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PAPER

A study on the static field of a point charge in three-dimensional electrostatics

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The static field potential in quantum electrostatics in the space of three dimensions (QED₃) produced by a point charge was investigated taking into account the polarization of the vacuum. The polarization operator is taken in the N^{-1} approximation which was previously used to study the dynamic violation of chiral symmetry in QED₃. An approximate analytical analysis is accompanied by numerical calculations; there is a good quantitative agreement between the results obtained using these methods. The case of massive dynamical fermions is considered. The limiting transition to the massless case is analyzed, as well as the confinement problem occurring in this model.

1. Introduction

Quantum electrostatics in the space of three dimensions (QED₃) has recently been the object of attention of many authors. The reasons for this interest are different. Among them, one can observe the presence of a dynamic violation of chiral symmetry (including the IT systems) [1–6] and confinement [7, 8] (understood as the growth of the potential modulus with increasing distance). These two phenomena are interrelated in this model. However, there are also other reasons for the interest in QED₃. Thus, breaking chiral symmetry accompanied by the appearance of the dynamic particle mass takes place in quantum field theory [9, 10]. In this case, a close analogy is observed between the processes of particle occurrence in the center of a strong field in the one-particle problem of relativistic quantum mechanics and the dynamic particle mass appearing in the quantum field approach [11–13]. In this respect, quantum electrostatics of three-dimensional space (QED₂₊₁) represents an example of a quantum field model in which, in a certain approach, chiral symmetry breaking, studied in ample detail in [14–17], is observed. This is not the case for the corresponding two-dimensional relativistic quantum mechanical problem that was studied only in a few works (see [18–21]). Moreover, a number of problems studied in detail in the spatial case [14–17] remained uninvestigated for the two-dimensional analog.

Particular attention was paid to the static field of the point charge in QED₃. We will note here that an explicit expression for the potential of such a field is necessary for solving a number of specific problems, in particular, for solving the corresponding quantum-mechanical problem mentioned above. On the other hand, as shown by relevant studies [22], it is important to take into account the polarization of the vacuum, since this can significantly change the character of the dependence of the potential on the distance between the point and the source. By contrast, an exact account of the vacuum polarization in the $N - 1$ approximation, carried out in [22] and being one of the simplest approaches, does not allow expressing this potential through known functions. In this case, the following ways can be used to obtain the desired expression for the potential:

- (a) numerical integration followed by interpolation;
- (b) approximation of the integrand by a simpler expression that makes it possible to calculate the integral explicitly, in a subsequent comparison with the result of numerical integration using the exact expression for the polarization operator.

In this paper the consideration of the static field of a point charge in QED₃ is discussed. As for the photon propagator, the same approximation is used here as in the literature [8]. Anyway, the novelty in comparison with the previous studies described in the literature of the field comprises the following:

- (a) studying approximations of various character within the linear fractional case for a function inverse to the polarization operator (earlier in [8]; for this purpose, only two-point approximation was used, see also [23]);
- (b) using numerical methods (in particular, Mathematica 9.0) when calculating the potential within the framework of the approximation used for the above function;
- (c) comparing the result obtained by means of numerical methods for the potential with the result obtained using approximations of various types within the linear-fractional approximation for a function inverse to the polarization operator;
- (d) analyzing the dependence of the investigated potential on the quantum coupling constant and the fermion mass in a sufficiently wide range of values of the indicated quantities.

2. The general expression for the static potential of the point charge field in QED₃

To find the static potential, we start from the standard expression [17]

$$A_0(\vec{r}) = iQ/(2\pi)^2 \int D_R^{(t)}(\vec{k}, 0) e^{i\vec{k}\cdot\vec{r}} d^2\vec{k}, \quad (1)$$

where Q —charge of the source, $D_R^{(t)}(\vec{k}, 0)$ —transverse part of the total photon propagator at zero frequency, regularized in a gauge-invariant manner. It can be represented as

$$D_R^{(t)}(\vec{k}, 0) = 1/(ik^2 \cdot \varphi(k)); \quad (2)$$

(here $k = |\vec{k}|$), and the function $\varphi(k)$, associated with the polarization operator in QED₃, was first calculated in the approximation considered in [22]. It has the following form:

$$\varphi(k) = 1 + \frac{\alpha m}{2\pi k^2} \cdot \left(1 + \frac{k^2 - 4m^2}{2mk} \cdot \text{arctg}(k/2m) \right) \quad (3)$$

where α stands for the dimensional coupling constant in QED₃, and m is the mass of the fermion. Further, introducing dimensionless quantities $x = k/2m$, $\beta = \alpha/(8\pi m)$, we represent the function φ as

$$\varphi(x) = 1 + \frac{\beta}{x^2} \cdot \left(1 + \frac{x^2 - 1}{x} \cdot \text{arctgx} \right). \quad (4)$$

