley & Kleinberg, 2010, pp. 191–193). As Philip D. Straffin (1993, p. 95) noted: ‘We would expect evolution to produce evolutionarily stable strategies. The argument is that a strategy which is not an ESS would have been invaded by another strategy. In the long run, only ESS’s should survive’.

In evolutionary games that do not go beyond biological applications, the role of the individual rationality of players is fulfilled by natural selection. Conversely, in games concerning the evolution of social norms (Bendor & Swistak, 2001), the adaptability of the

¹ These are the individuals coming from the outside (migrants) or representatives of the population born with new phenotypic traits (mutants).

analyzed behaviors is determined by social selection. As a result, the biological approach differs from the social one in the evolutionary process itself (Haman, 2014, p. 104). The former assumes that the disappearance of unfavorable behaviors takes place by the extinction of the carriers of inferior genes. Instead, the latter assumes that imitation is an essential mechanism responsible for shaping individual behavior². Objects of imitation (cultural or social replicators, memes³) are subject to vertical and horizontal transmission. The vertical direction can be equated with intergenerational transmission, while the horizontal direction with the pathways and dynamics of the spreading of infectious diseases (Cavalli-Sforza, 2001, pp. 180–182).

In Sections 2 and 3, we provide selected examples of the application of game theory models in studies on political violence. We used two types of 2×2 symmetric games in the analyses: the Prisoner’s Dilemma (PD) and Chicken (evolutionary biologists use the name Hawk–Dove game). When presenting the results of the analyses we preserved the division between classical and evolutionary games.

2. Classical Games Analyses

2.1. Preliminary Questions

According to the basic assumptions of the general model of symmetric 2×2 games,⁴ Bob (row player) and Alice (column player) choose one of two strategies independently of one another: C – cooperation or D – defection. The payoffs they can receive as a result of choosing C or D are shown in the game matrix below.

Table 1. General model of symmetrical 2×2 games

Bob/Alice	C	D
C	R, R	S, T
D	T, S	P, P

(x, y) = Bob’s payoff, Alice’s payoff

Source: own study.

² The ability to reproduce behavioral habits plays several important functions in the life of an individual: (1) it provides behavioral patterns consistent with the ethics of the reference group; (2) it improves significantly social acceptance, while satisfying the community instinct of an individual; (3) it facilitates self-identification; (4) it enables quick decision-making, especially vis-à-vis the shortage of time and information.

³ We refer here to the terminology used by Richard Dawkins (2006, pp. 189–201).

⁴ Symmetry depends on maintaining the proportion between individual payoffs.

Let us assume that: R stands for the reward for mutual cooperation; P means punishment for mutual defection; T is the temptation payoff to the player who defected while the opponent cooperated; S is the sucker’s payoff to the player who cooperated while the opponent defected.

In the PD game, the payoff values meet two equations: $T > R > P > S$ and $R > \frac{S+T}{2}$. The first equation makes it more profitable for players to refuse to cooperate, since $T > R$ and $P > S$. At the same time, there is a dilemma related to $R > P$, which makes the cooperation of both players more desirable than mutual defection. The second equation becomes more significant in repeated games. It ensures that simultaneous cooperation is more profitable than the expected value of payoff in situations in which there are equal opportunities for S or T. In a PD game, the D strategy is strictly dominant for both players, leading to equilibrium (D, D). It is not Pareto-optimal because there is a combination of strategies (C, C) that gives both players a more profitable result. Therefore, the payoff structure of PD game creates a conflict between individual rationality (domination criterion) and social rationality (Pareto optimality criterion) (Rapoport & Chammah, 1965).

In turn, the formal prerequisite for the occurrence of the interaction scheme appropriate for the Chicken game is meeting the following equation: $T > R > S > P$. It determines two Pareto-optimal Nash equilibria in pure strategies (C, D) and (D, C) and one Pareto-suboptimal equilibrium in mixed strategies. Importantly, no player has a dominant strategy in Chicken game, as is the case in PD game. This makes the situation of players much more complicated, as they do not have a simple indication of which equilibrium they should aim for. In this case, anticipation of the opponent’s decision will play a key role in strategy selection (Rapoport & Chammah, 1966).

