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# THE EFFECTIVE MLRSM MODEL AND LOW-ENERGY OBSERVABLES*

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We consider an effective Minimal Left–Right Symmetric Model. Instead of engaging all the dimension-6 operators, we choose only the relevant, in the limit that right-handed symmetry breaking scale is larger than the electroweak one, operators belonging to $\phi^2 X^2$ class. We adjudge the impact of those operators on the spectrum and vertices of the model, and then capture their contributions to the low-energy observables, *e.g.*, $\rho$, $\mathcal{G}_{\mathrm{F}}$, $\Theta_W$, and oblique parameters $S$, $T$, $U$.

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## 1. Introduction

We expect the discovery of new particles in the collider experiments in near future. This will inevitably lead us to access energy scales where new particles and new degrees of freedom (DOF) will be produced on-shell or indirectly seen in the precision studies.

There are several options being considered for future electron colliders [1], namely, FCC (Future Circular Collider) [2, 3], CLIC (Compact Linear Collider) [4, 5] — both at CERN, the ILC (International Linear Collider) [6–8]. The CEPC (Chinese Electron Positron Collider) [9, 10] in China is of the circular type and similarly to FCC is expected to collide electrons with positrons at 90–365 GeV centre-of-mass energies. The ILC collider planned in Japan could reach centre-of-mass collision energies of 1 TeV, while CLIC would cover the energies between 380 GeV and 3 TeV. In the case of FCC-ee, four running stages are considered [2, 11, 12], with a focus on $Z$, $W$, $H$, and top-quark production. With new colliders and physics opportunities at hand, it may happen that to capture new physics, the Standard Model

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Effective Field Theory (SMEFT) will not be sufficient. It will become pertinent that we extend the set of lighter DOFs beyond the SM ones by adding these newly discovered particles. This will also bring into picture new effective operators along with SMEFT ones. These new operators will also provide contributions to the low-energy observables, and thus it will be necessary to recompute these observables extending the SM electroweak pseudo-observable studies [13–15].

In this work, we consider the Minimal Left–Right Symmetric Model (MLRSM) itself as an effective theory, and compute the complete and independent set of dimension-6 effective operators using GrIP [16]. Then we invoke the following sequence of spontaneous symmetry breaking $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \to \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{em}}$. We compute the modified gauge boson spectrum and the modified interaction vertices after incorporating the dimension-6 operators.

In Section 2, we outline the full MLRSM Lagrangian highlighting its rich scalar sector, describe the pattern of symmetry breaking, and elucidate the computation of tree-level masses of the gauge bosons. This is followed by a discussion on dimension-6 operators and the field re-definitions necessitated by their incorporation in Section 3. Finally, in Section 4, we shed light on the procedure for recomputing low-energy observables once dimension-6 operators have been taken into account.

## 2. The MLRSM Lagrangian and model parameters

### *2.1. Model details*

The renormalizable Lagrangian for the Minimal Left–Right Symmetric Model (MLRSM) can be divided into the kinetic, scalar-potential, and Yukawa sectors [17–22]

$$\begin{aligned}\mathcal{L}_{\mathrm{kin}} &= -\frac{1}{4}\mathrm{Tr}[G^{\mu\nu}G_{\mu\nu}] - \frac{1}{4}\mathrm{Tr}[W_L^{\mu\nu}W_{\mu\nu,L}] - \frac{1}{4}\mathrm{Tr}[W_R^{\mu\nu}W_{\mu\nu,R}] - \frac{1}{4}(B^{\mu\nu}B_{\mu\nu}) \\ &+\mathrm{Tr}\left[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)\right] + \mathrm{Tr}\left[(D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)\right] + \mathrm{Tr}\left[(D_\mu\Phi)^\dagger(D^\mu\Phi)\right] \\ &+\bar{L}_L i\not{D} L_L + \bar{L}_R i\not{D} L_R + \bar{Q}_L i\not{D} Q_L + \bar{Q}_R i\not{D} Q_R\,. \end{aligned} \tag{1}$$

The internal symmetry group is $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ and the corresponding field strength tensors are following [21, 22]:

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f^{ABC} G^B_\mu G^C_\nu\,, \\ W^I_{L,\mu\nu} &= \partial_\mu W^I_{L,\nu} - \partial_\nu W^I_{L,\mu} + g_L \epsilon^{IJK} W^J_{L,\mu} W^K_{L,\nu}\,, \\ W^I_{R,\mu\nu} &= \partial_\mu W^I_{R,\nu} - \partial_\nu W^I_{R,\mu} + g_R \epsilon^{IJK} W^J_{R,\mu} W^K_{R,\nu}\,, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\,. \end{aligned}$$

The fermion sector includes leptons and quarks (SU(3) triplets). These can be further subdivided into left- and right-handed fields, where the former transform as doublets under $\mathrm{SU}(2)_L$, while the latter transform as doublets under $\mathrm{SU}(2)_R$. Each of these fields has non-trivial $\mathrm{U}(1)_{B-L}$ charges. Based on this, their covariant derivatives assume the following form [21, 22]:

$$D^\mu L_{L,R} = \left(\partial^\mu - ig_{L,R}\frac{\tau^I}{2}W^{I,\mu}_{L,R} - i\tilde{g}\frac{Y}{2}B^\mu\right)L_{L,R}, \quad (2)$$

$$D^\mu Q_{L,R} = \left(\partial^\mu - ig_3\frac{T^A}{2}G^A_\mu - ig_{L,R}\frac{\tau^I}{2}W^{I,\mu}_{L,R} - i\tilde{g}\frac{Y}{2}B^\mu\right)Q_{L,R}\,. \quad (3)$$

The scalar sector consists of a bi-doublet $\Phi$, (transforms as a doublet under $\mathrm{SU}(2)_L$ as well as $\mathrm{SU}(2)_R$), an $\mathrm{SU}(2)_L$ triplet $\Delta_L$, and an $\mathrm{SU}(2)_R$ triplet $\Delta_R$. The bi-doublet transforms trivially under $\mathrm{U}(1)_{B-L}$, whereas each of the triplets has 2 units of charge. The corresponding covariant derivatives are given as [21, 22]

$$\begin{aligned} D^\mu\Phi &= \partial^\mu\Phi - ig_L W^{I,\mu}_L\frac{\tau^I}{2}\Phi + ig_R\Phi\frac{\tau^I}{2}W^{I,\mu}_R, \\ D^\mu\Delta_{L,R} &= \partial^\mu\Delta_{L,R} - ig_{L,R}\left[\frac{\tau^I}{2}W^{I,\mu}_{L,R}, \Delta_{L,R}\right] - i\tilde{g}B^\mu\Delta_{L,R}\,. \end{aligned} \quad (4)$$