Substituting (2) and (3) into expression (1) and bearing in mind representation (4), as a result of integration over the angle, we have

$$A_0(\rho) = (Q/2\pi) \int_0^\infty \frac{J_0(x\rho)}{x \cdot \varphi(x)} dx, \quad (5)$$

where $\rho = 2mr$. We draw attention to the fact that the integral (5) diverges at the lower limit if $m \neq 0$. To avoid this divergence, we represent the function $1/\varphi(x) = f(x)$ as

$$f(x) = f(0) + f(x) - f(0), \quad (6)$$

where $f(0) = \frac{3}{3+4\beta}$.

Substituting the equality (6) into (5), we obtain

$$A_0(\rho) = (Q/2\pi) \left(\frac{3}{3+4\beta} \cdot L_1(\rho) + \frac{4\beta}{3+4\beta} \cdot L_2(\rho) \right). \quad (7)$$

Here

$$\begin{aligned} L_1(\rho) &= \int_0^\infty \frac{J_0(\rho x)}{x} \cdot dx; \\ L_2(\rho) &= \int_0^\infty \frac{J_0(\rho x)}{x} \cdot \frac{(x^3 - (3/4)(x + (x^2 - 1) \cdot \arctg x))}{x^3 + \beta(x + (x^2 - 1) \cdot \arctg x)} \cdot dx. \end{aligned} \quad (8)$$

3. Results and discussion

3.1. Approximation of the integrand

The integrals (5), (8) with function (4) are not expressed in terms of the known elementary and special functions; therefore, in the present paper, both the approximation of the integrand by a simpler expression and numerical integration are used for the calculation.

Let us first consider the calculation of the indicated integral by approximating the function $f(x)$, choosing the fractional-linear approximation as the simplest one. On the other hand, this choice is prompted by the type of the specified function when $m = 0$ [24]. In this case, the approximated function can be represented as

$$F(x) = \frac{a_1 + a_2 x}{b_1 + b_2 x}, \quad (9)$$

here constants a_1, a_2, b_1, b_2 are determined by the requirement of the functions $f(x)$ and $F(x)$ values coincidence at given points. Using in this case relations (5), (6), we obtain

$$A_0(\rho) = (Q/2\pi) \left(\frac{a_1}{b_1} \cdot L_1(\rho) + \left(a_2 - \frac{a_1 b_2}{b_1} \right) \cdot \tilde{L}_2 \right), \quad (10)$$

where L_1 is determined by the first of the relations (8), and \tilde{L}_2 is given by

$$\tilde{L}_2(\rho) = \int_0^\infty \frac{J_0(\rho x)}{b_1 + b_2 x} \cdot dx = \frac{\pi}{2b_2} \cdot \left(\mathbf{H}_0 \left(\rho \frac{b_1}{b_2} \right) - N_0 \left(\rho \frac{b_1}{b_2} \right) \right); \quad (11)$$

here \mathbf{H}_0 and N_0 are, respectively, the zero-order Struve and Neumann functions [25].

Giving a certain meaning to the divergent integral L_1 can be argued as follows [26]. We find the derivative $\frac{dL_1}{d\rho} = \frac{1}{\rho} \cdot \int_0^\infty J_1(\xi) \cdot d\xi = -\frac{1}{\rho}$; then, as a result of integrating the resulting expression, we obtain $L_1(\rho) = \ln C - \ln \rho$.

