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Review

Graviton Mass in the Era of Multi-Messenger Astronomy

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Abstract: The idea of massive graviton plays a fundamental role in modern physics as a landmark of most scenarios related to modified gravity theories. Limits on graviton mass can be obtained through different methods, using all the capabilities of multi-messenger astronomy available today. In this paper, we consider some emerging opportunities. In particular, modified relativistic dispersion relations of massive gravitons may lead to changes in the travel time of gravitational waves (GWs) emitted from distant astrophysical objects. Strong gravitational lensing of signals from a carefully selected class of extra-galactic sources such as compact object binaries (actually, binary neutron stars) is predicted to play an important role in this context. Comparing time delays between images of the lensed GW signal and its electromagnetic (EM) counterpart may be a new model-independent strategy (proposed by us in X.-L. Fan et al, 2017), which is especially promising in light of the fruitful observing runs of interferometric GW detectors, resulting in numerous GW signals. In addition to this direct, kinematic method, one can use an indirect, static method. In this approach, the non-zero graviton mass would modify estimates of the total cluster mass via a Yukawa term, influencing the Newtonian potential. In A. Piórkowska-Kurpas et al, 2022, using the X-COP galaxy cluster sample, we obtained $m_g < (4.99 - 6.79) \times 10^{-29}$ eV (at 95% C.L.), which is one of the best available constraints.

Keywords: graviton mass; gravitational waves; gravitational lensing

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1. Introduction

If our understanding that all fundamental interactions, including gravity, really do have their quantum versions is correct, then the carrier of gravity—a massless particle of spin 2 (the graviton)—should exist. This expectation has recently been supported by the experimental success of the standard model of elementary particles, crowned with the detection of the Higgs boson. Experimental confirmation of the graviton has not yet taken place (this is actually a highly difficult task), which opens up room for discussion concerning the real nature of the phenomenon of gravity. In this light, the idea of a massive graviton may play a fundamental role as a landmark of most scenarios related to the modification (or replacement) of general relativity as a theory of gravity. Such attempts are additionally highly motivated by so-called dark energy [3–8] and dark matter [9–17] problems in studies of the Universe, for which both the theoretical pillars underpinning our knowledge of nature at a fundamental level—general relativity and the standard model of elementary particles—have so far been unable to find any complete and satisfactory solution. In particular, due to the lack of conclusive data allowing one to uncover the true nature of the dark energy phenomenon, it is reasonable to describe it phenomenologically as a cosmological constant within the standard cosmological model for a flat Universe, with cold dark matter being taken into account [3–6,8]. This so-called ΛCDM scenario shows strong agreement with observational data and seems to be the simplest and the most useful one, incorporating dark energy and dark matter phenomena within a single framework. However, it cannot be treated as a final answer, which has motivated the emergence of a great number of alternative scenarios trying to find a way out from this impasse. One popular category of such attempts consists of models assuming particular modifications of the fundamental laws of gravity, allowing for the solving of dark energy and dark matter problems in a very clever and possibly natural manner (see, e.g., [18] and references therein). Others have postulated appropriate corrections to the Einstein field equations, implying the existence of gravitons with non-zero rest mass [19,20]. This may explain the present accelerated expansion of the Universe via the modification of gravity at large scales [21] or, on the other hand, the origin of the dark matter sector according to the fact that massive gravitons should also be considered themselves a source of gravity [22–25]. The presence of many alternatives to the standard theory and the continuous creation of new proposals opens a new window for the testing of physics at previously unreachable scales (i.e., the quantum gravity energy scale). In particular, even the smallest signal related to the non-zero mass of the graviton (or the lack of such a signal) would indicate an appropriate direction for seeking the generalization of general relativity to a more complete theory. For this reason, theories postulating the existence of massive gravitons have continuously been examined since the late 1930s, when the first approach to the subject of massive gravity was made by Fierz and Pauli [26], although they suffered from serious problems related, e.g., to additional degrees of freedom (see, e.g., [27] for a comprehensive review). Historically, Zwicky [28] made one of the first attempts to measure the mass of the graviton on the basis of galaxy cluster dynamical behavior. His bound of $m_g < 3.2 \times 10^{-31}$ eV looks impressive, especially when compared to the most recent ones based on Chandra cluster data in X-rays: $m_g < 10^{-29}$ eV [29,30] or $m_g < 10^{-30}$ eV [31,32]. This limit has been later corrected to $m_g < 1.1 \times 10^{-29}$ eV, with a more reliable value of the Hubble constant applied to independent cluster observations [33]). Dynamical properties of other astrophysical objects can also be used in this context (see, e.g., [34,35]) and in fact, this class of tests is among the most robust in the field, mainly due to the fact that they are independent of any particular massive gravity model [19,20]. Precise analysis of the shape of the GW signal from double compact object mergers [34,36–38] or comparisons between time delays for GW and EM gravitationally lensed signals from such sources [1,39,40] may support dynamical estimates of graviton mass, revealing the power of synergy between multi-messenger observations. Here, we explore this question more deeply. Particularly, in Section 2 we briefly explain how modified dispersion relations may help in constraining graviton mass with a time delay technique. The influence of massive gravitons on GW waveforms and how this effect may be used to obtain limits on graviton mass using recent LIGO/Virgo data are discussed in Section 3. The technique described in Section 2 may be used and actually improved with gravitationally lensed GW signals from binary neutron star systems, as is shown in Section 4. This method has several advantages (e.g., it enables us to circumvent the intrinsic time-lag problem between GW and EM signals emitted from the same source), making it especially interesting in light of the new generation of GW detectors. The concepts underlying the method based on the dynamical properties of galaxy clusters in the presence of a Yukawa potential have been presented in Section 5. Here, the most recent upper limit on the graviton's mass obtained using this method with the data for the X-COP cluster sample is also shown.

