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Gottlob Frege's Correspondence with Italian Mathematicians as a Material Sign of Culture of Science

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9th International Conference

of the European Society for the History of Science,

Centre for the History of Universities and Science at the University of Bologna
Italian Society for the History of Science

Bologna, 31st August - 3rd September 2020

A German mathematician, logician and philosopher from the University of Jena

Gottlob Frege (1848-1925)

collaborated for over twenty years with two Italian mathematicians interested in his scientific activity:
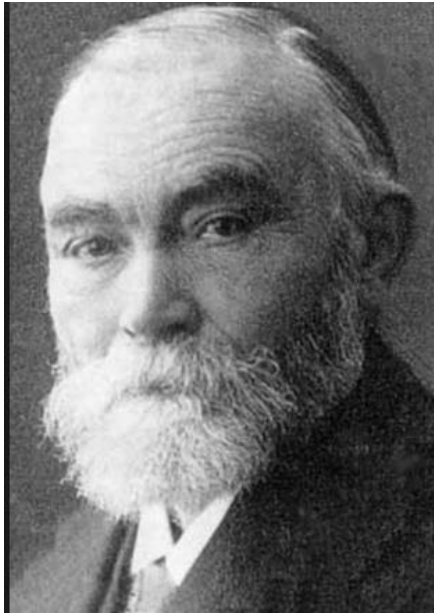
Giuseppe Peano (1859–1932)

Giovanni Vailati (1863–1909)

Their letters have survived and now are treated as material documents of their collaboration showing European culture of science at the turn of the century.
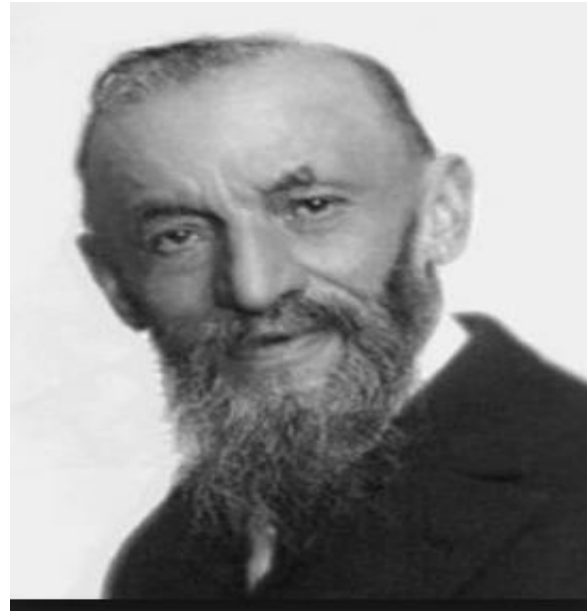
The persons in question:

Gottlob Frege (1848-1925)

The University of Jena;

his logic is called

concept script [*Begriffsschrift*]

Giuseppe Peano (1858–1932)

The University of Turin;

mathematical logic

In 1891 he founded a journal „Rivista di Matematica"

Giovanni Vailati (1863–1909)

Christian Gustav Adolph Mayer.

Graduated University of Turin;

Peano's student, and later co-worker (mainly in the area of history of logic)

Adolph Mayer (1839-1908)

Professor of mathematics in Leipzig

Material documents of their collaboration and inspiration

- **Preserved correspondence**: Frege – Peano, Frege – Vailati, Frege – Mayer, Vailati – Vacca.

- **Mayer wrote an introduction to A. Genocchi**: *Differentialrechnung und Grundzüge der Integralrechnung*. Ed. G. Peano. Transl. G. Bohlmann, A. Schepp. Introduction A. Mayer. Leipzig 1899.

- **Peano wrote a review of Frege's *Grundgesetze*** and Frege made comments on it. Published in „Rivista di matematica".

- Frege wrote **articles** and **parts of his books devoted to Peano's** mathematical logic.

- Peano referred to Frege's concept script in his **papers** and **public lectures**.

- They **exchanged** their papers.

Frege – Peano correspondence (after 1891-1903):

9 letters Peano – Frege; 3 letters Frege - Peano

I presented more substantial this topic two years ago during

8th International Conference of the European Society for the History of Science, London 2018 (September 14-17).