If we choose as the constant C the value ρ_0 which has the meaning of a dimensionless fundamental length, then for the desired integral we have

$$L_1(\rho) = \ln \frac{\rho_0}{\rho}. \quad (12)$$

To solve the problem, the two-point approximation was used in [3, 16, 17] (it was required that the above functions coincide at the point $x = 0$ and with $x \rightarrow \infty$). However, for determining all the constants of the two relations that result from the coincidence of the values of the functions in question at the indicated points, it turns out to be insufficient. In [3, 16, 17] there is no consideration of this question. Therefore, this gap is filled, namely, it is assumed that when $x \rightarrow \infty$ it coincides not only with the first but also the second terms in the expansion of the functions $f(x)$ and $F(x)$ by degree $1/x$. In this case, for the constants entering into the relation (6), we can obtain the following values:

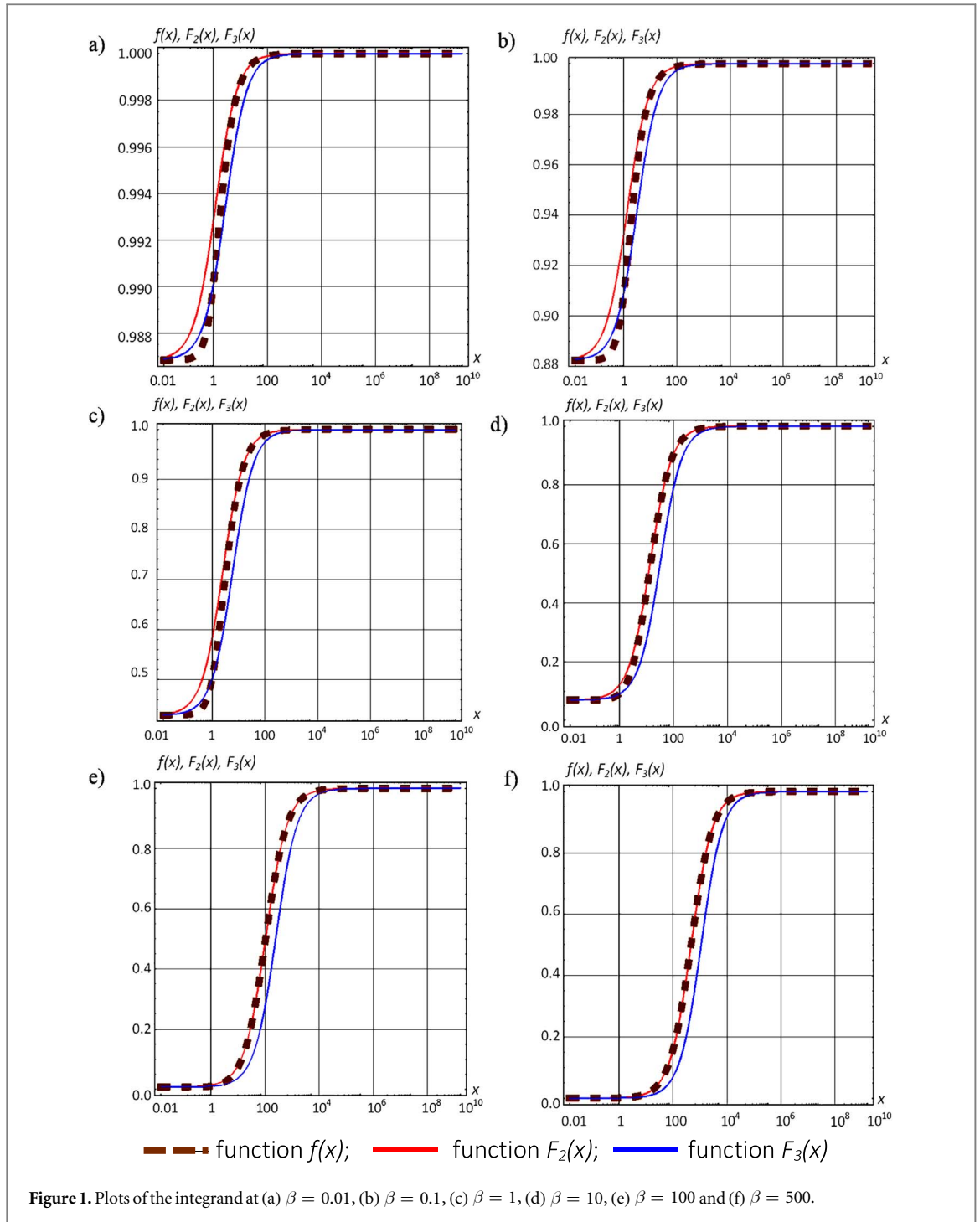
$$a_1 = 3; a_2 = 8/\pi; b_1 = 3 + 4\beta; b_2 = 8/\pi. \quad (13)$$

These values coincide with the results of [8, 23].