2.2. Models

We can illustrate the interaction patterns of the PD and Chicken games with an example. Let us assume that the game participants are two countries (identified as A and B) and the same terrorist organization carries out attacks on the territories of both of these countries. The latter are faced with the following decision: cooperation – strategy C – or refusal to cooperate – strategy D – regarding increasing financial resources for fighting the common enemy.

Table 2. Prisoner’s Dilemma

A\B	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

(x, y) = country A payoff, country B payoff

Source: own study.

Table 3. Chicken

A\B	C		D
C	1, 1	↓ → ↑	0, 2
D	2, 0	← ↑	-1, -1

(x, y) = country A payoff, country B payoff

Source: own study.

The analyzed games allow the following combinations of strategies:

- (C, C) – both countries increase expenditure on counter-terrorism.
- (C, D) – expenditure is only increased by country A.
- (D, C) – expenditure is only increased by country B.
- (D, D) – none of the countries adopts a higher budget for counter-terrorism.

2.3. Results

In the PD game, the Nash equilibrium comes from a pair of strategies (D, D) leading to a result (0, 0). Guided by the criterion of dominance (see the movement diagram in table 2), the players do not decide to increase their financial expenditure to fight the hostile terrorist organization. The reason for this choice is the fear of betrayal (defection) by a potential ally.

The Chicken game, conversely, has two Nash equilibria in pure strategies: (C, D) leading to the result (0, 2), and (D, C) leading to the result (2, 0) (see the movement diagram in table 3). These can be interpreted as follows: it is better to play C at the risk that the other side will take advantage of us than to go on a limb and play D. When selecting the second solution, we can gamble on getting the full reward, but every mistake in the assessment of the opponent’s intent will end the game with the worst possible outcome, i.e. (D, D) which means loss for both players.

In addition to equilibria in pure strategies, the Chicken game also has one equilibrium in mixed strategies. Since payoffs are symmetric, it is enough to calculate the mixed strategy for one of the players. Let us suppose, therefore, that country A chooses strategy C with probability p , and strategy D with probability $(1 - p)$. Assuming $U_B(C) \sim U_B(D)$ (indifference of country B with regard to payoffs related to choosing either C or D), we look for the p which fulfils the following:

$$1p + 0(1 - p) = 2p - 1(1 - p) \quad (1)$$

which results in $p = \frac{1}{2}$. Mixed equilibrium, therefore, requires each player to use both C and D with a frequency of $\frac{1}{2}$. In this case, the expected value of each player’s payoff will be $\frac{1}{2}$ of the unit of utility.

So far, we have assumed that players cannot communicate with each other. Let us now allow for the procedure, which makes it possible to establish a mutually acceptable result. This procedure is the Nash arbitration scheme. The values of payoffs from tables 2 and 3 represented on the Cartesian plane constitute two polygons (figure 1a and figure 1b). Both are symmetrical to line $y = x$ passing through point $SQ = (0, 0)$.⁵ This gives rise to the conclusion that the Nash solution will be positioned on this line (Axiom 3). Furthermore, this solution must belong to the negotiation set (Axiom 1) and have the highest value of $(x - 0)(y - 0)$.

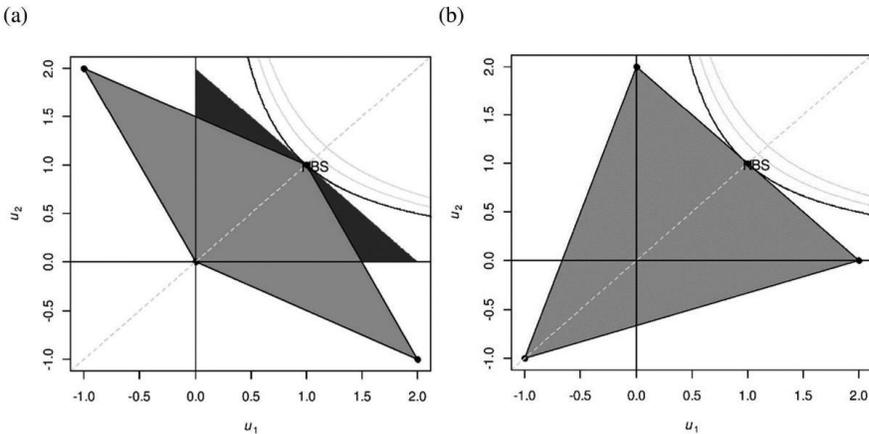


Fig. 1 Nash arbitration solution for games: Prisoner’s Dilemma (a) and Chicken (b)

The utility of country A is presented on the x-axis and the utility of country B on the y-axis.