The scalar potential has a rich structure which is highlighted below

$$\begin{aligned} &V(\Delta_L,\Delta_R,\Phi) = -\mu_1^2\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right) - \mu_2^2\left(\mathrm{Tr}\left[\tilde{\Phi}\Phi^\dagger\right] + \mathrm{Tr}\left[\tilde{\Phi}^\dagger\Phi\right]\right) \\ &-\mu_3^2\left(\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right] + \mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right]\right) + \lambda_1\mathrm{Tr}\left[\Phi\Phi^\dagger\right]^2 \\ &+\lambda_2\mathrm{Tr}\left[\tilde{\Phi}\Phi^\dagger\right]^2 + \lambda_3\mathrm{Tr}\left[\tilde{\Phi}\Phi^\dagger\right]\mathrm{Tr}\left[\tilde{\Phi}^\dagger\Phi\right] + \lambda_2\mathrm{Tr}\left[\tilde{\Phi}^\dagger\Phi\right]^2 \\ &+\lambda_4\mathrm{Tr}\left[\Phi\Phi^\dagger\right]\left(\mathrm{Tr}\left[\tilde{\Phi}\Phi^\dagger\right] + \mathrm{Tr}\left[\tilde{\Phi}^\dagger\Phi\right]\right) \\ &+\rho_1\left(\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right]^2 + \mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right]^2\right) \\ &+\rho_2\left(\mathrm{Tr}\left[\Delta_R\Delta_R\right]\mathrm{Tr}\left[\left(\Delta_R\right)^\dagger\left(\Delta_R\right)^\dagger\right] + \mathrm{Tr}\left[\Delta_L\Delta_L\right]\mathrm{Tr}\left[\left(\Delta_L\right)^\dagger\left(\Delta_L\right)^\dagger\right]\right) \\ &+\rho_3\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right]\mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right] \\ &+\rho_4\left(\mathrm{Tr}\left[\Delta_R\Delta_R\right]\mathrm{Tr}\left[\left(\Delta_L\right)^\dagger\left(\Delta_L\right)^\dagger\right] + \mathrm{Tr}\left[\Delta_L\Delta_L\right]\mathrm{Tr}\left[\left(\Delta_R\right)^\dagger\left(\Delta_R\right)^\dagger\right]\right) \\ &+\alpha_1\left(\mathrm{Tr}\left[\Phi\Phi^\dagger\right]\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right] + \mathrm{Tr}\left[\Phi\Phi^\dagger\right]\mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right]\right) \\ &+\alpha_2\left(\mathrm{Tr}\left[\Phi\tilde{\Phi}^\dagger\right]\mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right] + \mathrm{Tr}\left[\Phi^\dagger\tilde{\Phi}\right]\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right]\right. \end{aligned}$$

$$
\begin{aligned}
&+\mathrm{Tr}\left[\Phi^\dagger\tilde{\Phi}\right]\mathrm{Tr}\left[\Delta_R\left(\Delta_R\right)^\dagger\right]+\mathrm{Tr}\left[\tilde{\Phi}^\dagger\Phi\right]\mathrm{Tr}\left[\Delta_L\left(\Delta_L\right)^\dagger\right]\Big)\\
&+\alpha_3\left(\mathrm{Tr}\left[\Phi\Phi^\dagger\Delta_L\left(\Delta_L\right)^\dagger\right]+\mathrm{Tr}\left[\Phi^\dagger\Phi\Delta_R\left(\Delta_R\right)^\dagger\right]\right)\\
&+\beta_1\left(\mathrm{Tr}\left[\Phi\Delta_R\Phi^\dagger\left(\Delta_L\right)^\dagger\right]+\mathrm{Tr}\left[\Phi^\dagger\Delta_L\Phi\left(\Delta_R\right)^\dagger\right]\right)\\
&+\beta_2\left(\mathrm{Tr}\left[\tilde{\Phi}\Delta_R\Phi^\dagger\left(\Delta_L\right)^\dagger\right]+\mathrm{Tr}\left[\tilde{\Phi}^\dagger\Delta_L\Phi\left(\Delta_R\right)^\dagger\right]\right)\\
&+\beta_3\left(\mathrm{Tr}\left[\Phi^\dagger\Delta_L\tilde{\Phi}\left(\Delta_R\right)^\dagger\right]+\mathrm{Tr}\left[\Phi\Delta_R\tilde{\Phi}^\dagger\left(\Delta_L\right)^\dagger\right]\right)\,. \qquad (5)
\end{aligned}
$$

The Yukawa sector describes interaction between the fermion and scalar sectors. It consists of both the Dirac-like and Majorana-like terms. It is interesting to note that there is no interaction between the quarks and the triplet scalars

$$
\begin{aligned}
\mathcal{L}_{\mathrm{Y}} &= -\left\{y_{\mathrm{D}}\ \bar{L}_L\Phi L_R+\tilde{y}_{\mathrm{D}}\ \bar{L}_L\tilde{\Phi}L_R+\mathrm{h.c.}\right\}\\
&\quad-\left\{y_q\bar{Q}_L\Phi Q_R+\tilde{y}_q\bar{Q}_L\tilde{\Phi}Q_R+\mathrm{h.c.}\right\}\\
&\quad-y_{\mathrm{M}}\left\{L_L^TCi\tau_2\Delta_L L_L+\ L_R^TCi\tau_2\Delta_R L_R+\mathrm{h.c.}\right\}. \qquad (6)
\end{aligned}
$$

### *2.2. Symmetry breaking in MLRSM*

Symmetry breaking occurs in two steps with $\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\to\mathrm{U}(1)_Y$ occurring first, followed by $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\to\mathrm{U}(1)_{\mathrm{em}}$. During the course of these events, the neutral scalars encapsulated within the multiplets acquire vacuum expectation values (vevs) and fluctuations around the vevs can be represented as

$$
\begin{aligned}
\Phi &= \begin{pmatrix}(\kappa_1+h_1+ia_1)/\sqrt{2} & \phi_1^+\\ \phi_2^- & (\kappa_2+h_2+ia_2)/\sqrt{2}\end{pmatrix},\\
\Delta_{L,R} &= \begin{pmatrix}\delta^+/\sqrt{2} & \delta^{++}\\ \left(v+\delta^{0r}+i\delta^{0i}\right)/\sqrt{2} & -\delta^+/\sqrt{2}\end{pmatrix}_{L,R}. \qquad (7)
\end{aligned}
$$

### *2.3. Gauge boson mass spectrum*

The charged and neutral gauge boson mass matrices have the forms

$$
\tilde{m}_W^2 = \frac{g^2}{4}\begin{pmatrix}\kappa_+^2+2v_L^2 & -2\kappa_1\kappa_2\\ -2\kappa_1\kappa_2 & \kappa_+^2+2v_R^2\end{pmatrix}, \qquad (8)
$$

$$
\tilde{m}_0^2 = \frac{1}{2}\begin{pmatrix}\frac{1}{2}g^2\left(\kappa_+^2+4v_L^2\right) & -\frac{1}{2}g^2\kappa_+^2 & -2g\tilde{g}v_L^2\\ -\frac{1}{2}g^2\kappa_+^2 & \frac{1}{2}g^2\left(\kappa_+^2+4v_L^2\right) & -2g\tilde{g}v_R^2\\ -2g\tilde{g}v_L^2 & -2g\tilde{g}v_R^2 & 2\tilde{g}^2\left(v_L^2+v_R^2\right)\end{pmatrix}. \qquad (9)
$$