It is quite natural in this case to use also the three-point approximation, proceeding from the coincidence of the values of the functions $f(x)$ and $F(x)$ in points $x = 0; x = 1$ as well as $x \rightarrow \infty$. Then for constants a_1, a_2, b_1, b_2 we have

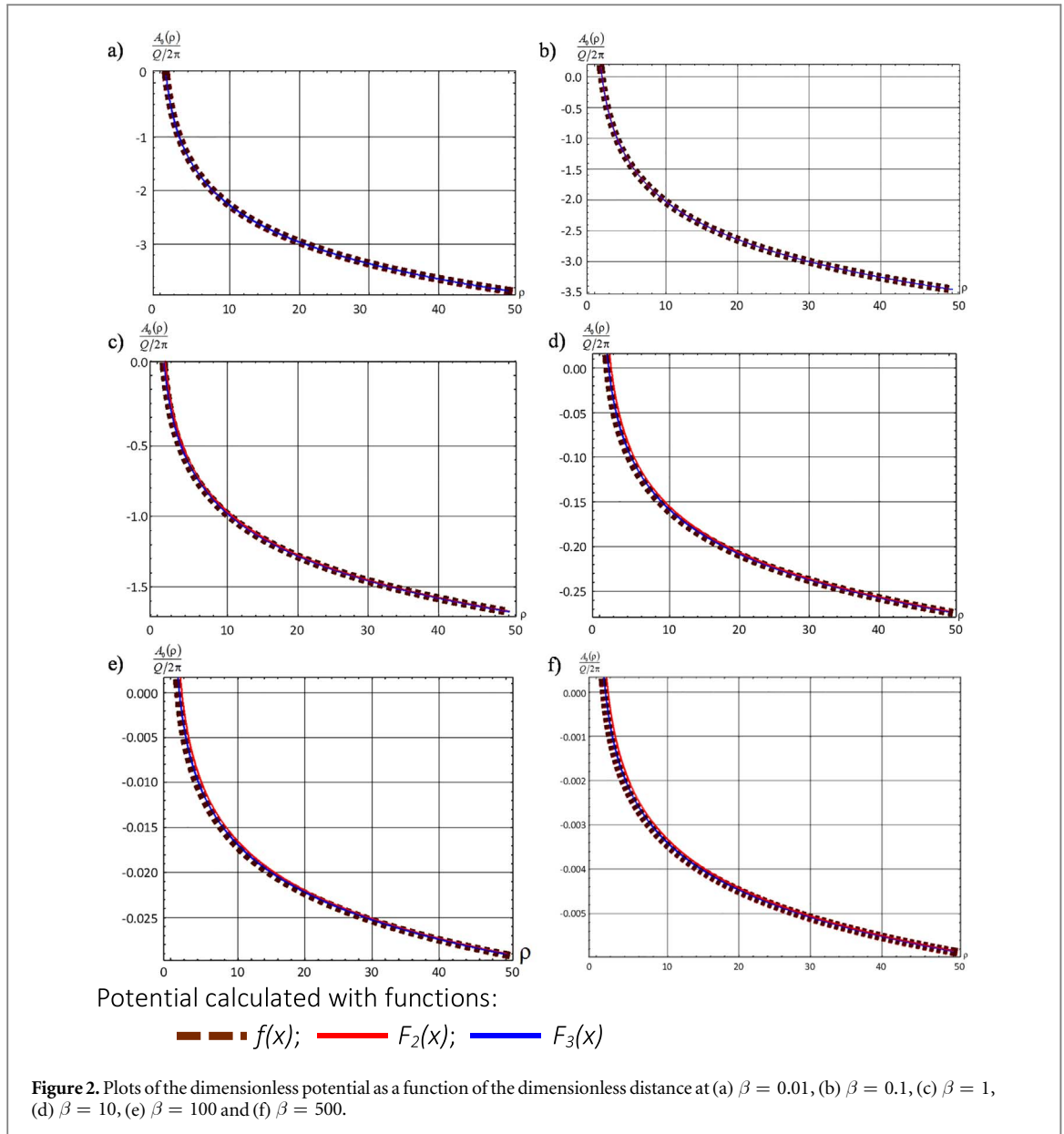
$$a_1 = 3; a_2 = 1; b_1 = 3 + 4\beta; b_2 = 1. \quad (14)$$

Figure 1 shows the plots of the functions $f(x)$ and $F(x)$, corresponding to two-point ($F_2(x)$) and three-point ($F_3(x)$), approximating (functions $f(x)$ correspond to the dotted line, the functions $F_2(x)$ and $F_3(x)$ are plotted with red and blue lines, respectively). In the process of numerical calculations, the computational package of Mathematica 9.0 was used.



