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Review

New Perspectives for Multifrequency GW Astronomy: Strong Gravitational Lensing of GW [†]

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Abstract: Direct detection of gravitational waves was for a long time the holy grail of observational astronomy. The situation changed in 2015 with the first registration of a gravitational wave signal (GW150914) by laboratory interferometers on Earth. Now, successful operating runs of LIGO/Virgo gravitational wave detectors, resulting in numerous observations of gravitational wave signals from coalescing double compact objects (mainly binary black hole mergers) with the first evidence of a coalescing binary neutron star system, has elevated multimessenger astronomy to an unprecedented stage. Double compact objects (binary black hole systems, mixed black hole–neutron star systems, and double neutron star systems) are the main targets of future ground-based and space-borne gravitational wave detectors, opening the possibility for multifrequency gravitational wave studies and yielding very rich statistics of such sources. This, in turn, makes it possible that certain, non-negligible amounts of double compact objects will have a chance of being strongly lensed. In this paper, we will discuss new perspectives for future detections of gravitational wave signals in the case of strong gravitational lensing. First, the expected rates of lensed gravitational wave signals will be presented. Multifrequency detections of lensed gravitational wave events will demand different treatments at different frequencies, i.e., wave approach vs. geometric optics approach. New possibilities emerging from such multifrequency detections will also be discussed.

Keywords: gravitational lensing; gravitational waves: sources; gravitational waves: experiments



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1. Introduction

Successful operating runs of LIGO/Virgo gravitational wave (GW) interferometers have recently brought us numerous (more than 50 events already observed) direct detections of GW signals from coalescing double compact objects (DCOs). These are overwhelmingly binary black hole (BH–BH) mergers [1] (including GW150914—the first ever laboratory registration of a GW signal [2]) but the first evidence of coalescing binary neutron star (NS–NS) system [3] and probably the first mixed black hole–neutron star (BH–NS) merger [4] has also been recorded. Consequently, multimessenger astronomy has been elevated to an unprecedented stage that is particularly visible in the case of NS–NS coalescence (GW170817) when the electromagnetic (EM) counterpart to a GW signal was identified at different EM wavelengths [2,5]. In this light, the possibility that data collected by LIGO/Virgo detectors (and the recently started KAGRA Observatory) would be in the future supported by a new generation of ground-based GW interferometers such as the Einstein Telescope (ET) [6] and space-borne GW detectors seems to be extremely promising. In fact, we expect that increased sensitivity of such planned GW detectors will bring us rich statistics and thus big catalog of DCO events up to cosmological distances. Apart of an improved sensitivity, future space missions like LISA [7] and DECihertz Interferometer Gravitational wave Observatory (DECIGO [8,9] and its smaller scale version B-DECIGO [10]) are designed to probe the GW spectrum to frequencies lower

than 1 Hz, which are inaccessible for ground-based detectors due to irremovable seismic noise. This would allow the observation of DCOs in the inspiraling phase for weeks to years before they become observable from the ground when entering the merger stage. In particular, DECIGO will cover mHz to hHz GW frequencies, providing an intersection with the low-frequency LISA window and the hHz range of ground-based detectors, which opens the possibility for multifrequency gravitational wave studies. On the other hand, reach statistics of GW events observed by ET or DECIGO allow us to expect that certain, non-negligible amounts of DCOs will have a chance to be gravitationally lensed. In this paper, we explore this possibility more deeply. In particular, predictions concerning expected detection rates of lensed GW signals from DCO systems in the case of ET and DECIGO/B-DECIGO detectors will be discussed in more detail.

2. Method

To calculate expected detection rates for unlensed and lensed DCO systems in the case of planned ET and DECIGO/B-DECIGO missions, it is necessary to first investigate two main components underlying our studies: DCO merger rates, and detector sensitivity. At this point, we would like to emphasize that our calculations are based on a procedure that will be only briefly summarized here. More information concerning the method presented in this section can be found in our previous papers [11–13].

2.1. DCO Merger Rate

The intrinsic DCO merger rate (i.e., intrinsic inspiral rates as a function of source redshift) can be calculated based on the source evolution modeled according to the analytical formula (valid only for NS–NS systems; see [11] and references therein). We opted, however, for a different approach, in which we take values of intrinsic inspiral rates as forecasted using the StarTrack code [14] (data available at <https://www.syntheticuniverse.org>) for each DCO type and for a given redshift slice from $z = 0.04$ to $z = 17$. These detailed population synthesis calculations are based on several thoroughly motivated assumptions concerning star formation rates, galaxy mass distributions, stellar populations, and metallicities, as well as on two galaxy metallicity evolution scenarios with redshift (assigned in [14] as “high-end” and “low-end”) reflecting the varied chemical composition of the universe. DCO formation depends on several processes that are uncertain to some degree, such as the physics of the common envelope phase of the binary system evolution or the supernova explosion mechanism. Thus, StarTrack population synthesis calculations have been performed for four scenarios: the standard, and three of its modifications (optimistic common envelope, delayed supernova explosion, and high BH natal kicks; more detail can be found in [12–14]).