2. Time Delay Technique in Probing Graviton Mass

An obvious physical implication of a non-zero graviton rest mass is the fact that this particle will no longer travel at the speed of light. This entails a phenomenological approach to finding constraints on graviton mass via a modified dispersion relation, which is extremely popular, for example, in testing the consequences of the Lorentz symmetry breakdown contemplated within many quantum gravity models (see, e.g., [41,42] and references therein). A widely accepted point of view within these frameworks is that at extremely high energies of the order of the Planck energy scale ($E_{Pl} \sim 10^{19}$ GeV), at which we expect that new physics (i.e., quantum gravity) should start to manifest itself, the standard dispersion relation for relativistic particles,

$$E^2 = m^2c^4 + p^2c^2, \tag{1}$$

where E, p and m are energy, momentum and the mass of a given particle, with c being the speed of light in a vacuum, should rather be replaced by a more general (currently unknown) function of the particle's energy and momentum [42,43]. The low-energetic Taylor expansion of this function

$$E^2 = m^2c^4 + p^2c^2 + f(E, p, m; E_{Pl}) \tag{2}$$

will lead to tiny deviations from a standard case, ruled by Equation (1), as quantum gravity effects, if they exist, should be extremely small at currently accessible energies in the light of the experimental success of the well-known standard theory. The specific structure of deformation in Formula (2) depends on a particular quantum gravity model (details can be found in [44,45]), but all of them agree with the fact that such a dispersion relation may lead to changes in the travel times of signals emitted from distant sources, thereby opening opportunities for time-of-flight measurements. There is a common expectation that the farther the source is, the stronger this exotic signal could be. We simply expect that non-standard effects accumulate on the path between the source and the observer, amplifying the signal to detectable levels [42]. Therefore, in this context, the most attractive objects are high-energy extra-galactic cosmological sources such as active galactic nuclei, gamma-ray bursts and, recently, double compact object mergers. These sources are also promising due to their extremely regular (double compact object mergers) or fine-scale (gamma-ray bursts, active galactic nuclei) time structures, which are required for the time delay technique to be robust. As highly energetic astrophysical objects, pulsars can also be considered as a tool for testing quantum gravity with the time-of-flight method (see, e.g., [46] and references therein). Even through the present catalogs mainly contain galactic pulsars, the quasi-periodicity of their signals may help to improve the accuracy of such estimates—expected tiny non-standard quantum gravity effects should be more visible, and thus easier to detect for regular pulses. Such searches may be conducted between signals emitted in different energy channels (e.g., low and high energies; see, for example, [47,48] and references therein) and/or between different particle types (e.g., EM waves and photons for graviton mass estimation [1,39,40] or EM waves and neutrinos allowing one to constrain the quantum gravity energy scale within the assumption of Lorentz invariance violation; see [49]), thus motivating the need to take advantage of all the present-day capabilities of multi-messenger astronomy. This is especially evident when the dispersion relation for massive graviton is used in the context of putting constraints on the graviton's mass, m_g:

$$E^2 = m_g^2c^4 + p^2c^2. \tag{3}$$

In this case, the speed of the GWs associated with gravitons traveling along radial geodesics in the flat Friedman–Robertson–Walker (FRW) model (i.e., with the metric $ds^2 = c^2dt^2 - a(t)^2\left[dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right]$) would be [1,34]

$$v_{\mathrm{GW}} = \frac{c}{a}\left[1 - \frac{1}{2}\frac{m_g^2c^2a^2}{p_r^2}\right], \tag{4}$$

where a is the scale factor in the FRW model and p_r is the graviton's four-momentum (actually its covariant component). The resulting difference in the time of arrival could be calculated as follows. The comoving distance to the source, determined based on the EM signal, is $r_\gamma = \int_{t_e}^{t_0}\frac{c}{a(t)}dt = c\int_0^z\frac{dz}{H(z)}$, where t_e, t_0 are the emission and detection times, respectively. For the massive graviton, one has $r_{\mathrm{GW}} = \int_{t_e}^{t_0} v(t)dt$. This means that if the massive graviton is registered together with the photon, the GW source should be closer by $\Delta r_{\mathrm{GW}} = r_\gamma - r_{\mathrm{GW}}$. Since they are emitted from the same source, the result would be the arrival-time difference $\Delta t_{\mathrm{GW}} = \Delta r_{\mathrm{GW}}/c$. Finally, the difference in the time of arrival will be:

$$\Delta t_{\mathrm{GW}} = \frac{1}{2}\frac{m_g^2c^2}{p_r^2}\int_{t_e}^{t_0} a(t)dt. \tag{5}$$

Here, $H(z) = H_0 h(z)$ is expansion rate of the Universe, with H_0 being the Hubble constant, representing the current rate of this expansion and $h(z)$ being a dimensionless form of it, depending on a particular cosmological scenario (e.g., in the flat ΛCDM model, we have $h(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$, where Ω_m represents the present value of the matter energy density of the Universe as a fraction of its critical density). Thus, we expect the GW signal to be detected with an extremely small delay compared to its EM counterpart, provided that both are emitted without an intrinsic time-lag at the source (see detailed discussion in [1]). This requirement seriously limits the usefulness of the above method.

3. Constraints on Graviton Mass with GW Signals

One may easily notice that Equation (3) leads to GW speed being a function of energy or, equivalently, of the frequencyof the GW signal associated with the massive graviton—see Equation (4) where $p_r^2 = a^2(t_e)(E^2/c^2 - m_g^2 c^2)$; detailed calculations and discussion can be found in [1,34]. This opens up the possibility of using the deformed dispersion relation for the graviton, given by Equation (3), in a manner that is different from that proposed in the previous section (i.e., Section 2), allowing one to overcome the problem associated with the unknown intrinsic time lag between GWs and EM waves emitted from the same source—a major obstacle when one wants to think of constraining graviton mass via the time-of-flight method. In this case, a low-frequency GW signal (corresponding to low-energy gravitons) should travel slightly slower compared to a high-frequency one (related to high-energy gravitons). Low-frequency GW signals from double compact objects are detected earlier than those of higher frequencies, forming a characteristic time structure, the so-called 'chirp'. When the graviton is massive, this may cause a tiny squeezing of the GW pattern in contrast to the standard case, which may translate into a change in the phase of the observed GW waveform [34]. This observation was used by the LIGO Team to achieve limits on graviton mass as early as the first ever laboratory detection of GW signals via interferometric detectors, assigned as GW150914 [36]. They obtained a very strong upper bound on graviton mass of $m_g \leq 1.2 \times 10^{-22}$ eV$/c^2$ at a 90% confidence level [37], which was subsequently confirmed by another group [50]. More recently, we have entered a new era in which GW astronomy can be treated as a powerful tool not only in astrophysics (see, e.g., [51]; note also that LIGO/Virgo findings have proven the real existence of double black holes in nature, which before GW150914 were a matter of many debates), but also for cosmological investigations (i.e., the use of GW signals as standard sirens in a compatible way to other distance measurements based on the conception of standard candles and standard rulers; see [52–54]). We are now in the time of successful operating runs of LIGO/Virgo GW detectors, recently joined by the KAGRA observatory, resulting in numerous records of GW signals from coalescing double compact objects. A recent statistical analysis of 24 GW events by the LIGO/Virgo Collaboration shifted the limit on the graviton's mass by one order of magnitude (i.e., $m_g \leq 1.76 \times 10^{-23}$ eV$/c^2$ at a 90% confidence level) [38].