The analysis of the curves presented in figure 1 as well as formulae (9), (13), (14) shows the following:

- (1) Both the two-point approximation and its three-point analogue, in general terms, from a qualitative point of view correctly convey the behavior of the function in question throughout the investigated region of photon pulses.
- (2) For small momenta in the case of small β ; the accuracy of the three-point approximation proves to be higher than the two-point approximation; this accuracy decreases with increasing constant β .
- (3) For sufficiently large β the accuracy of the two-point and three-point approximations in the region of both small and large momenta turn out to be comparable.
- (4) With the increase in constants, β the output of both functions to the asymptotic mode of values occurs later than at its small values.



3.2. Calculation of the static potential of the point charge field in QED₃

We proceed to—calculating the potential which is of interest to us, using, first of all, the results of approximation. Using formulae (10)–(14), for the two-point approximation we have

$$A_0(\rho) = \frac{Q}{2\pi} \left(\frac{3}{3+4\beta} \cdot \ln \frac{\rho_0}{\rho} + \frac{4\beta}{3+4\beta} \cdot \frac{\pi}{2} \cdot \left(\mathbf{H}_0 \left(\frac{\pi}{8} (3+4\beta) \cdot \rho \right) - N_0 \left(\frac{\pi}{8} (3+4\beta) \cdot \rho \right) \right) \right). \quad (15)$$

Similarly, we obtain for the three-point approximation

$$A_0(\rho) = \frac{Q}{2\pi} \left(\frac{3}{3+4\beta} \cdot \ln \frac{\rho_0}{\rho} + \frac{4\beta}{3+4\beta} \cdot \frac{\pi}{2} \cdot \left(\mathbf{H}_0((3+4\beta) \cdot \rho) - N_0((3+4\beta) \cdot \rho) \right) \right). \quad (16)$$

Dependence of potential $A_0(\rho)$ on the dimensionless distance at different β is shown in figure 2.

Turning to these figures and comparing the results obtained using approximating functions, as well as those obtained on the basis of the exact function (4), we reach the conclusions listed below.

- (1) The approximations in question describe the qualitative behavior of the potential $A_0(\rho)$ well in the investigated interval of distances; we also observe a good quantitative agreement between the results of the approximation and the exact result for the dependence of the potential under consideration on the distance in the investigated range of distances.
- (2) With increasing the β constant, the accuracy of the approximation improves.
- (3) The accuracy of the approximation proves to be higher in the regions of both the small and large distances.
- (4) In the investigated range of distances, the accuracy of the three-point approximation is higher than the two-point approximation.

The last remark needs additional comments. Thus, it is known that when $m = 0$ ($\beta \rightarrow \infty$) the integral (5) converges and is computed in a closed form. In this case we have [24]

$$A_0(r) = \frac{Q}{4} \left(\mathbf{H}_0 \left(\frac{\alpha r}{8} \right) - N_0 \left(\frac{\alpha r}{8} \right) \right). \quad (17)$$

This relation can be obtained by passing to the limit from the finite mass of the fermion to the case $m \rightarrow 0$ in formula (15), but not (16), whereas with regard to item 4) of the conclusions of the section under consideration we should expect the opposite. This fact is a consequence of the divergence of the integral (5), which implies the order of the transition $m \rightarrow 0$ and integration over the photon momenta.

From what has been said, the following follows. If it is necessary to use the expression for potential $A_0(\rho)$ in the analytical form, in a wide range of values of the constant β of values $\beta \ll 1$ and up to the values $\beta \gg 1$, but finite, three-point approximation is preferable (formula (16)). When $\beta \rightarrow \infty$ ($m = 0$), the formula (17) should be used.

Thus, when analyzing the potential under consideration, if it is an analytic expression, it is reasonable to start from the relation (16) to which we turn. Here, we consider the possibility of weakening the condition for the disappearance of confinement, understood as the disappearance of the field at infinity. It is known [4, 15–17, 25–27] that in QED₃ confinement takes place if $m \neq 0$ (this can be easily seen from expression (16)). If $m = 0$, confinement disappears.