Source: own study using R program.

In the case of PD game, the negotiation set is made up of the sections connecting points $(0, 1\frac{1}{2})$ with $(1, 1)$ and $(1, 1)$ with $(1\frac{1}{2}, 0)$. The first section is described by the equation:

$$y = -\frac{1}{2}x + 1\frac{1}{2} \tag{2}$$

Therefore, the expression to be maximized is:

$$(x - 0) (-\frac{1}{2}x + 1\frac{1}{2} - 0) = -\frac{1}{2}x^2 + 1\frac{1}{2}x \tag{3}$$

The product reaches its maximum value when $x = 1\frac{1}{2}$ and $y = \frac{3}{4}$. The second section is described by the equation:

$$y = -2x + 3 \tag{4}$$

⁵ SQ is created by security levels for players.

The expression to be maximized takes the following form:

$$(x - 0)(-2x + 3 - 0) = -2x^2 + 3x. \tag{5}$$

The product reaches its maximum value when $x = \frac{3}{4}$ and $y = 1\frac{1}{4}$. Since for both sections the results are outside the negotiation set, the arbitration solution is the closest point of the negotiation set, i.e. (C, C) with coordinates (1, 1).

In Chicken game, the situation is much less complex. The equation of the negotiation set (the section connecting points (0, 2) and (2, 0) takes the form:

$$y = -x + 2. \tag{6}$$

In view of the expression which should be maximized:

$$(x - 0)(-x + 2 - 0) = -x^2 + 2x. \tag{7}$$

As in the previous example, the product reaches its maximum when $x = 1$ and $y = 1$.

The fairness of the Nash arbitration solution in the games under review is closely related to the symmetry of payoffs. The result of the arbitration on the coordinate system is equally distant to the best result for country A and the best result for country B.

3. Evolutionary Games Analyses

3.1. Preliminary Questions

In evolutionary games, the R , T , S and P payoffs can be considered in terms of benefits (b) and costs (c) resulting from the selection of a specific strategy. In the evolutionary Prisoner's Dilemma:

- $R = b - c$. The payoff for cooperation is the difference between benefits and costs.
- $T = b$. The temptation payoff involves only the benefits.
- $S = -c$. The sucker's payoff involves only the costs.
- $P = 0$. The payoff for mutual defection maintains the *status quo*.

In addition, the following equation must also be met: $b > c > 0$. Conversely, for the Hawk-Dove game (the evolutionary version of the Chicken game):

- $R = \frac{b}{2}$. The payoff for cooperation equals half of the benefits.
- $T = b$. The temptation payoff involves only the benefits.
- $S = 0$. The sucker's payoff maintains the *status quo*.
- $P = \frac{b-c}{2}$. The payoff for mutual defection is determined by the difference between half of the benefits and half of the costs.

The game should meet the equation: $c > b > 0$.

3.2. Models

Let us assume that the subject of analysis will be the interactions taking place within the population⁶ formed by representatives of the political elite of country X. Let us also assume that this consists of 10,000 individuals. The population is non-spatial, so at any time a player is equally likely to come into conflict over values, access to goods or access to power with any other representative of the population. The players can choose between two strategies: C – cooperation (conciliatory attitude) – or D – defection (application of solutions escalating the conflict, e.g. physical violence, blackmail, intimidation or acts meant to discredit the opponent in the eyes of the public).⁷ Next, let us assume that changes in the state (proportion) of the population will be observed over the duration of 50 generations (stages of the evolutionary process). Measurements will be carried out for population variants with the initial involvement of co-operating players at the levels of: 5%, 50% and 95%. In the case of PD, parameters b and c were 2 and 1, and for Hawk-Dove – 2 and 4 respectively. These models assume asynchronous population updates.

3.3. Results

In the PD game, the Nash equilibrium comes from a pair of strategies (D, D). Since D is a strictly dominant strategy ($(b > b - c)$ and $(0 > -c)$), it is also an evolutionarily stable strategy (Schechter & Gintis, 2016, pp. 209–210). This means that: (1) the population composed of aggressors will not be destabilized by the appearance of a small group of cooperating mutants; and (2) the appearance of even the smallest group of aggressors in the population of co-operators will ultimately marginalize the latter. These population states reflect the simulation results presented in figure 2(a).