Although at first glance, results suggest that GWs propagate without dispersion (which is equivalent to saying that the graviton, if it exists, is massless), there is still room for exploration. Specifically, we have the following bound on the Compton wavelength $\lambda_g = \frac{h}{m_g c}$ of the massive graviton (see [34] and comprehensive discussion in [20]):

$$\lambda_g > 5 \cdot 10^{11}\mathrm{km}\left(\frac{r_0}{200\mathrm{Mpc}}\frac{100\mathrm{Hz}}{f}\,\rho\right)^{1/2}, \tag{6}$$

where r_0 is a characteristic distance parameter (i.e., a distance which can be probed by a given detector with a given sensitivity) and f and ρ are, respectively, the frequency and the signal-to-noise (S/N) ratio of the GW signal. Hence, this bound can be extended with the use of more sensitive detectors probing a larger volume of the Universe. In particular, the new generation of GW detectors, such as the Einstein Telescope (ET; [55]), the Laser Interferometer Space Antenna (LISA; [56]) and the DECihertz Interferometer Gravitational wave Observatory (DECIGO; [57–59]), are extremely promising in this context. For example, ET polynomial approximation to the noise curve in its initial configuration gives $r_0 = 1527$ Mpc, which may be increased to $r_0 = 1918$ Mpc when the advanced 'xylophone' configuration is realized [55]. The noise spectrum of DECIGO [60] (and the smaller-scale project B-DECIGO [61,62]) would result in $r_0 = 6709$ Mpc ($r_0 = 535$ Mpc for B-DECIGO)—a volume that is about 64 times larger than that of ET. Such greatly improved sensitivity (for comparison, the range of aLIGO is of the order of 1 Gpc for double black hole systems and about 70–80 Mpc for binary neutron stars [63]) will be achieved through the building of planned detectors on a triangular basis (a single detector unit with a triangular geometry is equivalent to two standard L-shaped interferometers rotated by 45° [60]). Moreover, ET is planned to be built underground, and LISA and DECIGO in space, which will significantly reduce (or even remove in the case of space-borne interferometers) seismic noise, thereby increasing the capabilities of these detectors for the registration of GW signals at lower frequencies (lower than about 1 Hz). This, in turn, will allow multi-frequency GW astronomy to come into play: double compact objects might be observed in the inspiralling phase for weeks up to years before coalescence (see discussion in, e.g., [62]). In particular, DECIGO is planned to work at the mHz frequency range, filling the gap between the low-frequency LISA and the high-frequency LIGO/Virgo and KAGRA observations, which is especially important in the light of the method discussed in the next section (Section 4).

4. Gravitational Lensing of GWs for Graviton Mass Estimates

Predictions for the detection of GW signals within ET, LISA and DECIGO/B-DECIGO projects allow us to expect rich statistics from such events in the future, making it probable that many of them will undergo the phenomenon of gravitational lensing (see, e.g., [64–66]). Gravitational lensing of GWs is expected in the light of the similarity between the mathematical formalism of GWs and EM waves, i.e., when the Einstein field equations in terms of metric perturbations within the weak field regime are compared to EM field equations in terms of the Lorentz gauge. Thus, the propagation of GWs and EM waves is similar, particularly within the geometric-optics regime (GWs also travel along null geodesics). Light emitted from a distant astrophysical source, when deflected by the presence of a massive object (the lens), will produce multiple, magnified and distorted images of the source. This phenomenon is known as strong gravitational lensing, in opposition to weak lensing and microlensing, both of which are revealed in different source-lens-observer configurations and time scales [67]. The typical image separation is set by the so-called Einstein radius θ_E, given within the singular isothermal sphere (SIS) model of the mass distribution in the lensing galaxy (strongly supported by galaxy lensing studies,according to which the population of lenses consists of massive ellipticals [15]) according to the following simple formula (e.g., [67,68]):