2.2. Detector Sensitivity

The second question that must be taken into account prior to calculations concerning DCO detection rates is the sensitivity of a particular detector for which analysis is performed. ET and DECIGO design is planned based on the triangular geometry of interferometers. ET is proposed as three nested detectors forming an equilateral triangle with arm lengths of 10 km within an underground infrastructure [15]. By contrast, DECIGO will be composed of four clusters put into the heliocentric orbit, where each cluster will consist of three drag-free spacecrafts in an equilateral triangle configuration with arm lengths of 1000 km [16]. Even the modest project B-DECIGO (in comparison to the original design of DECIGO) still makes a spectacular impression: it will consist of three satellites forming a 100 km equilateral triangle [10,17]. The increased size of interferometer length arms and planned implementation of several new technologies will translate into a greatly improved sensitivity of ET or DECIGO also according to its property (see [16]) that a single triangular detector unit is equivalent to two standard L-shaped interferometers rotated by 45° . A detector noise power spectrum allows the estimation of the so-called characteristic distance parameter r_0 of a given detector—a distance that can be probed by a given detector with

a given sensitivity. The polynomial approximation to the ET noise curve in the initial configuration gives $r_0 = 1527$ Mpc (and may be increased to $r_0 = 1918$ Mpc for advanced “xylophone” configuration) [15]. On the other hand, DECIGO and B-DECIGO noise spectrum [16–18] gives estimated values of the characteristic distance parameters respectively: $r_0 = 6709$ Mpc and $r_0 = 535$ Mpc, which means that DECIGO in full original design would be able to probe about 64 times larger volume than ET (see discussion in [13]).

2.3. Detection Rates for Unlensed Events

The main goal of our studies is to make predictions concerning the rates at which ET and DECIGO/B-DECIGO would be able to observe gravitationally lensed DCO sources. As a first step, we need to estimate yearly detection rates of unlensed GW sources originating at redshift z_s and producing signal with a S/N ratio exceeding the detector’s threshold $r_0 = 8$ [11,12,16,18]. For this, we consider the matched filtering S/N ratio for a single detector according to [19], which allows the consideration in our calculations of a usually non-optimal random orientation of the DCO system with respect to the detector. Another problem is that the confusion noise from unresolved binaries may severely deteriorate the ability for a given detector to see GW signals from DCO systems. Our calculations concerning this issue (see detailed analysis supported by Figure 1 in [13]) show that DECIGO sensitivity will be significantly affected mainly by unresolved BH–BH systems (B-DECIGO will be affected much less) and thus we have to modify the DECIGO (and B-DECIGO) noise spectrum such that it will include confusion noise from unresolved systems separately for each kind of DCO and for each binary system evolution scenario considered in this work according to StarTrack simulations. This procedure leads to the effective change of characteristic distance parameter r_0 values for DECIGO and B-DECIGO (differently for each DCO type and evolutionary scenario). It should be noticed here that all our estimations described in this and the next section have been made within flat Λ CDM cosmology with $H_0 = 70 \text{ kms}^{-1}\text{Mpc}^{-1}$ and $\Omega_m = 0.3$, which is compatible with the assumptions underlying the StarTrack population synthesis simulation.

2.4. Detection Rates for Lensed Events

The similarity of a mathematical description between GWs and EM waves (i.e., Einstein field equations in terms of metric perturbations within weak field regime vs. EM field equations in the Lorentz gauge) leads to a resemblance in their properties, which in turn allows us to expect that GWs should experience the same geometric optics effects as EM waves. In particular, GWs also travel along null geodesics and thus we expect that they should undergo the gravitational lensing phenomenon. Strong gravitational lensing occurs when the light emitted from a distant source encounters a massive object (a galaxy or cluster of galaxies) on its way to an observer on the Earth. Consequently, we observe multiple, magnified, and distorted images of the source. In addition, a non-zero time delay between images occurs as a result of two phenomena: geometrical difference between light paths from different images, and the influence of the gravitational potential of the lens on travel time of the light known as Shapiro time delay [20,21]. In the case of GW strong gravitational lensing, assuming that the mass distribution in the lensing galaxy follows the singular isothermal sphere (SIS) model strongly supported by galaxy lensing studies according to which the population of lenses comprises massive elliptical galaxies [22], we expect to observe two time-delayed waveforms with different amplitudes but of similar temporal structures (i.e., frequency drift) [11–13]. Although the ET will register two different merger signals, separated by lensing time delay, DECIGO will likely detect the interference pattern between unresolved images [23] (argumentation can be found in the next section). This highlights the importance of multifrequency observations. In our analysis, we follow SIS assumptions for mass distribution in lensing galaxies. We also consider the case when both images, brighter I_+ and fainter one I_- , produce signals visible by detector (i.e., detected SNR: r_+ and r_- exceed detector’s threshold $r_0 = 8$). Then, we calculate the total optical depth for lensing based on velocity dispersion distribution of stars in lensing galaxies

according to the modified Schechter function (see discussion, e.g., in [13]) with parameters representing the population of elliptical galaxies. Since DECIGO/B-DECIGO missions are planned for 4 years continuous operation, we should also correct our results accordingly: the finite duration time of observations may be relevant to those signals that come near the beginning or the end of the survey (they may be missed if we register only one image due to time delay, and thus we would not be able to judge properly if this signal were lensed).