Here, $\kappa_\pm^2 = \kappa_1^2 \pm \kappa_2^2$. To find the physical basis corresponding to $\mathrm{SU}(2)_{L,R}$, $\mathrm{U}(1)_{B-L}$ gauge bosons, we need to diagonalize the mass matrices. The following rotation matrices connect the physical (charged: $W_1^{\pm\mu}, W_2^{\pm\mu}$, neutral: $Z_1^\mu, Z_2^\mu, A^\mu$), and unphysical basis of gauge bosons

$$\begin{pmatrix} W_L^{\pm\mu} \\ W_R^{\pm\mu} \end{pmatrix} = \mathcal{R}_a \begin{pmatrix} W_1^{\pm\mu} \\ W_2^{\pm\mu} \end{pmatrix}, \qquad \begin{pmatrix} W_{3L}^\mu \\ W_{3R}^\mu \\ B^\mu \end{pmatrix} = \mathcal{R}_b \begin{pmatrix} Z_1^\mu \\ Z_2^\mu \\ A^\mu \end{pmatrix}. \tag{10}$$

Here, $W_{L,R}^{\pm\mu} = \frac{1}{\sqrt{2}} \left( W_{L,R}^{1\mu} \mp i W_{L,R}^{2\mu} \right)$. The rotation matrices are given as

$$\mathcal{R}_a = \begin{pmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{pmatrix},$$
$$\mathcal{R}_b = \begin{pmatrix} \cos\theta_W \cos\theta_2 & \cos\theta_W \sin\theta_2 & \sin\theta_W \\ -\sin\theta_W \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2 & -\sin\theta_W \sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 & \cos\theta_W \sin\theta_1 \\ -\sin\theta_W \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 & -\sin\theta_W \cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2 & \cos\theta_W \cos\theta_1 \end{pmatrix}. \tag{11}$$

The angles $\xi$, $\theta_W$, $\theta_1$, and $\theta_2$ can be described in terms of the model parameters as

$$\cos\theta_1 = \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}, \qquad \sin\theta_1 = \tan\theta_W, \qquad g = \frac{e}{\sin\theta_W},$$
$$\tilde{g} = \frac{e}{\sqrt{\cos 2\theta_W}}, \qquad \tan 2\xi = \frac{-2\kappa_1\kappa_2}{v_R^2 - v_L^2}, \qquad \tan 2\theta_2 = \frac{a}{b}, \tag{12}$$

where

$$a = \left( \frac{1}{4} g^2 \kappa_+^2 - \tilde{g}^2 v_L^2 \right) \sqrt{\cos 2\theta_W}, \tag{13}$$

$$b = \left( \frac{1}{4} g^2 \kappa_+^2 \sin^2\theta_W + \frac{1}{2} g^2 v_L^2 - \frac{1}{2} \tilde{g}^2 v_L^2 \sin^2\theta_W - \frac{1}{2} \left( g^2 + \tilde{g}^2 \right) v_R^2 \cos^2\theta_W \right). \tag{14}$$

Note: $\theta_W$ is the SM weak mixing angle. The physical gauge boson spectrum can be written as [21–23]

$$M_{W_{1,2}}^2 = \frac{g^2}{4} \left[ \kappa_+^2 + v_L^2 + v_R^2 \mp \sqrt{\left( v_R^2 - v_L^2 \right)^2 + 4\kappa_1^2 \kappa_2^2} \right], \tag{15}$$

$$M_{Z_{1,2}}^2 = \frac{1}{4} g^2 \kappa_+^2 + \frac{1}{2} \left( g^2 + \tilde{g}^2 \right) \left( v_L^2 + v_R^2 \right) \mp \frac{1}{\cos^2\theta_W} \sqrt{a^2 + b^2}. \tag{16}$$

## 3. Effective operators of dimension-6

We had delineated an elegant procedure for constructing higher mass dimensional operators in [16]. Treating MLRSM itself as an effective theory in [22], we then utilized the method to explicitly construct and classify a complete set of independent operators of mass dimension-6.

The basis of operators can be broadly subdivided into the following eight classes at mass dimension-6:

$$\phi^6\,, \quad X^3\,, \quad \phi^2 X^2\,, \quad \phi^4\mathcal{D}^2\,, \quad \psi^2\phi^2\mathcal{D}\,, \quad \psi^2\phi X\,, \quad \psi^2\phi^3\,, \quad \psi^4\,. \tag{17}$$

The impact of the operators of each class on the mass spectra, low-energy observables, and their contribution to rare processes has been discussed in meticulous detail in [22]. In this article, however, we devote our attention to a specific subset of the $\phi^2 X^2$ class of operators and study how these operators necessitate the re-definition of gauge bosons and ultimately offer $\Lambda^2$ suppressed corrections to observables such as the weak mixing angle ($\theta_W$), $\rho$-parameter, the Fermi constant ($\mathcal{G}_\mathrm{F}$), and the oblique $(S,T,U)$ parameters [24].

We also work within the approximation that $v_R \gg v_L, \kappa_{1,2}$, which means we will only treat $v_R$ as non-zero, while setting all other vacuum expectation values to zero. With this in mind, the relevant dimension-6 operators of the $\phi^2 X^2$ class are the following:

$$\begin{aligned}
&\mathcal{O}_{\Delta W}^{RrW_LW_L} : \mathrm{Tr}\big[\Delta_R^\dagger \Delta_R W_{L\mu\nu} W_L^{\mu\nu}\big]\,, \qquad \mathcal{O}_{\Delta W}^{RrW_RW_R} : \mathrm{Tr}\big[\Delta_R^\dagger \Delta_R W_{R\mu\nu} W_R^{\mu\nu}\big]\,,\\
&\mathcal{O}_{\Delta W}^{RW_RrW_R} : \mathrm{Tr}\big[\Delta_R W_{R\mu\nu} \Delta_R^\dagger W_R^{\mu\nu}\big]\,, \qquad \mathcal{O}_{\Delta W_RB}^{Rr} : \mathrm{Tr}\big[\Delta_R^\dagger W_R^{\mu\nu} \Delta_R\big] B_{\mu\nu}\,,\\
&\mathcal{O}_{\Delta B}^{Rr} : \mathrm{Tr}\big[\Delta_R^\dagger \Delta_R\big] B_{\mu\nu} B^{\mu\nu}\,.
\end{aligned} \tag{18}$$

In the above equation, the naming scheme for the operators and the corresponding Wilson coefficients have been borrowed from Ref. [22]. Since the operators contain a trace over the matrix product of $\Delta_{L,R}$, $\Delta_{L<R}^\dagger$ and $W_{L,R}$ fields, the symbols in the superscript describe the order in which the $2\times 2$ matrices appear in the product. More specifically, $R$ is a mnemonic for $\Delta_R^\dagger$, whereas $r$ represents $\Delta_R$, with $W_L$ or $W_R$ referring to the gauge boson involved in the operator. If the operator involves a $B_{\mu\nu}$, we add no symbol for that, since it is not a $2\times 2$ matrix. The subscript highlights the scalar and the gauge boson involved in the operator, this was important because in [22], we had additional operators involving the scalar bidoublet $\Phi$.