$$\theta_E = 4\pi \frac{\sigma_v^2}{c^2} \frac{D_{ls}}{D_s}, \tag{7}$$

where σ_v^2 is the one-dimensional velocity dispersion of stars in the lensing galaxy and D_s and D_{ls} are, respectively, the distance from the observer to the source and between the lens and the source. In this case, we observe two images (of different magnifications: μ_+ and μ_-) formed on the opposite sides of the lens with angular separation $\theta_\pm = \beta \pm \theta_E$, where β is the angular position of the source with respect to the optical axis. The characteristic property of strong gravitational lensing is the occurrence of a time delay between images as a direct consequence of two factors: a geometrical one (light rays from different images travel along paths of different lengths) and the Shapiro delay caused by the gravitational potential of the lens [67,68]. The formula for the lensing time delay in the SIS model as follows: [68]:

$$\Delta t_{SIS,EM} = \frac{1+z_l}{2c} \frac{D_l D_s}{D_{ls}} \left(\theta_+^2 - \theta_-^2\right), \tag{8}$$

which can then be converted to

$$\Delta t_{SIS,EM} = \frac{2(1+z_l)}{c}\frac{D_l D_s}{D_{ls}}\theta_E \beta, \tag{9}$$

revealing an explicit dependence of the lensing time delay on the Einstein radius θ_E. In Equations (8) and (9) z_l is the redshift of the lens. Similarly to gravitationally lensed EM signals, strong lensing applied to GWs (contemplated as previously within the SIS model) would result in two time-delayed waveforms with different magnifications (i.e., of different amplitudes) and the same 'chirp' structure (i.e., frequency drift in time) [64–66,69]. Predictions concerning the magnitude of the time delay between lensed GW signals are of the order of a few seconds for ground-based GW detectors, up to even a few months for space-borne GW interferometers [53]. This shows how simultaneous multi-frequency observations are important for GW astronomy: they would enhance our identification ability for lensed GW signals for a long time before double compact objects enter the merger stage, thus allowing for increased capabilities in the detection of lensed transient events in different messenger windows (e.g., in GWs and EM waves).

The Einstein radius of lensed GW signals in the standard case of a massless graviton follows the same formula as that for light (i.e., θ_E given by Equation (7)). If the graviton becomes massive, this would no longer be the case. One should rather use a new form for image separation [70], which, in the SIS model, can be expressed as:

$$\theta_{E,GW} = \theta_E \left(1 + \frac{m_g^2 c^4}{2E^2}\right). \tag{10}$$

As a result, the time delay between lensed images of a given source producing both GW and EM signals (e.g., double compact objects at the time of coalescence, specifically, a binary neutron star merger) would be different depending on whether the source is measured in the GW or EM window [1]:

$$\Delta t_{SIS,EM} - \Delta t_{SIS,GW} = \Delta t_{SIS,EM}\frac{m_g^2 c^4}{2E^2} F_{lens}(z_l, z_s), \tag{11}$$

where $F_{lens}(z_l, z_s)$ is a minor ($\sim O(1)$) factor, weakly depending on the particular lens model and cosmological scenario. From Equation (11), it is clear that strongly lensed GWs may be used as a alternative tool to obtain the upper limit on the graviton mass:

$$\frac{m_g^2 c^4}{2E^2} \leq \frac{\delta T}{\Delta t_{EM} F_l}, \tag{12}$$

with δT being the timing accuracy of the determination of time delays (see our paper [1] for more details and discussion). Although this method is less restrictive than the travel time technique described in Section 2 (the cumulation process of tiny effects take place on the distance between the lens and the observer, not on the whole distance to the source), it has, however, several advantages. First, it is based on a phenomenological approach anchored on the modification of the dispersion relation and is thus independent of a particular quantum gravity model. Second, as the time delay is produced near the lens plane, which is rather close to the observer (compared to the distance between the source and the observer or other cosmological distances), the method also does not depend strongly on the background cosmological model. Finally, thanks to its differential nature, it is absolutely free from any intrinsic time lags between EM and GW signal emissions originating in the source.