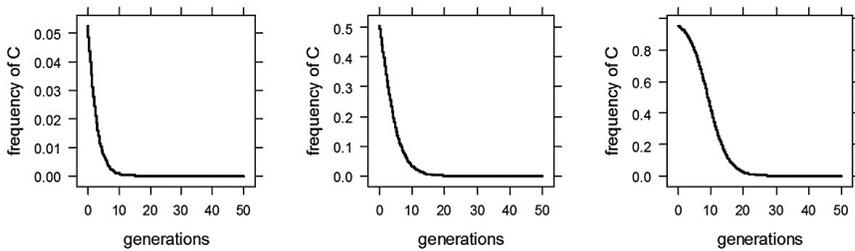
In case of an initial frequency of strategy C at the level of 5%, strategy D dominated the population after just a few generations. Where the co-operators initially constituted half of the population, the aggressors gained dominance after over a dozen generations. In the last case, i.e. when strategy C was initially chosen by 95% of the players, the aggressors dominated the population after over twenty generations. In the analyzed variants, the behavior of the aggressor may be perceived by the players as falling within the social norm.

In the Hawk-Dove game, strategy C is the best response to strategy D and vice versa – strategy D provides the best response to strategy C. The structure of game payoffs, therefore, generates two asymmetrical Nash equilibria in pure strategies (C, D) and (D, C). In addition,

⁶ The use of biological terms such as ‘population’, ‘generation’ or ‘mutation’ in sociological or political analyses conducted using tools of evolutionary game theory is entirely based on convention.

⁷ What is important, the set of alternative strategies must provide a separate and comprehensive population classification at all times. In addition, the sum of changes in strategy frequencies cannot be different from zero (Poleszczuk, 2004, pp. 23–24).

(a) $b = 2, c = 1$



(b) $b = 2, c = 4$

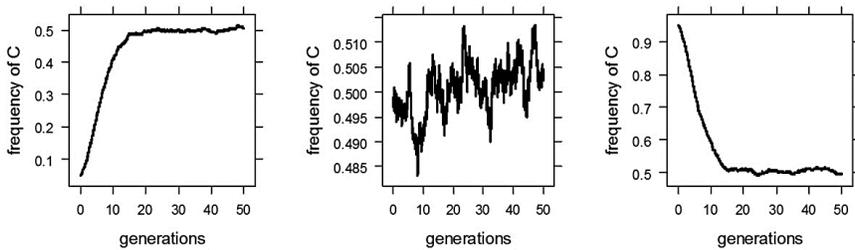


Fig. 2. Evolution of cooperation in games: Prisoner's Dilemma (a) and Hawk-Dove (b). Simulations for populations with an initial frequency of cooperative strategy at levels: 5%, 50% and 95%. Population size = 10,000; number of generations = 50.

Source: own study using R program.

the game also has one symmetrical equilibrium in mixed strategies, which corresponds to mixed ESS. Therefore, if both players choose C and D with the probability provided by mixed equilibrium, the composition of the population will stabilize temporarily⁸ (Schechter & Gintis, 2016, pp. 209–210).

In the first variant, 5% of the initial population composition is made up of co-operators and 95% of defectors. This means that we will encounter a rival pursuing strategy C with a probability of 0,05, and a rival choosing strategy D with a probability of 0,95. The expected payoff resulting from the use of a cooperation strategy will equal $0,05 \cdot 1 + 0,95 \cdot 0 = 0,05$, while players choosing a non-cooperative strategy may expect a payoff of $0,05 \cdot 2 + 0,95 \cdot (-1) = -0,85$. In such a population, it is better to be a co-operator than a defector exposed to the destructive effects of a fight with other defectors. As a consequence, the share of the population of C-playing individuals will increase until it reaches an evolutionarily stable state (balanced average values of payoffs) (see figure 2b). After reversing the initial proportions, i.e. in a population composed of 95% co-operators and only 5% defectors, the former will

⁸ Until a new mutation (behavior) occurs.