### *3.1. Gauge field redefinitions due to $\phi^2 X^2$ operators*

The gauge kinetic terms, Eq. (1), get modified in the presence of $\phi^2 X^2$ operators as

$$\mathcal{L}^{(4)+(6)}_{\text{gauge,kin}} = -\begin{pmatrix}\partial_\mu W^-_{L\nu} & \partial_\mu W^-_{R\nu}\end{pmatrix}\begin{pmatrix}1-\frac{2\Theta_{W_{LL}}}{\Lambda^2} & 0\\ 0 & 1-\frac{2\Theta_{W_{RR}}}{\Lambda^2}\end{pmatrix}\begin{pmatrix}\partial^\mu W^{+\nu}_L\\ \partial^\mu W^{+\nu}_R\end{pmatrix}$$
$$-\frac{1}{2}\begin{pmatrix}\partial_\mu W_{3L\nu}\\ \partial_\mu W_{3R\nu}\\ \partial_\mu B_\nu\end{pmatrix}^T\begin{pmatrix}1-\frac{2\Theta_{3L3L}}{\Lambda^2} & 0 & 0\\ 0 & 1-\frac{2\Theta_{3R3R}}{\Lambda^2} & -\frac{2\Theta_{3RB}}{\Lambda^2}\\ 0 & -\frac{2\Theta_{3RB}}{\Lambda^2} & 1-\frac{2\Theta_{BB}}{\Lambda^2}\end{pmatrix}\begin{pmatrix}\partial^\mu W^\nu_{3L}\\ \partial^\mu W^\nu_{3R}\\ \partial^\mu B^\nu\end{pmatrix}.$$

Here, the parameters are given as

$$\begin{aligned}\Theta_{W_{LL}} &= v_R^2\mathcal{C}^{RrW_LW_L}_{\Delta W}, & \Theta_{3R3R} &= v_R^2\left(\mathcal{C}^{RrW_RW_R}_{\Delta W}-\mathcal{C}^{RW_RrW_R}_{\Delta W}\right),\\ \Theta_{W_{RR}} &= v_R^2\mathcal{C}^{RrW_RW_R}_{\Delta W}, & \Theta_{3RB} &= -\frac{1}{2}v_R^2\mathcal{C}^{Rr}_{\Delta W_RB},\\ \Theta_{3L3L} &= v_R^2\mathcal{C}^{RrW_LW_L}_{\Delta W}, & \Theta_{BB} &= v_R^2\mathcal{C}^{Rr}_{\Delta B}.\end{aligned} \tag{19}$$

The $\mathcal{C}_i$s are Wilson coefficients corresponding to the operators listed in Eq. (18). The redefined gauge fields are written as

$$\begin{aligned}W^{\pm\mu}_L &\to \left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right)W^{\pm\mu}_L, & W^\mu_{3R} &\to \left(1+\frac{\Theta_{3R3R}}{\Lambda^2}\right)W^\mu_{3R}+\frac{\Theta_{3RB}}{\Lambda^2}B^\mu,\\ W^{\pm\mu}_R &\to \left(1+\frac{\Theta_{W_{RR}}}{\Lambda^2}\right)W^{\pm\mu}_R, & B^\mu &\to \left(1+\frac{\Theta_{BB}}{\Lambda^2}\right)B^\mu+\frac{\Theta_{3RB}}{\Lambda^2}W^\mu_{3R},\\ W^\mu_{3L} &\to \left(1+\frac{\Theta_{3L3L}}{\Lambda^2}\right)W^\mu_{3L}.\end{aligned} \tag{20}$$

## 4. Impact on low-energy observables

In Table 1, we show the observables we aim to study within our effective operators framework. Their exact dependence on the leading $\Lambda$ scale is given in the formulas gathered below.

### *4.1. Weak mixing angle*

The weak mixing angle $\theta_W$ is measured using low-energy data of nucleon–neutrino scattering → scattering driven by charged and neutral currents. One can define $\widehat{R}$ as [25]

$$\widehat{R} = \frac{\sigma^{\nu\text{NC}}-\sigma^{\bar{\nu}\text{NC}}}{\sigma^{\nu\text{CC}}-\sigma^{\bar{\nu}\text{CC}}}, \tag{21}$$

Table 1. Low-energy observables and their origin in effective Lagrangian influenced by $\phi^2 X^2$ operators.

| Observables | Contributing terms |
|---|---|
| $\theta_W$, $\mathcal{G}_{\rm F}$, $\rho$ | $\mathcal{L}_{\psi}^{\rm kin}\left(\psi, A'_\mu\right)$ |
| Oblique parameters $(S,T,U)$ | $\mathcal{L}_{A}^{\rm kin}\left(A'_\mu\right) + \mathcal{L}_{\phi}^{\rm kin}\left(\phi', A'_\mu\right) + \frac{1}{\Lambda^2}\phi^2 X^2$ |

This can be recast in terms of the weak mixing angle [25]

$$\widehat{R} = \frac{1}{2} - \sin^2\bar{\theta}_{\rm w}\,. \tag{22}$$

To examine the $\sigma^{\nu(\bar{\nu}){\rm NC}}$ and $\sigma^{\nu(\bar{\nu}){\rm CC}} \to$ cross sections of neutrino (antineutrino)–nucleon scattering through neutral (NC) and charged (CC) currents, the effective NC and CC Lagrangians must be defined.