The title of the presentation was:

UNITY AND DISUNITY BETWEEN GOTTLOB FREGE

AND GIUSEPPE PEANO

ON THE BASIS OF THEIR CORRESPONDENCE

IN THE YEARS 1891-1903

You can find the presentation on this website:

https://rebus.us.edu.pl/handle/20.500.12128/15173

Vailati – Frege correspondence (before 1904 – 1906)

- Vailati – Frege, before 17.03.1904, with Vailati's paper on definition. Lost document.
- Frege – Vailati, before 17.03.1904, with Frege's three papers. Lost document.
- Vailati – Frege, 17.03.1904, Como, a letter in French.
- Frege – Vailati, 20.03.1904, Jena, a postcard in German.
- Vailati – Frege, 05.05.1906, Hamburg, a postcard in French. Frege – Vailati, 06.05.1906, Jena, a postcard in German.

Frege's preserved letter to Adolph Mayer on Peano

Frege – Mayer correspondence started on June 8th 1896, one month after Frege's lecture on Peano's logic in Lübeck (Germany).

The subject-matter of the correspondence was publication of Frege's article on Peano's mathematical logic.

The Frege's first letter to Mayer, June 8th 1896, has not been published yet, it will appear in Polish translation of Frege's letters:

Gottlob Frege: *Korespondencja naukowa. Wydanie krytyczne.* Eds. Gabriela Besler, Joanna Zwierzyńska. Katowice, Wydawnictwo Uniwersytetu Śląskiego 2020, November.

The letter was found by Professor Gottfried Gabriel and he agreed to publish it in the Polish edition.

Gottlob Frege to Adolph Mayer

Jena, 8th June 1896

Dear Colleague,

On numerous occasions, Mr Peano from Turin urged me to publish something on his *logique mathématique.* Since he himself reviewed my *Grundlagen der Arithmetik* and sent me some of his essays, I also feel somewhat obliged to do so. When I then recently talked to my colleague Thomae about where I could publish an essay on this topic, he suggested, that you may be inclined to submit it to the *Königl. Sächs. Gesellschaft der Wissenschaften (Royal Saxony Society of Science).* Given that he repeatedly forwarded me your kind regards, which I hereby gratefully return, I dare to trouble you with this request, and I enclose the essay of which I hope that it is suitable for the above-mentioned purpose. Further, I take the liberty to enclose a printed article which also concerns logical issues that seem to me of fundamental importance for a scientific symbolic language.

In the hope that the enclosed material is not undesirable, I remain respectfully yours.

Sincerely yours, G. Frege

translated by
Philip A. Ebert
University of Stirling

Jena, den 8. Juni 1886

Sehr geehrter Herr Kollege!

Herr Peano in Turin mahnt mich immer wieder, etwas über seine logique mathématique zu veröffentlichen. Und da er selber meine Grundlagen der Arithmetik angezeigt und mir manche von seinen Aufsätzen zugeschickt hat, so fühle ich mich dazu auch einigermaßen verpflichtet. Als ich nun neulich mit dem Kollegen Thomae darüber sprach, wo ich wohl eine Abhandlung über diesen Gegenstand veröffentlichen könnte, meinte er, dass Sie vielleicht die Güte haben würden, sie der Königl. Sächs. Gesellschaft der Wissenschaften vorzulegen. Und da er mir wiederholt freundliche Grüsse von Ihnen bestellt hat, die ich hiermit bestens erwidere, so wage ich es, Sie mit dieser Bitte zu belästigen, und lege die Abhandlung bei, von der ich hoffe, dass Sie für den genannten Zweck geeignet sein wird. Ich erlaube mir noch einen gedruckten Aufsatz beizufügen, der gleichfalls logische Fragen behandelt, die mir für eine wissenschaftliche Symbolik von grundlegender Wichtigkeit zu sein scheinen.

Indem ich hoffe, dass Ihnen diese Zusendung nicht unerwünscht sein wird, verbleibe ich mit Hochachtung.

Ihr ergebenster, G. Frege

Based on Frege's letter to Mayer we know on Frege – Peano collaboration:

- Peano **encouraged** Frege to publish something on his mathematical logic in Germany. It was Carl Joannes Thomae who suggested publishing it in Mayer's journal „Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse."

- Frege **felt obliged** to do it because Peano published a review of *Grundgesetze der Arithmetik* (1893). Frege made a mistake writing that *Grundlagen der Arithmetik* was reviewed by Peano.

- Peano was in contact with Mayer before the middle of 1896 because there is a sentence in Frege's letter to Mayer: '**he [Peano] repeatedly forwarded me your kind regards**.'

- I don't know whether Peano – Mayer correspondence survived.

- I don't know what an additional paper was sent by Frege to Peano.

Mayer responded positively to Frege's request for publishing his article on Peano's mathematical logic

and he read Frege's paper during the meeting of

Königlich Sächsische Gesellschaft der Wissenschaften zu Leipzig

which took place on July 6th 1896.