be able to expect a payoff of $0,95 \cdot 1 + 0,05 \cdot 0 = 0,95$, and the average payoff of the latter will be $0,95 \cdot 2 + 0,05 \cdot (-1) = 1,85$. In this variant, it is the individuals playing strategy D who will be in a more favourable situation, which will result in their faster reproduction rate (quicker replication of the strategy). The population will reach an evolutionarily stable state when half of the population is composed of conciliation players and half of aggressors. In this configuration, the average payoffs resulting from the use of strategies C and D will be equated with $0,5 \cdot 1 + 0,5 \cdot 0 = 0,5$ and $0,5 \cdot 2 + 0,5 \cdot (-1) = 0,5$, respectively. The interaction scheme of the Hawk-Dove game shows that none of the analysed behaviours will gain a longer lasting advantage in the population. Evolutionary pressure will lead to a situation in which they coexist, while even minor deviations from the evolutionarily stable state will be corrected.

4. Summary and Discussion

It would be difficult to imagine researching highly complex phenomena without the continuous improvement of the methods providing the most accurate results possible and whose implementation is carried out under conditions of limited resources, i.e. time, financial resources, technology. The group of research activities, which constitutes a compromise between cognitive requirements and the available material base, is, undoubtedly, modelling.

Models provide simplified representations of observed phenomena. The researcher using models usually starts with the basic variant, i.e. the construction in which irrelevant features and details are omitted. Then, as the study progresses, the reduced model becomes more concrete. A concrete option is temporarily accepted if it explains the observed facts with a better approximation.

Modelling is applied both to material structures, e.g. astronomical objects, as well as non-material structures, e.g. concepts of theoretical mathematics or natural languages. Each model is a thought structure in the initial stages of design. In the final form, however, depending on the research goals and adopted assumptions, it may be expressed by objects characterized by specific physical parameters – material models – or with the use of symbols and concepts – abstract models.

Mathematical models are the most representative group of abstract models. They enable us to penetrate deep into the studied processes, bringing out the features and relationships that have so far escaped researchers. The application of mathematical models may be dictated both by cognitive and aesthetic considerations. Justyna Brzezińska (2015, p. 154) noted: ‘The construction of models describing complex and multidimensional phenomena is characterized by logic, transparency and mathematical elegance of the record. Many researchers favor simplicity because models are easier to understand, present and verify empirically on the basis of data.’

One of the classes of mathematical models is game theory models. They enable the analysis of strategic situations, i.e. situations of interdependence between two or more players in which the effects of players' strategy choices are co-determined by strategies selected by other players (Załuski, 2016, p. 277). In a much broader context, the game theory models help to maintain the internal consistency of the theory. 'Logically inconsistent theories are clearly problematic. Since *any* conclusion can be derived from them, inconsistent theories can explain *any* empirical observation. Inconsistent theories, therefore, are non-falsifiable and of little practical use. When used properly, formal structures, like game theory, can help in the identification of flawed theory' (Zagare, 2008, p. 55).

Game theory is successfully applied in studies regarding a number of phenomena within the scope of interest of social sciences. These include: processes of forming political coalitions, processes of political transformation, electoral behaviors, legislative behaviors and political violence. We made the last of these phenomena the subject of the research discussed in this article. We carried out sample analyses of single-shot classical and evolutionary models. The former involve singular interactions between rational units, while the latter involve populations of multiple individuals (not necessarily rational) playing the same game. Classical models assumed interactions between two countries fighting against a hostile terrorist organization. In the evolutionary models the simulations covered interactions between representatives of the political elite of a hypothetical X state. Prisoner's Dilemma and Chicken (Hawk-Dove) interaction schemes were used in the studies. The simplicity of the models selected for analysis, and thus the absence of the need to use complex mathematical formalisms, make this article an accessible source of knowledge on the methodological foundations for constructing game theory models of violent interactions in politics.

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References:

- Apter, D.E. (1997). "Political Violence in Analytical Perspective". In D.E. Apter (Ed.), *The Legitimization of Violence* (1–32). New York: New York University Press.
- Bendor, J., Swistak, P. (2001). "The Evolution of Norms". *American Journal of Sociology*, 106(6), 1493–1545.
- Berkovitz, L.D., & Dresher, M. (1959). "A Game-Theory Analysis of Tactical Air War". *Operations Research*, 7, 599–620. DOI: <https://doi.org/10.1287/opre.7.5.599>.
- Bonta, J., Law, M., & Hanson, K. (1998). "The Prediction of Criminal and Violent Recidivism among Mentally Disordered Offenders: A Meta-Analysis". *Psychological Bulletin*, 132(2), 123–142.
- Brzezińska, J. (2015). *Analiza logarytmiczno-liniowa. Teoria i zastosowania z wykorzystaniem programu R*. Warsaw: Wydawnictwo C.H. Beck.