What would make our method useful is its ability to detect gravitationally lensed high-energy transient sources such as gamma-ray bursts or supernovae in parallel in both GW and EM windows. The possibility of the gravitational lensing of supernovae and its use as a tool for cosmology was originally discussed by Sjur Refsdal in [71], but with the lack of observational confirmation, this remained for a long time in the area of theoretical speculations. Now, the situation has changed, with successful lensing surveys resulting in rich catalogs of strong lensing systems (see, e.g., [72–74] and references therein). As has been expected, for the majority of these cases, the lens is an elliptical galaxy (early-type galaxies are the most massive ones and thus dominate the statistics of the lensing phenomenon), supporting the use of the SIS model, as discussed above. The first observation of lensed supernova [75] showed us that this is only the tip of the iceberg and shifted the idea of Refsdal into a realistic one. On the other hand, the first detection of a binary neutron star merger, assigned as GW170817, accompanied by the short gamma-ray burst signal GRB 170817A (and its afterglow followed up at other wavelengths) identified as its EM counterpart, leads us to believe that our method discussed here will be useful in the near future. This is especially true in light of the future GW detectors such as ET and DECIGO, combined with the forthcoming surveys such as LSST (Legacy Survey of Space and Time) at Vera C. Rubin Observatory [76] or Joint Dark Energy Mission (JDEM, [77]), as well as impressive planned missions such as Transient High-Energy Sky and Early Universe Surveyor (THESEUS, [78]).

5. Graviton Mass from Dynamical Properties of Galaxy Clusters

The existence of a massive graviton may be associated with the weakening of gravitational interaction at large distances due to a Yukawa-like potential (see, e.g., [20,34]):

$$g_Y(r) = \frac{GM}{r} \exp\left[-\frac{r}{\lambda_g}\right]\left(\frac{1}{\lambda_g} + \frac{1}{r}\right), \tag{13}$$

which recovers the classical Newtonian gravitational potential

$$g(r) = \frac{GM}{r^2} \tag{14}$$

as $m_g \to 0$ (i.e., $\lambda_g \to \infty$). Because at small scales (e.g., the scale of the Solar System) the departure from the Newtonian case (Equation (14)), if it really exists, should be extremely tiny, one needs galactic and extragalactic scales to use Equation (13) in the testing of non-standard effects related with the graviton's mass. For example, the bounds of the solar system yield $m_g < 10^{-23}$ eV [34] and the recent limit of $m_g < 2.9 \times 10^{-21}$ eV has been derived for the S2 star orbit at the Galactic Center as a result of comparison between observational data and simulations [35]. Another idea is to use galaxy clusters, which are actually the largest gravitationally bound structures in the Universe. Cluster dynamical properties may be used in the method based on the Yukawa potential via replacing the mass M in Equations (14) and (13), respectively, by the total mass of a galaxy cluster measured within a given radius r in standard massless graviton physics (let us assign it as $M_{\rm tot}(< r)$) and when the massive graviton scenario is imposed (marked as $M^Y_{\rm tot}(< r)$). This, in practice, brings the whole issue down to a comparison between these two masses (i.e., $M_{\rm tot}(< r)$ vs. $M^Y_{\rm tot}(< r)$). To see how the existence of massive graviton modifies cluster mass estimates, it is essential to know how such masses can be measured. This can be accomplished indirectly through the projected (i.e., integrated along the line-of-sight) X-ray surface brightness over a given frequency band [79–81]. Clusters contain baryons, mainly in the form of the hot intracluster medium (galaxies account only for a small percentage of the cluster mass, and the rest goes to the mysterious dark part of it) with temperatures in the range of 20–100$\times 10^6$K, determined by the gravitational potential well depth. This temperature is sufficient for intracluster medium electrons to radiate in X-rays via bremsstrahlung, making the whole cluster perfectly visible at these frequencies. In particular, by measuring X-ray surface brightness, one is able to reconstruct intracluster medium temperature $T_{\rm gas}$ and density $n_{\rm gas}$ profiles within a given radius beyond the core radius ($r < r_c$) of the cluster, and in turn—its total mass $M_{\rm tot}$. The assumption here is that intracluster medium follows a spherical symmetry distribution and, as a perfect gas with the well-known equation of state

$$P_{\text{gas}} = n_{\text{gas}} k_B T_{\text{gas}} = \frac{\rho_{\text{gas}}(r)}{\mu m_u} k_B T_{\text{gas}}, \tag{15}$$

where $\mu \sim 0.6$ is the mean molecular weight (in atomic mass unit $m_u = 1.66 \times 10^{-24}$ g) for ionized plasma [81,82], remains in a hydrostatic equilibrium with the cluster gravitational potential

$$\frac{dP_{\text{gas}}}{dr} = -\rho_{\text{gas}}(r) g(r). \tag{16}$$

Putting the standard formula for Newtonian gravitational potential given by Equation (14) into Equation (16), one may obtain the total mass of the cluster (within a radius r):

$$M_{\text{tot}}(< r) = -\frac{k_B}{\mu m_u} \frac{r T_{\text{gas}}(r)}{G} \left(\frac{d \ln \rho_{\text{gas}}(r)}{d \ln r} + \frac{d \ln T_{\text{gas}}(r)}{d \ln r} \right). \tag{17}$$