Lagrangian containing charged current interactions is

$$\mathcal{L}_{\nu Q}^{\rm CC} \supset \frac{g^2}{2\mathcal{M}^2_{W_{1,2}}}\bar{e}_L\gamma^\mu\nu_L\left(\epsilon^{1,2}_{(e\nu)_L*(ud)_L}\bar{u}_L\gamma^\mu d_L + \epsilon^{1,2}_{(e\nu)_L*(ud)_R}\bar{u}_R\gamma^\mu d_R\right) + {\rm h.c.} \tag{23}$$

Here, $\epsilon^{1,2}_{(e\nu)_L*(ud)_L} = \epsilon^{1,2}_{(e\nu)_L}\cdot\epsilon^{1,2}_{(ud)_L}$ and $\epsilon^{1,2}_{(e\nu)_L*(ud)_R} = \epsilon^{1,2}_{(e\nu)_L}\cdot\epsilon^{1,2}_{(ud)_R}$ with

$$\begin{aligned}
\epsilon^1_{(e\nu)_L},\,\epsilon^1_{(ud)_L} &= \cos\xi\left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right), \qquad \epsilon^2_{(e\nu)_L},\,\epsilon^2_{(ud)_L} = \sin\xi\left(1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right),\\
\epsilon^1_{(e\nu)_R},\,\epsilon^1_{(ud)_R} &= -\sin\xi\left(1+\frac{\Theta_{W_{RR}}}{\Lambda^2}\right), \quad \epsilon^2_{(e\nu)_R},\,\epsilon^2_{(ud)_R} = \cos\xi\left(1+\frac{\Theta_{W_{RR}}}{\Lambda^2}\right).
\end{aligned} \tag{24}$$

Lagrangian containing neutral current interactions is

$$\begin{aligned}
\mathcal{L}_{\nu Q}^{\rm NC} &\supset \frac{g^2}{4\cos^2\theta_W\mathcal{M}^2_{Z_{1,2}}}\bar{\nu}_L\gamma^\mu\nu_L\\
&\times\left(\zeta^{1,2}_{\nu_L*u_L}\bar{u}_L\gamma^\mu u_L+\zeta^{1,2}_{\nu_L*u_R}\bar{u}_R\gamma^\mu u_R+\zeta^{1,2}_{\nu_L*d_L}\bar{d}_L\gamma^\mu d_L+\zeta^{1,2}_{\nu_L*d_R}\bar{d}_R\gamma^\mu d_R\right).
\end{aligned} \tag{25}$$

Here, $\zeta^{1,2}_{\nu_L*f} = \zeta^{1,2}_{\nu_L}\cdot\zeta^{1,2}_f$, with $f\in\{u_L,u_R,d_L,d_R\}$ and

$$\begin{aligned}
\zeta^1_{f_L} &= \left[a^f_L\cos\theta_2 + b^f_L\sin\theta_2\right], \qquad f=\nu,e,u,d\,,\\
\zeta^1_{f_R} &= \left[a^f_R\cos\theta_2 + b^f_R\sin\theta_2\right], \qquad f=e,u,d\,,\\
\zeta^2_{f_L} &= \left[a^f_L\sin\theta_2 - b^f_L\cos\theta_2\right], \qquad f=\nu,e,u,d\,,\\
\zeta^2_{f_R} &= \left[a^f_R\sin\theta_2 - b^f_R\cos\theta_2\right], \qquad f=e,u,d\,.
\end{aligned} \tag{26}$$

The "$a$" and "$b$" parameters can be factored into parts derived from the renormalizable Lagrangian and those derived from the effective Lagrangian as follows:

$$a_L^f = \left(a_L^f\right)_{\rm tree}\left[1+\frac{1}{\Lambda^2}\left(a_L^f\right)_{\rm eff}\right], \qquad b_L^f = \left(b_L^f\right)_{\rm tree}\left[1+\frac{1}{\Lambda^2}\left(b_L^f\right)_{\rm eff}\right],$$
$$a_R^f = \left(a_R^f\right)_{\rm tree}\left[1+\frac{1}{\Lambda^2}\left(a_R^f\right)_{\rm eff}\right], \qquad b_R^f = \left(b_R^f\right)_{\rm tree}\left[1+\frac{1}{\Lambda^2}\left(b_R^f\right)_{\rm eff}\right], \quad (27)$$

$$\left(a_L^f\right)_{\rm tree} = \left(A_1^{fL}\cos^2\theta_W - A_2^{fL}\sin^2\theta_W\right),$$
$$\left(b_L^f\right)_{\rm tree} = A_2^{fL}\frac{\sin^2\theta_W}{\sqrt{\cos 2\theta_W}},$$
$$\left(a_R^f\right)_{\rm tree} = -\left(A_1^{fR}+A_2^{fR}\right)\sin^2\theta_W,$$
$$\left(b_R^f\right)_{\rm tree} = \left(-A_1^{fR}\sqrt{\cos 2\theta_W} + A_2^{fR}\frac{\sin^2\theta_W}{\sqrt{\cos 2\theta_W}}\right), \quad (28)$$

$$\left(a_L^f\right)_{\rm eff} = \frac{A_1^{fL}\cot^2\theta_W \Theta_{3L3L} - A_2^{fL}\left(\frac{\sin\theta_W}{\sqrt{\cos 2\theta_W}}\Theta_{3RB}+\Theta_{BB}\right)}{A_1^{fL}\cot^2\theta_W - A_2^{fL}}, \quad (29)$$

$$\left(b_L^f\right)_{\rm eff} = \Theta_{BB} - \frac{\sqrt{\cos 2\theta_W}}{\sin\theta_W}\Theta_{3RB}, \quad (30)$$

$$\left(a_R^f\right)_{\rm eff} = \frac{A_1^{fR}\left(\Theta_{3R3R}+\csc\theta_W\sqrt{\cos 2\theta_W}\Theta_{3RB}\right)+A_2^{fR}\left(\frac{\sin\theta_W}{\sqrt{\cos 2\theta_W}}\Theta_{3RB}+\Theta_{BB}\right)}{A_1^{fR}+A_2^{fR}}, \quad (31)$$

$$\left(b_R^f\right)_{\rm eff} = \frac{A_1^{fR}\left(-\sqrt{\cos 2\theta_W}\,\Theta_{3R3R}+\sin\theta_W\,\Theta_{3RB}\right)}{-A_1^{fR}\sqrt{\cos 2\theta_W}+A_2^{fR}\frac{\sin^2\theta_W}{\sqrt{\cos 2\theta_W}}} + \frac{A_2^{fR}\left(-\sin\theta_W\,\Theta_{3RB}+\frac{\sin^2\theta_W}{\sqrt{\cos 2\theta_W}}\,\Theta_{BB}\right)}{-A_1^{fR}\sqrt{\cos 2\theta_W}+A_2^{fR}\frac{\sin^2\theta_W}{\sqrt{\cos 2\theta_W}}}, \quad (32)$$

where

$$A_1^{iL,R} = 2T_{3L,R}^i, \qquad A_2^{iL,R} = 2\left(Q^i - T_{3L,R}^i\right); \qquad i = u,d,\nu,e. \quad (33)$$

$T_{3L}, T_{3R}$, and $Q$ are quantum numbers of the fermions corresponding to the $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$, and $\mathrm{U}(1)_{\rm em}$ gauge groups respectively.