- Bueno de Mesquita, E. (2005). "Conciliation, Counterterrorism, and Patterns of Terrorist Violence". *International Organization*, 59(1), 145–176. DOI: <https://doi.org/10.1017/S0020818305050022>.
- Cavalli-Sforza, L.L. (2001). *Genes, Peoples and Languages*. London: Penguin Books.
- Danskin, J.M. (1968). "A Helicopter Versus Submarine Search Game". *Operations Research*, 16(3), 509–517. DOI: <https://doi.org/10.1287/opre.16.3.509>.
- Dawkins, R. (2006). *The Selfish Gene*. Oxford: Oxford University Press.
- Denninghoff, K.R., Knox, L., Cunningham, R., Partain, S. (2002). "Emergency Medicine: Competencies for Youth Violence Prevention and Control". *Academic Emergency Medicine*, 9(9), 947–956.
- Easley, D., Kleinberg, J. (2010). *Networks, Crowds, and Markets: Reasoning about a Highly Connected World*. Cambridge: Cambridge University Press.
- Euchner, Ch. C. (1996). *Extraordinary Politics: How Protest and Dissent Are Changing American Democracy*. Boulder, CO: Westview Press.
- Farmer, P.E., Nizeye, B., Stulac, S., Keshavjee, S. (2006). "Structural Violence and Clinical Medicine". *PLoS Med*, 3(10), e449. DOI: <https://doi.org/10.1371/journal.pmed.0030449>.
- Fry, D.P. (2006). *The Human Potential for Peace: An Anthropological Challenge to Assumptions about War and Violence*. Oxford: Oxford University Press.
- Gin, N.E., Rucker, L., Frayne, S., Cygan, R., Allan Hubbel, F. (1991). "Prevalence of Domestic Violence Among Patients in Three Ambulatory Care Internal Medicine Clinics". *Journal of General Internal Medicine*, 6(4), 317–322.
- Gordon, L. (1988). *Heroes of Their Own Lives: The Politics and History of Family Violence, Boston, 1880–1960*. New York: Viking.
- Haman, J. (2014). *Gry wokół nas. Socjolog i teoria gier*. Warsaw: Wydawnictwo Naukowe Scholar.
- Kalyvas, S.N. (2006). *The Logic of Violence in Civil War*. Cambridge: Cambridge University Press.
- Lapan, H.E., Sandler, T. (1993). "Terrorism and Signaling". *European Journal of Political Economy*, 9(3), 83–397. DOI: [https://doi.org/10.1016/0176-2680\(93\)90006-G](https://doi.org/10.1016/0176-2680(93)90006-G).
- Le Billon, P. (2001). "The Political Ecology of War: Natural Resources and Armed Conflicts". *Political Geography*, 20, 561–584. DOI: [https://doi.org/10.1016/S0962-6298\(01\)00015-4](https://doi.org/10.1016/S0962-6298(01)00015-4).
- Malešević, S. (2010). *The Sociology of War and Violence*. Cambridge: Cambridge University Press.
- Maynard Smith, J., Price, G.R. (1973). "The Logic of Animal Conflict". *Nature*, 246, 15–18. DOI: <https://doi.org/10.1038/246015a0>.
- Maynard Smith, J. (1982). *Evolution and the Theory of Games*. Cambridge: Cambridge University Press.
- Michaels, J.L. (2017). "Social Forces Sustaining the Israeli-Palestinian Tensions: A Dynamical Psychology Perspective". *International Journal of Conflict and Violence*, 11, 1–17. DOI: <https://doi.org/10.4119/UNIBI/ijcv.255>.
- Mider, D. (2013). "The Anatomy of Violence: A Study of the Literature". *Aggression and Violent Behavior*, 18(6), 702–708.
- Myerson, R.B. (1997). *Game Theory: Analysis of Conflict*. Cambridge-London: Harvard University Press.
- Nash, J.F. (1950). "Equilibrium Points in N-Person Games". *Proceedings of the National Academy of Sciences (USA)*, 36 (1), pp. 48–49. DOI: <https://doi.org/10.1073/pnas.36.1.48>.
- Nash, J.F. (1950). "The Bargaining Problem". *Econometrica*, 18(2), 155–162.
- Nash, J.F. (1951). "Non-Cooperative Games". *Annals of Mathematics*, 54(2), 286–295. DOI: <https://doi.org/10.2307/1969529>.
- Nieburg, H.L. (1969). *Political Violence: The Behavioral Process*. New York: St. Martin's Press.
- Poleszczuk, J. (2004). *Ewolucyjna teoria interakcji społecznych*. Warsaw: Wydawnictwo Naukowe Scholar.