From this formula, it is clear that the temperature and density profile measurements are crucial for the hydrostatic mass determination of galaxy clusters. On the other hand, using the non-standard Yukawa potential (Equation (13)) instead of the Newtonian one (Equation (14)) in Equation (16) to represente hydrostatic equilibrium, the formula for the dynamical total mass of the cluster would be

$$M^{Y}_{\text{tot}}(< r) = M_{\text{tot}}(< r) \exp\left[\frac{r}{\lambda_g}\right] \left(\frac{\lambda_g}{r + \lambda_g} \right), \tag{18}$$

where $M_{\text{tot}}(< r)$ is the total cluster mass given by the Equation (17). Thus, the comparison between cluster total masses—M^{obs}_{tot} obtained observationally within the standard case of massless graviton (Equation (17))—with those calculated for the massive graviton scenario ($M^{Y}_{\text{tot}}{}^{obs}$; Equation (18)) in the presence of the Yukawa potential (Equation (13))—would lead to the lower limit on the Compton wavelength λ_g (which can then be translated easily into the upper limit on the graviton mass m_g) obtained via the minimization procedure of the χ^2 objective function:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{M^{Y}_{\text{tot},i}(r; \lambda_g) - M^{obs}_{\text{tot},i}(r)}{\sigma_{M^{obs}_{\text{tot},i}}} \right)^2, \tag{19}$$

where summation is over the size of the sample used (see [2] for more details). We applied this method in [2] to the data obtained within the XMM-Newton Cluster Outskirts Project (X-COP) [82] on the basis of the joint analysis of XMM-Newton observations in the X-rays and the Sunyaev–Zel'dovich effect measured by the Planck satellite. Cluster mass profiles reconstructed alone (i.e., without auxiliary data) are biased by many systematic effects such as the presence of gas inhomogeneities, which affect gas density measurements, or X-ray background contamination reducing signal-to-noise ratio [82]. The thermal Sunyaev–Zel'dovich effect is a direct consequence of the inverse Compton scattering when the low-energy photons of the cosmic microwave background (CMB) interact with the high-energy electrons present in the intracluster medium; the latter lose energy to the CMB photons, changing slightly the CMB blackbody spectrum (i.e., the apparent change in the CMB temperature distribution $\Delta T_{\text{CMB}} / T_{\text{CMB}}$) in a very characteristic way [83,84]. Due to its specificity, this tiny effect can be measured with a high degree of accuracy, leading to improved precision in the cluster data. In particular, the thermal Sunyaev–Zel'dovich effect allows one to probe cluster outskirts in a perfectly complementary way to X-ray observations, which rather probe the denser regions of the intracluster medium. This property is particularly valuable for cluster mass estimation (see the Equation (17)): the apparent change in the CMB temperature distribution caused by the thermal Sunyaev–

Zel'dovich effect depends on the so-called (dimensionless) Compton parameter given by the formula:

$$y = \frac{\sigma_T k_B}{m_e c^2} \int T_e(l) n_e(l) \mathrm{d}l, \tag{20}$$

which, integrated along the line of sight, may help to recover the pressure profile of the electrons in the intracluster medium under the perfect gas assumption (see Equation (15)):

$$y = \frac{\sigma_T k_B}{m_e c^2} \int T_e(l) n_e(l) \mathrm{d}l = \frac{\sigma_T}{m_e c^2} \int P_e(l) \mathrm{d}l. \tag{21}$$

Here, σ_T is the cross-section for Thomson scattering (see [85,86] for more details). The X-COP sample contains 12 low-redshift ($0.04 < z < 0.1$) massive galaxy clusters for which hydrostatic mass profiles, corrected for a non-thermal contribution to intracluster medium pressure [87], was recovered with an unprecedented accuracy (relative median error of 3% at R_{500} and 6% at R_{200} (see detailed discussion in [82]). The resulting bound is $m_g < (3.49 - 4.77) \times 10^{-29}$ eV at 68% confidence level and $m_g < (4.99 - 6.99) \times 10^{-29}$ eV at 95% confidence level (see, Table 1 of [2]), which is one of the strongest results compared to other dynamical tests of Yukawa terms applied for galaxy clusters (see, e.g., [29–32]).