Afterwards, Frege's paper was published as

G. FREGE: *Über die Begriffsschrift des Herrn Peano und meine eigene.* „Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaft zu Leipzig Mathematisch-Physische Klasse" 1896, Bd. 48, p. 361–378; *Kleine Schriften*, pp. 220–233.

ANGELO GENOCCHI

DIFFERENTIALRECHNUNG

UND

GRUNDZÜGE DER INTEGRALRECHNUNG

HERAUSGEGEBEN VON

GIUSEPPE PEANO

AUTORISIERTE DEUTSCHE ÜBERSETZUNG

VON

G. BOHLMANN UND A. SCHEPP

MIT EINEM VORWORT VON A. MAYER

LEIPZIG

DRUCK UND VERLAG VON B. G. TEUBNER

1899

- It was three years after Frege's letter to Mayer when a translation of Genocchi's book appeared in Germany with Peano's five articles as appendices:

- Translators of Genocchi and Peano's papers, Bohlmann and Schepp wrote in the introduction that it was Adolph Mayer who strived for the German translations.

Gottlob Frege – Giovanni Vailati collaboration

- Until now I have not found a paper devoted to Frege – Vailati collaboration, however, it is mentioned in

P. P. Cantù, M. De Zan: *Life and Work of Giovanni Vailati*. In: *Logic and Pragmatism. Selected Essays by Giovanni Vailati*. Eds. C. Arrighi, P. Cantù, m. De Zan, P. Suppes. Transl. C. Arrighi. Stanford, Center for the Study of Language and Information, 2010.

- In 1888 Vailati, called '**the Philosopher**' by his classmates, graduated in mathematics.

- Both Frege and Vailati were philosophers while Peano was denied competence in this field, (Kennedy, Peano, p. 63).

- Vailati **collaborated with Peano** on the edition of *Logique mathématique* (1897), where Frege is mentioned as an author of some laws.

- After reading **Russell**'s book ***The Principles of Mathematics*** (1903) **Giovanni Vacca** encouraged Vailati to study Frege's papers. However, Vacca treated Frege's and Russell's logic as less profound than **Peano's Formulario** (see Vacca – Vailati, 06.12.1903).

- Vailati was in contact with many academics. For example, he wrote letters in German about Euclidean geometry with **Franz Brentano** between 1900 and 1902.

- Frege wrote his letters to Vailati in German while Vailati in **French**. Frege was praised by his French correspondent Eugène Ballue for having a good command of French.

There are two main topics in Vailati's letter to Frege:

1. Geometry, especially Hilbert's geometry.
2. Theory of definition.

Geometry:

- Frege and Vailati did not discuss mathematical logic but **Hilbert's geometry**.

- Frege and Vailati taught geometry and published on geometry. Vailati was an **assistant lecturer in projective geometry** (see P. Cantù, M. De Zan: *Life and Work of Giovanni Vailati*, pp. XXIV, XXVI).

- Vailati wrote that he agreed with Frege

'[…] regarding Hilbert's exposition, as it relates to the axioms, as absolutely incoherent.' (Vailati – Frege, 17.03.1904).

However Vailati called Frege's criticism of Hilbert's *Grundlagen der Geometrie* **not satisfactory** (see Vailati – Vacca, 22.04.1904).

Hilbert's geometry

- 1. **Understanding of axioms**

Hilbert (as well Frege) understood axioms as 'fundamental facts of our intuition.' Vailati wrote that such understanding of axioms

'plainly contradicts the use he [Hilbert] makes of them and the sense (or better *lack of sense*) which he in effect attributes to them in his treatise.' (Vailati – Frege, 17.03.1904)

What Frege could have answered is not obvious because Frege understood axioms as 'fundamental facts of our intuition' as well, and for him 'sense' was a term from his philosophy of language.

- 2. Hilbert's '[…] irrational devotion to Kantian philosophical jargon […]' (Vailati – Frege, 17.03.1904) on **nature of geometrical judgements**.

Vailati did not agree with Kant's point of view on geometry, but Frege assumed Kant's view that geometrical judgements are synthetic *a priori*. It means that Frege and Vailati did not agree on the nature of geometrical judgements.

- 3. **Definition and existence of an object**

Vailati claimed that existence of an object follows from its definition, as it was established by Aristotle.

Hilbert assumed a similar point of view:

'[…] if the arbitrarily given axioms do not contradict one another with all their consequences, then they are true and the things defined by the axioms exist.' (Hilbert – Frege, 29.12.1899)

For **Frege** existence is a second order concept:

'When I say

'There is at least one square root of 4'

I do not predicate anything of 2 or −2, but of the concept square root of 4.' (Frege – Liebmann, 25.08.1900)

Theory of definition

- Vailati referred to Hilbert's geometry and from this point of view criticised Hilbert's definitions by axioms.