Incorporating all these corrections, we can express $\widehat{R}$ as

$$\widehat{R} = \frac{4\mathcal{M}^4_{W_1}}{\cos^4\theta_W \mathcal{M}^4_{Z_1}} \left[ \frac{\left(\zeta^1_{\nu_L * u_L}\right)^2 + \left(\zeta^1_{\nu_L * d_L}\right)^2 - \left(\zeta^1_{\nu_L * u_R}\right)^2 - \left(\zeta^1_{\nu_L * d_R}\right)^2}{\left(\epsilon^1_{(\nu e)_L * (ud)_L}\right)^2 - \left(\epsilon^1_{(\nu e)_L * (ud)_R}\right)^2} \right] . \tag{34}$$

To express the corrected quantities as a sum of the tree-level result and a $\Lambda^2$ suppressed correction, we express each quantity on the right-hand side of Eq. (34) in terms of the sum of the tree-level value and the corresponding correction due to dimension-6 operators.

We must emphasize again that we are only taking into account the impact of $\phi^2 X^2$ class of operators and we are using the approximation $v_R \gg v_L, \kappa_{1,2}$, due to which we have set $v_L, \kappa_{1,2}$ to zero. This allows us to write[1]

$$\begin{aligned} \mathcal{M}^2_{W_1} &= M^2_{W_1}\left[1+\frac{1}{\Lambda^2}T_1\right] , \quad T_1 = \frac{v_R^2}{M^2_{W_1}}\, g^2\, \Theta_{W_{RR}} \sin^2\xi \, , \\ \mathcal{M}^2_{Z_1} &= M^2_{Z_1}\left[1+\frac{1}{\Lambda^2}T_2\right] , \quad T_2 = \frac{v_R^2}{M^2_{Z_1}}\left(t^{(1)}\Theta_{BB}+t^{(2)}\Theta_{3RB}+t^{(3)}\Theta_{3R3R}\right) , \end{aligned} \tag{35}$$

where

$$\begin{aligned} t^{(1)} &= -g\tilde{g}\, f_2(\theta_W,\theta_1,\theta_2) + 2\tilde{g}^2\, f_3(\theta_W,\theta_1,\theta_2)\, , \\ t^{(2)} &= -2g\tilde{g}\, \left(f_1(\theta_W,\theta_1,\theta_2) + f_3(\theta_W,\theta_1,\theta_2)\right) + \left(g^2+\tilde{g}^2\right)\, f_2(\theta_W,\theta_1,\theta_2)\, , \\ t^{(3)} &= 2g^2\, f_1(\theta_W,\theta_1,\theta_2) - g\tilde{g}\, f_2(\theta_W,\theta_1,\theta_2)\, . \end{aligned} \tag{36}$$

Here, $f_i(\theta_W,\theta_1,\theta_2)$ are trigonometric functions given as

$$\begin{aligned} f_1(\theta_W,\theta_1,\theta_2) &= \cos^2\theta_1\, \sin^2\theta_2 + \frac{1}{2}\sin\theta_W\, \sin 2\theta_1\, \sin 2\theta_2 \\ &\quad + \sin^2\theta_W\, \sin^2\theta_1 \cos^2\theta_2\, , \\ f_2(\theta_W,\theta_1,\theta_2) &= -\sin 2\theta_1\, \sin^2\theta_2 + \sin\theta_W\, \cos 2\theta_1\, \sin 2\theta_2 \\ &\quad + \sin^2\theta_W\, \sin 2\theta_1 \cos^2\theta_2\, , \\ f_3(\theta_W,\theta_1,\theta_2) &= \sin^2\theta_1\, \sin^2\theta_2 - \frac{1}{2}\sin\theta_W\, \sin 2\theta_1\, \sin 2\theta_2 \\ &\quad + \sin^2\theta_W\, \cos^2\theta_1 \cos^2\theta_2\, . \end{aligned} \tag{37}$$

[1] The full expressions for each of the above quantities taking into account the effect of all dimension-6 operators and also using non-zero $v_L, \kappa_{1,2}$ have been discussed in great detail in [22]. In a full analysis, the angles $\xi$ and $\theta_2$ also receive corrections due to the inclusion of dimension-6 operators, but those corrections are further weighted by a factor of $\frac{1}{\Lambda^2}$, hence we have ignored them here.

For the term inside $[\cdots]$ on the right-hand side of Eq. (34), the denominator can be recast into

$$\left(\epsilon^{1}_{(\nu e)_L*(ud)_L}\right)^2 - \left(\epsilon^{1}_{(\nu e)_L*(ud)_R}\right)^2 = \cos^2\xi\,\cos 2\xi\left[1+\frac{T_3}{\Lambda^2}\right], \tag{38}$$

with

$$T_3 = \left(\frac{1+2\cos 2\xi}{\cos 2\xi}\right)\Theta_{W_{LL}} - \left(\frac{1-\cos 2\xi}{2\cos 2\xi}\right)\Theta_{W_{RR}}\,. \tag{39}$$

To simplify the numerator, we first substitute Eq. (27) into Eq. (26) to rewrite

$$\zeta^1_{f_L} = (\zeta^1_{f_L})_{\text{tree}} + \frac{1}{\Lambda^2}(\zeta^1_{f_L})_{\text{eff}}\,, \tag{40}$$

where

$$\begin{aligned}\left(\zeta^1_{f_{L,R}}\right)_{\text{tree}} &= \left(a^f_{L,R}\right)_{\text{tree}}\cos\theta_2 + \left(b^f_{L,R}\right)_{\text{tree}}\sin\theta_2\,,\\ \left(\zeta^1_{f_{L,R}}\right)_{\text{eff}} &= \left(a^f_{L,R}\right)_{\text{eff}}\cos\theta_2 + \left(b^f_{L,R}\right)_{\text{eff}}\sin\theta_2\,.\end{aligned} \tag{41}$$

After some simplification, we can rewrite

$$\begin{aligned}&\left(\zeta^1_{\nu_L*u_L}\right)^2 + \left(\zeta^1_{\nu_L*d_L}\right)^2 - \left(\zeta^1_{\nu_L*u_R}\right)^2 - \left(\zeta^1_{\nu_L*d_R}\right)^2 =\\ &\left(\zeta^1_{\nu_L}\right)^2_{\text{tree}}\left[\left(\zeta^1_{u_L}\right)^2_{\text{tree}} + \left(\zeta^1_{d_L}\right)^2_{\text{tree}} - \left(\zeta^1_{u_R}\right)^2_{\text{tree}} - \left(\zeta^1_{d_R}\right)^2_{\text{tree}}\right]\left[1+\frac{T_4}{\Lambda^2}\right],\end{aligned} \tag{42}$$

with

$$T_4 = 2\left(\frac{\left(\zeta^1_{\nu_L}\right)_{\text{eff}}}{\left(\zeta^1_{\nu_L}\right)^2_{\text{tree}}} + \frac{\left(\zeta^1_{u_L}\right)_{\text{eff}} + \left(\zeta^1_{d_L}\right)_{\text{eff}} - \left(\zeta^1_{u_R}\right)_{\text{eff}} - \left(\zeta^1_{d_R}\right)_{\text{eff}}}{\left(\zeta^1_{u_L}\right)^2_{\text{tree}} + \left(\zeta^1_{d_L}\right)^2_{\text{tree}} - \left(\zeta^1_{u_R}\right)^2_{\text{tree}} - \left(\zeta^1_{d_R}\right)^2_{\text{tree}}}\right). \tag{43}$$