3. Results and Discussion

According to analysis described in the previous section, the expected yearly detection rates of resolvable inspiraling DCO systems producing GW signals above detector threshold are in the order of 10^4 – 10^6 for ET, 10^2 – 10^5 for DECIGO, and 10 – 10^5 for B-DECIGO, depending on a particular type of DCO system, population synthesis scenario (standard, optimistic CE, delayed SN, or high BH kicks), and galaxy metallicity evolution model (“high-end” or “low-end”). The results concerning DCO detection rates are collected for ET in Tables 1 and 2 of [12] and for DECIGO/B-DECIGO in Table 2 of [13]. The expected DCO strong gravitational lensing rates have been assessed under the assumption that a GW source would be detectable without lensing (both images: I_+ and I_- produce signals with a S/N ratio exceeding the detector’s threshold of $r_0 = 8$) and taking into account a planned finite duration time of the survey: first 1, 5, and 10 years of ET operation and 4 years of DECIGO nominal duration. The results, as in the case of the unlensed detection rates described above, depend on the type of DCO system, binary evolution scenario, and galaxy metallicities. Consequently, we obtain that ET would be able to detect 50–100 lensed GW events per year (mainly BH–BH systems) which is consistent with the corresponding results obtained if one considers the Earth rotation effect on DCO lensing rates [24]. On the other hand, DECIGO and B-DECIGO yearly detection rates of lensed GW signals from DCO inspirals are significantly reduced in comparison to ET predictions. Due to the contamination of unresolved systems, DECIGO and B-DECIGO could register about 50 lensed GW events per year, but it would be only signals from BH–BH inspiraling systems—the chance that DECIGO or B-DECIGO will detect lensed NS–NS or mixed BH–NS inspirals is negligible [13]. These results are reported in detail in Tables 3 and 4 of [12] for ET and Table 3 of [13] for DECIGO/B-DECIGO.

It should be noted, however, that in the case of gravitational lensing, a description made with light-ray formalism may be inappropriate. In fact, lensing acts like a diffraction barrier with a narrow slit (actually strong lensing phenomena can be contemplated as an equivalent to double-slit interference of EM waves), and thus there exist some conditions when the wave nature of GW becomes relevant. In general, if GW wavelength λ_{GW} is much shorter than the lens mass scale (e.g., Schwarzschild radius of the lens mass) the wave nature of the GW may be ignored, and gravitational lensing may be safely analyzed within a geometric optics limit as in the case of the light. By contrast, if λ_{GW} is much longer than the Schwarzschild radius of the lens mass, then the GW wave nature becomes relevant and the wave optics limit should be used instead of geometric optics [23,25]. Because GW signals from DCO systems are coherent, diffraction and interference could be embedded into amplification of the wave intensity calculated in the case of monochromatic waves propagating in the presence of weak gravitational potential according to the Helmholtz equation [20,25–27], influencing our predictions concerning strong GW lensing. It can be shown [25] that in the situation where diffraction becomes important, it makes lensing inefficient in the sense that the magnification starts to be small. This problem is especially significant in the era of multifrequency detections of lensed GWs. In the case of space-borne interferometers such as LISA or DECIGO $\lambda_{\text{GW}} \sim 1\text{AU}$ indicating that different formalisms (geometric optics vs. wave optics) should be used for lensed GW events observed with different detectors operating in different frequency domains. In particular, in the case of strong GW lensing observed by a DECIGO detector, interference effects between unresolved multiple images may be observable as a beat pattern in a time domain, which seems to be very promising for lens mass and time delay distance measurements [23].

4. Conclusions

In this paper, we have discussed new perspectives for multifrequency GW astronomy in the case of GW strong gravitational lensing. A new generation of ground-based detectors such as ET or space-borne missions such as DECIGO/B-DECIGO will considerably enrich GW statistics making it highly probable that some GW signals from DCO inspiral systems will undergo strong gravitational lensing. Our robust prediction is that ET would be able to register up to 100 strongly lensed GW events per year. On the other hand, we predicted that DECIGO would be highly contaminated due to stochastic noise from unresolved DCO systems influencing detection rates of lensed GW signals seen by this detector. In particular, DECIGO/B-DECIGO will not be able to see any lensed NS–NS or BH–NS systems, but they could be able to detect yearly up to $O(10)$ strongly lensed BH–BH inspirals. Space interferometers will be able to significantly complement global network composed of ground-based GW detectors, opening new possibilities for multifrequency observations of GW events. In particular, DECIGO missions dedicated to the exploration of the decihertz GW frequency range would be able to observe DCO inspiral events for days to months before they enter the frequency band observable by GW detectors on Earth. However, in the case of GW observations carried via space interferometers, wave phenomena such as diffraction and interference of GWs from lensed images should be considered.

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