- Vailati distinguished two kinds of definitions:

1. Direct (by means of conditions satisfied by defined object);

2. Indirect (by means of relation between given objects), later called **definition by abstraction**. This kind of defining was also used by Frege in his definition of number given in *Grundlagen der Arithmetik* (1884, § 107):

The Number which belongs to the concept *F* is the extension of the concept „concept equal to the concept *F*" where a concept *F* is called equal to a concept *G* if there exists the possibility of one-one correlation referred to above.

The quotation from Vailati's paper precisely describes Frege's way of defining number:

So, if two objects are in a certain relationship, and such relationship has properties similar to those of the equivalence or similarity, the supposition that they should actually resemble each other in some aspects may lead, and in fact has led in many cases, to the discovery of new properties in the objects in question, and to the realization of which are, among those properties, those whose common possession correlates with, or determine, the existence of their relationship.

Vailati: *Sull'importanza delle Ricerche relative alla Storia delle Scienze*, 1897 [*On the Importance of Research Regarding the History of Science*, p. 83].

Vailati and Deutschen Mathematiker-Vereinigung

- In a letter of 17.03.1904 Vailati asked Frege whether he as a non-German person can join Deutschen Mathematiker-Vereinigung.

- Three days later Frege answered that non-Germans are also members of this society and suggested contacting August Gutzmer, an editor of Jahresbericht der Deutschen Mathematiker-Vereinigung.

- 1905 – 1907 neither Vailati nor Peano were members of Deutschen Mathematiker-Vereinigung. You can see it on the list on its members published every year in Jahresbericht der Deutschen Mathematiker-Vereinigung.

Frege – Vailati meeting in Jena

- From Hamburg May 5th 1906 Vailati wrote to Frege asking to meet him in Jena.
- In a postcard sent the next day Frege invited him to have dinner together in Frege's home. At that time Frege had been a widower for two years, and he would adopt a boy Alfred two years later.
- Actually, the Frege's postcard to Vailati written on May 6th 1906 is the last preserved material document of Frege's collaboration with Italian mathematicians.

From Vailati's letter to Vacca we know they rather met.

Until now no other material evidence that they really met and if so, what topics they discussed have been found.

What subjects could Frege have discussed with Vailati during their meeting in 1906 in Jena?

- Criticism of Hilbert's geometry;
- Kant's theory of mathematical knowledge, especially nature of geometrical judgements;
- Language, philosophy of language;
- Historical contributions to subsequent editions of Peano's *Formulario*, where Frege was mentioned and later omitted;
- Positive function of errors as Vailati could comment Frege's antinomy;
- Theory of definition, definition in mathematics, definition by abstraction.

Vailati on positive function of errors as possible comment to Frege's antinomy in his logical system:

An erroneous assertion, an inconclusive argument from a scientist of the past can be worthy of consideration as much as a discovery or an ingenious intuition, if they are equally useful in shedding light on the causes that have accelerated or delayed the progress of human knowledge, or in revealing how our intellectual faculties operate. **Every error shows us an obstacle to avoid, while not every discovery shows us a path to follow.**

Vailati: *Sull'importanza delle Ricerche relative alla Storia delle Scienze*, 1897 [*On the Importance of Research Regarding the History of Science*, p. 5].

# What happened after Frege – Vailati meeting in Jena in 1906:

- In 1906 Frege published **another paper on geometry**

G. Frege: Über die Grundlagen der Geometrie [1906]. „Jahresbericht der Deutschen Mathematiker-Vereinigung" 1906, Bd. 15, pp. 293–309, 377–403, 423–430;

In this long article Frege referred only to Korselt and Hilbert. Actually, Vailati is not mentioned in any of Frege's papers.

- From Vailati's letter to Vacca we know that during his three or four months' stay in Florence in 1906 since May Vailati wanted to write **a paper on Frege**. A journal "Loria" was considered. However, he did not manage it.