Substituting Eqs. (35), (38), and (42) into Eq. (34), we can express $\widehat{R}$ as a sum of the tree-level value and a $\Lambda^2$ suppressed correction as follows:

$$\begin{aligned}\widehat{R} &= R\times\left[1+\frac{1}{\Lambda^2}T_1\right]^2\left[1+\frac{1}{\Lambda^2}T_2\right]^{-2}\left[1+\frac{1}{\Lambda^2}T_3\right]^{-1}\left[1+\frac{1}{\Lambda^2}T_4\right]\\ &\simeq R\times\left[1+\frac{1}{\Lambda^2}\left(2T_1-2T_2-T_3+T_4\right)\right].\end{aligned} \tag{44}$$

The term inside the parenthesis can ultimately be expressed as a function of Wilson coefficients. From this, we can obtain the expression for the corrected weak mixing angle as

$$\sin^2\bar\theta_{\text{w}} = \frac{1}{2}-\widehat{R} = \left(\frac{1}{2}-R\right)\left[1+\frac{T_5}{\Lambda^2}\right] = \sin^2\theta_W\left[1+\frac{T_5}{\Lambda^2}\right]. \tag{45}$$

Here, $T_5 = -R\csc^2\theta_W\,(2T_1-2T_2-T_3+T_4)$.

### *4.2. Fermi constant*

$$
\begin{aligned}
(\mathcal{G}_{\rm F})_{\rm EFT} &= \frac{g^2}{4\sqrt{2}\,\mathcal{M}^2_{W_1}}\left[\epsilon^{1,e}_{(1,e)_L}+\epsilon^{1,e}_{(1,\mu)_L}\right] \\
&= \mathcal{G}_{\rm F}\times\left[1+\frac{\Theta_{W_{LL}}}{\Lambda^2}\right]\left[1+\frac{T_1}{\Lambda^2}\right]^{-1} \\
&\simeq \mathcal{G}_{\rm F}\times\left[1+\frac{\Theta_{W_{LL}}-T_1}{\Lambda^2}\right]. \qquad (46)
\end{aligned}
$$

### *4.3. ρ parameter*

$$
\begin{aligned}
\bar{\rho} &= \frac{\mathcal{M}^2_{W_1}}{\mathcal{M}^2_{Z_1}\left(1-\sin^2\bar{\theta}_{\rm w}\right)} \\
&= \rho\times\left[1+\frac{1}{\Lambda^2}\,T_1\right]\left[1+\frac{1}{\Lambda^2}\,T_2\right]^{-1}\left[1-\frac{\tan^2\theta_W}{\Lambda^2}\,T_5\right]^{-1} \\
&\simeq \rho\times\left[1+\frac{1}{\Lambda^2}\,\left(T_1-T_2+\tan^2\theta_W\,T_5\right)\right]. \qquad (47)
\end{aligned}
$$

### *4.4. Oblique parameters*

The two-point vector boson correlation functions are [24, 26, 27]

$$
i\Pi^{\mu\nu}_{V_iV_j}\left(p^2\right)=i\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)\Pi_{V_iV_j}\left(p^2\right)+\left(i\frac{p^\mu p^\nu}{p^2}\ \text{terms}\right), \qquad (48)
$$

where $V_i\in\{W_1,W_2,W_3,B\}$ or $\{W,Z,\gamma\}$ (unphysical or physical basis) and $p^\mu$ is the external momentum. Here, $\gamma$ represents the photon field $A_\mu$.

Expanding $\Pi_{V_iV_j}(p^2)$ as a polynomial in external momentum $(p^2)$ [24, 26, 27],

$$
\Pi_{V_iV_j}\left(p^2\right)=\left[\Pi_0+\Pi_2p^2+\Pi_4p^4+\mathcal{O}\left(p^6\right)\right]_{V_iV_j}. \qquad (49)
$$

Oblique parameters $S,T,U$ encapsulate the radiative corrections to the tree-level correlation functions [24]. These can be defined in both unphysical and physical gauge boson basis as [24, 26–28]

$$\begin{aligned} S &= -\frac{4\cos\theta_W \sin\theta_W}{\alpha} \Pi'_{W_3 B}(0) \\ &= -\frac{4\cos^2\theta_W \sin^2\theta_W}{m_Z^2 \alpha}\Big[\Pi_{ZZ}\left(m_Z^2\right) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}\left(m_Z^2\right) \\ &\quad -\frac{\cos^2\theta_W - \sin^2\theta_W}{\cos\theta_W \sin\theta_W}\left(\Pi_{\gamma Z}\left(m_Z^2\right) - \Pi_{\gamma Z}(0)\right)\Big], \end{aligned} \tag{50}$$

$$\begin{aligned} T &= \frac{1}{\alpha m_W^2}\left(\Pi_{W_1 W_1}(0) - \Pi_{W_3 W_3}(0)\right) \\ &= \frac{1}{\alpha}\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{2\sin\theta_W \Pi_{\gamma Z}(0)}{\cos\theta_W m_Z^2}\right], \end{aligned} \tag{51}$$

$$\begin{aligned} U &= \frac{4\sin^2\theta_W}{\alpha}\left(\Pi'_{W_1 W_1}(0) - \Pi'_{W_3 W_3}(0)\right) \\ &= -\frac{4\sin^2\theta_W}{\alpha}\Bigg[\left(\frac{\Pi_{WW}\left(m_W^2\right) - \Pi_{WW}(0)}{m_W^2}\right) \\ &\quad -\cos^2\theta_W\left(\frac{\Pi_{ZZ}\left(m_Z^2\right) - \Pi_{ZZ}(0)}{m_Z^2}\right) \\ &\quad -2\cos\theta_W \sin\theta_W\left(\frac{\Pi_{\gamma Z}\left(m_Z^2\right) - \Pi_{\gamma Z}(0)}{m_Z^2}\right) - \sin^2\theta_W \frac{\Pi_{\gamma\gamma}\left(m_Z^2\right)}{m_Z^2}\Bigg], \end{aligned} \tag{52}$$

where $\alpha = e^2/4\pi$ is the fine-structure constant.