- Powell, R. (2002). "Bargaining Theory and International Conflict". *Annual Review of Political Science*, 5, 1–30.
- Raaflaub, K.A. (Ed.). (2007). *War and Peace in the Ancient World*. Malden, MA: Blackwell Publishing.
- Rak, J. (2017). "Intrastate, Regional, and Colonial Contributions to Post-2008 Cultures of Political Violence". *Polish Political Science Yearbook* 46(1), 1–8. DOI: <http://dx.doi.org/10.15804/ppsy2017118>.
- Rapoport, A., Chammah, A.M. (1966). "The Game of Chicken". *American Behavioral Scientist*, 10(3), 10–28.
- Rapoport, A., Chammah, A.M. (1965). *Prisoner's Dilemma: A Study in Conflict and Cooperation*. Ann Arbor: University of Michigan Press.
- Riechmann, T. (2014). *Spieltheorie*. München: Verlag Franz Vahlen.
- Sampson, R.J. (1987). "Urban Black Violence: The Effect of Male Joblessness and Family Disruption". *American Journal of Sociology*, 93(2), 348–383.
- Sandler, T., Arce, D.G. (2003). "Terrorism and Game Theory". *Simulation & Gaming*, 34(3), 319–337.
- Schabas, W.A. (2000). *Genocide in International Law: The Crimes of Crimes*. Cambridge: Cambridge University Press.
- Schecter, S., Gintis, H. (2016). *Game Theory in Action: An Introduction to Classical and Evolutionary Models*. Princeton–Oxford: Princeton University Press.
- Schelling, T.C. (1966). *Arms and Influence*. New Haven, CT: Yale University Press.
- Schmidt, B.E., Schröder, I.W. (Eds.). (2001). *Anthropology of Violence and Conflict*. London/New York: Routledge.
- Seidler, G., Groszyk, H., Pieniżek, A. (2010). *Wprowadzenie do nauki o państwie i prawie*. Wydawnictwo UMCS: Lublin.
- Sousa, C.A. (2013). "Political Violence, Collective Functioning and Health: A Review of the Literature". *Medicine, Conflict, and Survival*, 29(3), 169–197.
- Springer, S., Le Billon, P. (2016). "Violence and Space: An Introduction to the Geographies of Violence". *Political Geography*, 52, pp. 1–3.
- Straffin, P.D. (1993). *Game Theory and Strategy*. Washington, DC: Mathematical Association of America.
- Toft, M.D. (2003). *The Geography of Ethnic Violence: Identity, Interests, and the Indivisibility of Territory*. Princeton/Oxford: Princeton University Press.
- Vincent, T.L., Brown, J.S. (2005). *Evolutionary Game Theory, Natural Selection, and Darwinian Dynamics*. Cambridge: Cambridge University Press.
- Von Neumann, J., & Morgenstern, O. (1944). *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.
- Walby, S. (2013). "Violence and Society: Introduction to an Emerging Field of Sociology". *Current Sociology*, 61(2), 95–111.
- Walker, L.E. (1997). "Psychology and Domestic Violence Around the World". *American Psychologist*, 54(1), pp. 21–29.
- Zagare, F.C. (2008). "Game Theory". In P.D. Williams (Ed.), *Security Studies: An Introduction* (44–58). London/New York: Routledge.
- Załoski, W. (2016). "Racjonalność i teoria gier". In M. Gorazda, Ł. Hardt & T. Kwarciński (Eds.), *Metaekonomia. Zagadnienia z filozofii ekonomii* (277–302). Kraków: Copernicus Center Press.
- Zimring, F.E. (1998). "Toward a Jurisprudence of Youth Violence". *Crime and Justice*, 24, 477–501.