In 1911, two years after Vailati's death, all his papers appeared in Leipzig's publishing house Johann Ambrosius Barth Verlag and Florenze Successori B. Seeber, over 1,000 pages. Frege is not mentioned there.
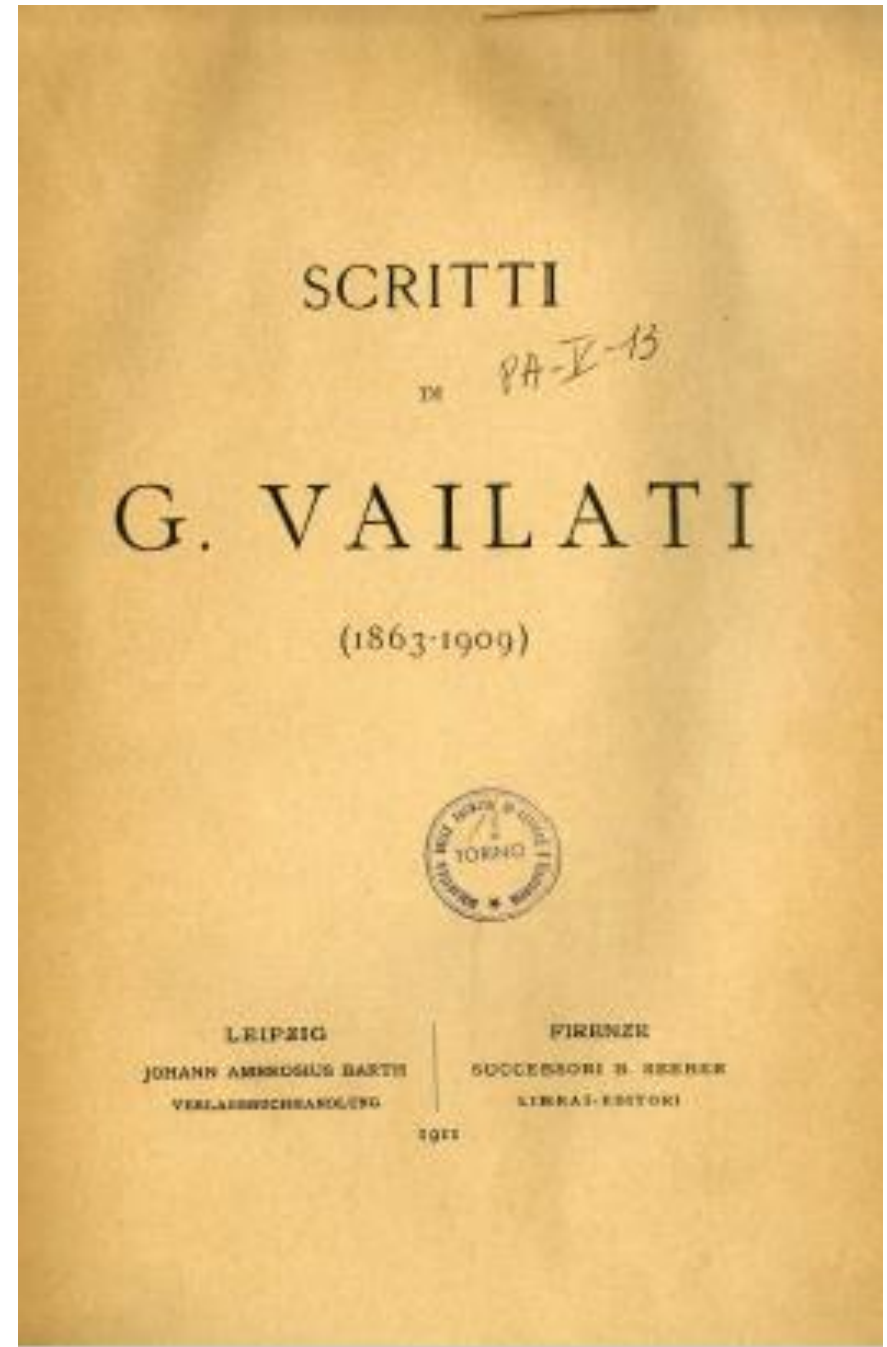
Conclusions:

- Recently prof. Gottfried Gabriel has found an important letter from **Frege to Mayer**. It shows some more details about Peano's relationship with the German academic society.

- Maybe **Vailati's** contacts with Frege were also helpful in publishing a 1,000-pages book with Vailati's papers in Leipzig/ Florence.

- Vailati did not agree with Frege's criticism of Hilbert's geometry. But it was Frege who went down in the **history of geometry**, not Vailati.

- Vailati is mentioned in none of Frege's papers. Similarly, Frege is never mentioned in Vailati's papers. Nevertheless, they worked on similar topics, exchanged letters and papers, met in person.

- There are still **letters and other documents to be found**. We need them to establish topics of Frege – Vailati discussion in Frege's house in 1906. That discussion encouraged Vailati to write an article on Frege to an Italian journal but it seems that it was not written.

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Thank you for your attenion and **if you have any questions**, I´ll be happy to answer **them**.