6. Summary and Discussion of Perspectives

An effective phenomenology based on the modification of relativistic dispersion relations for massive gravitons according to Equation (3) may help to provide constraints on its mass. This can be achieved via a careful GW signal analysis: non-zero graviton mass may cause shape distortion of the observed GW waveform (see Section 3). On the other hand, tiny deviations from the standard case (i.e., massless graviton physics) influence the travel time of the GW signal, making it slightly different when compared to, e.g., the time of flight of the photons emitted from the same source at the same time (Section 2). In particular, the comparison of time delays between images of strongly lensed GW and EM signals from a binary neutron star coalescence has great potential in limiting graviton mass regardless of a particular massive gravity model, background cosmology and intrinsic time-lag problems, as has been discussed in Section 4. This method is especially attractive from the perspective of the planned third generation of GW detectors such as ET and future cosmic interferometers such as LISA or DECIGO/B-DECIGO. Predictions concerning the expected yearly rates at which such detectors would be able to observe double compact object sources are of the order of $10^4 - 10^6$ for ET, $10^2 - 10^6$ for DECIGO and $10^3 - 10^5$ for the smaller-scale B-DECIGO project, depending on a specific type of double compact object system, population synthesis scenario and galaxy metallicity evolution (see Tables 1 and 2 of [65], collecting double compact object detection rates for ET and Table 2 of [66] for DECIGO/B-DECIGO). With the assumption that the GW source would be detectable without lensing (i.e., both images will produce signals that can be registered above the detector threshold) and taking into account the planned finite operation time of the survey (particularly, the first 1, 5 and 10 years of ET and the 4 years of DECIGO's nominal duration), the expected yearly detection rates of gravitationally lensed GW signals are promising: ET would be able to detect $50 - 100$ lensed GW events per year; DECIGO/B-DECIGO could register about 50 lensed GW events each year (such predictions also depend on the type of double compact object system, the binary evolution scenario and the galaxy metallicities; more details can be found in [66]). These would be mainly binary black hole systems, especially in the DECIGO/B-DECIGO case, in which the contamination of unresolved double compact object systems dramatically lowers the ability for the detection of the lensed binary neutron star or mixed, neutron star-black hole coalescences (see results reported in Tables 3 and 4 of [65] for ET and Table 3 of [66] for DECIGO/B-DECIGO). From the perspective of the possible application of the method based on time delay difference between GW and EM signals from double compact object mergers as a tool for graviton mass estimates (Section 4), gravitationally lensed binary neutron star mergers, which are expected to produce transient EM signals observable as short gamma-ray bursts, are required. The first evidence for a coalescing binary neutron star system (assigned as GW170817 [88]) has been announced along with the identification of its electromagnetic counterpart at different wavelengths [89], thereby shifting the above proposal towards a more realistic one, especially in the ET era, when we should be able to register a few of these events per year [64,65,69,90]. Another advantage is that at the operating time of the ET and DECIGO projects, GW emissions associated with a neutron star binary system would be registered a long time before its coalescence becomes visible in the EM window as a prompt (short) gamma-ray burst signal, thus translating into better timing accuracy and thus better precision in our graviton mass estimates. An alternative to methods based on modified dispersion relations for massive gravitons could be dynamical tests searching for deviations (compared to the standard case) in galaxy cluster total mass estimates caused by a non-zero graviton mass within the Yukawa potential (Equation (13); Section 5), which gives one of the most stringent bounds on graviton mass, of the order of $(10^{-29} - 10^{-30})$ eV [2]. In practice (see Section 5), this method is based on the comparison of the cluster masses calculated for the massive graviton scenario under the assumption that the intracluster medium follows a perfect gas equation of state, has spherical symmetry, and remains in hydrostatic equilibrium with the gravitational potential of the cluster to those, obtained observationally for the standard massless graviton scenario, and with the same assumption made in relation to hydrostatic equilibrium, invoking the Navarro–Frenk–White profile, and thus being a reasonable proposal for the distribution of dark matter within a cluster of galaxies [91,92] (see also discussion in [82]). In this light, current and planned X-ray missions (e.g., eRosita and Athena) seem to be particularly promising. Thanks to their upgraded technology, they would result in data on hundreds of thousands of galaxy clusters, obtained with impressive angular and spectral resolution [93–95], leading to accurate total mass estimates and thus translating into an improvement of the existing limits on graviton mass and supporting tests anchored within the GW domain.

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