Analyzing the graphs shown in figure 2, one can see the following:

- (a) when the virtual fermion mass is not equal to zero and the coupling constant is fixed in the considered range of coupling constants and distances, the potential under study is a monotonously decreasing function of the distance from the observation point to the source;
- (b) at a fixed value of the indicated distance, a weak dependence of the potential on the coupling constant is observed at its small values, starting to increase significantly with the growth of this constant from the values of $\beta > 1$.

An investigation of the dependence of the potential on the distance can be performed in more detail, if we choose the value of the distance scale

$$r_m = 3\pi m/c_0. \quad (18)$$

where

$$c_0 = \frac{1}{1 + \frac{4}{3}\beta} \quad (19)$$

Then, as shown in [8], we have:

$$A_0(r) = \frac{Q}{2\pi} \ln(4c_0/3\pi mr), \text{ if } r \ll r_m;$$

$$A_0(r) = c_0 \cdot \frac{Q}{2\pi} \ln(4c_0/3\pi mr), \text{ if } r \gg r_m. \quad (20)$$

Thus, it can be observed that at large distances the role of the charge which is the source of the field, is the magnitude $c_0 \cdot Q$. In other words, the magnitude c_0 in our case is the renormalization constant, the appearance of which is associated with the creation of virtual massive fermions in a vacuum. In addition, we see, that potential $A_0(r)$ being monotonically decreasing function of the distance between a field source and observation point changes its sign if noticed distance increases.

Let us consider now the situation when virtual fermions are massless. Here the natural scale of distances is the quantity

$$r_m = 8/\alpha \quad (21)$$

Then we have

$$\begin{aligned} A_0(r) &= \frac{Q}{2\pi} \ln(8/\alpha r), \text{ if } r \ll r_m; \\ A_0(r) &= c_0 \cdot \frac{Q}{2\pi} \cdot 8/\alpha r, \text{ if } r \gg r_m. \end{aligned} \quad (22)$$

We notice that in this case, the potential depending from the distance between a field source and an observation point is different for different distances (with an increasing distance). In addition, we had established that in the case under consideration the potential keeps its sign. The physical aspects of this phenomenon are discussed in detail in [8]. Briefly, in this case, a pole appears in the photon propagator in the space-like region of momenta, which leads to the appearance of mass in the photon. The presence of an imaginary mass, on the one hand, leads to instability of the bulk state, which is responsible for screening at large distances, and, on the other hand, means its instability. This effect manifests itself in a faster decrease compared to the case of massive vacuum fermions of the studied potential with distance from the point under consideration fields to the source.

Since in the problem under consideration, there are three dimensional parameters which are the dimensional fundamental length r_0 , the mass of the fermion m and the coupling constant α , it is possible to construct various dimensionless combinations with them which can play the role of characteristic parameters for the corresponding problems. In particular, here, when we try to weaken the condition for the disappearance of confinement in the sense indicated above, we use the value introduced previously $\beta = \frac{\alpha}{8\pi m}$ and, instead of the traditional condition for this case $\beta \rightarrow \infty$, it is assumed that

$$\beta \gg 1. \quad (23)$$

Using expression (16), and also writing out the principal term of the asymptotic representation of the difference $\mathbf{H}_0(z) - N_0(z)$ [25], as a condition of screening (lack of confinement), it is necessary to take

$$3 \ln \frac{r_0}{r} + \frac{4\beta}{6mr + (\alpha r)/r} > 0. \quad (24)$$

Taking into account condition (18), and also that $r \gg r_0$; from the relation (19), we obtain

$$r < r_0 \cdot e^{1/(6mr)}, \quad (25)$$

from which it follows that the screening is impossible and, hence, the presence of confinement only for strictly nonzero values of the loop fermion masses.

4. Conclusions

When calculating the potential of the static charge field in QED₃ in N^{-1} approximation, the possibility of using a fractional-linear approximation for a function associated with a polarization operator is investigated. When comparing the application of different variants of fractional-linear approximation for the considered function with the result of numerical integration, it turns out that the best approximation gives a three-point approximation. On the basis of this approximation, an analytical expression is obtained for the required potential and it is shown that when the mass of loop fermions vanishes, this expression does not go over into the known exact expression; the reason for this discrepancy is analyzed. On the basis of the expression for the potential obtained with the help of the three-point approximation, the possibility of weakening the condition for the disappearance of confinement was investigated and it is shown that in the approximation considered this possibility is absent.

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