Here, for unphysical basis

$$\Pi'_{V_i V_j}(0) = \left.\frac{\mathrm{d}\Pi_{V_i V_j}}{\mathrm{d}p^2}\right|_{p^2=0}, \qquad \Pi_{W_1 W_1}\left(p^2\right) = \Pi_{W_2 W_2}\left(p^2\right),$$

and for physical basis, we have

$$\Pi'_{V_i V_j}(0) = \left.\left(\frac{\Pi_{V_i V_j}\left(p^2\right) - \Pi_{V_i V_J}(0)}{p^2}\right)\right|_{p^2=0}, \qquad \Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0\,.$$

Through the redefinition of gauge fields, the $\phi^2 X^2$ operators provide contributions to the oblique parameters. In the unphysical basis, we compute

$$\Pi_{W_L W_L}\left(p^2\right) = p^2\left(1 + 2\frac{\Theta_{W_{LL}}}{\Lambda^2}\right), \tag{53}$$

$$\Pi_{W_R W_R}\left(p^2\right) = p^2\left(1 + 2\frac{\Theta_{W_{RR}}}{\Lambda^2}\right) + \frac{1}{\Lambda^2} g^2 v_R^2 \Theta_{W_{RR}}, \tag{54}$$

$$\varPi_{W_{3L}W_{3L}}\left(p^2\right) = p^2\left(1+2\frac{\varTheta_{3L3L}}{\varLambda^2}\right), \tag{55}$$

$$\varPi_{W_{3R}W_{3R}}\left(p^2\right) = p^2\left(1+2\frac{\varTheta_{3R3R}}{\varLambda^2}\right)+\frac{2v_R^2}{\varLambda^2}\left(g^2\varTheta_{3R3R}-g\tilde{g}\varTheta_{3RB}\right), \tag{56}$$

$$\begin{aligned}\varPi_{W_{3RB}}\left(p^2\right) &= p^2\frac{\varTheta_{3RB}}{\varLambda^2}-g\tilde{g}v_R^2\\ &\quad -\frac{v_R^2}{\varLambda^2}\left(g\tilde{g}\varTheta_{3R3R}-\varTheta_{3RB}\left(g^2+(\tilde{g})^2\right)+g\tilde{g}\varTheta_{BB}\right),\end{aligned} \tag{57}$$

$$\varPi_{BB}\left(p^2\right) = g\tilde{g}v_R^2-\frac{\tilde{g}v_R^2}{\varLambda^2}\left(2g\varTheta_{3RB}-2\tilde{g}\varTheta_{BB}\right). \tag{58}$$

Using the rotation matrices $\mathcal{R}_{a,b}$, see Eq. (11), we can define these $\varPi_{V_iV_j}(p^2)$ in the physical basis as follows:

$$\begin{aligned}
\varPi_{W_1W_1} &= \varPi_{W_LW_L}\left(\mathcal{R}_a^{11}\right)^2+\varPi_{W_RW_R}\left(\mathcal{R}_a^{21}\right)^2,\\
\varPi_{W_1W_2} &= \varPi_{W_LW_L}\mathcal{R}_a^{11}\mathcal{R}_{12}^a+\varPi_{W_RW_R}\mathcal{R}_{21}^a\mathcal{R}_{22}^a,\\
\varPi_{W_2W_2} &= \varPi_{W_LW_L}\left(\mathcal{R}_{12}^a\right)^2+\varPi_{W_RW_R}\left(\mathcal{R}_{22}^a\right)^2,\\
\varPi_{Z_1Z_1} &= \varPi_{BB}\left(\mathcal{R}_{31}^b\right)^2+\varPi_{W_{3L}W_{3L}}\left(\mathcal{R}_{11}^b\right)^2+2\varPi_{W_{3R}B}\mathcal{R}_{21}^b\mathcal{R}_{31}^b\\
&\quad +\varPi_{W_{3R}W_{3R}}\left(\mathcal{R}_{21}^b\right)^2,\\
\varPi_{Z_2Z_2} &= \varPi_{BB}\left(\mathcal{R}_{32}^b\right)^2+\varPi_{W_{3L}W_{3L}}\left(\mathcal{R}_{12}^b\right)^2+2\varPi_{W_{3R}B}\mathcal{R}_{22}^b\mathcal{R}_{32}^b\\
&\quad +\varPi_{W_{3R}W_{3R}}\left(\mathcal{R}_{22}^b\right)^2,\\
\varPi_{Z_1Z_2} &= \varPi_{BB}\mathcal{R}_{31}^b\mathcal{R}_{32}^b+\varPi_{W_{3L}W_{3L}}\mathcal{R}_{11}^b\mathcal{R}_{12}^b+\varPi_{W_{3R}B}\left(\mathcal{R}_{21}^b\mathcal{R}_{32}^b+\mathcal{R}_{22}^b\mathcal{R}_{31}^b\right)\\
&\quad +\varPi_{W_{3R}W_{3R}}\mathcal{R}_{21}^b\mathcal{R}_{22}^b,\\
\varPi_{\gamma\gamma} &= \varPi_{BB}\left(\mathcal{R}_{33}^b\right)^2+\varPi_{W_{3L}W_{3L}}\left(\mathcal{R}_{13}^b\right)^2+2\varPi_{W_{3R}B}\mathcal{R}_{23}^b\mathcal{R}_{33}^b\\
&\quad +\varPi_{W_{3R}W_{3R}}\left(\mathcal{R}_{23}^b\right)^2,\\
\varPi_{Z_1\gamma} &= \varPi_{BB}\mathcal{R}_{31}^b\mathcal{R}_{33}^b+\varPi_{W_{3L}W_{3L}}\mathcal{R}_{11}^b\mathcal{R}_{13}^b+\varPi_{W_{3R}B}\left(\mathcal{R}_{21}^b\mathcal{R}_{33}^b+\mathcal{R}_{23}^b\mathcal{R}_{31}^b\right)\\
&\quad +\varPi_{W_{3R}W_{3R}}\mathcal{R}_{21}^b\mathcal{R}_{23}^b,\\
\varPi_{Z_2\gamma} &= \varPi_{BB}\mathcal{R}_{32}^b\mathcal{R}_{33}^b+\varPi_{W_{3L}W_{3L}}\mathcal{R}_{12}^b\mathcal{R}_{13}^b+\varPi_{W_{3R}B}\left(\mathcal{R}_{22}^b\mathcal{R}_{33}^b+\mathcal{R}_{23}^b\mathcal{R}_{32}^b\right)\\
&\quad +\varPi_{W_{3R}W_{3R}}\mathcal{R}_{22}^b\mathcal{R}_{23}^b.
\end{aligned} \tag{59}$$

The oblique parameters $(S,T,U)$ for MLRSM-EFT can be constructed by substituting Eq. (59) into Eqs. (50), (51), (52).

## 5. Conclusions

In this article, we have considered the MLRSM as an effective theory. Instead of studying the impact of the full dimension-6 effective operator basis on the mass spectra and observables, we have focused on a subset of the $\phi^2 X^2$ class of operators. We have computed the corrected values of low energy observables such as $\rho$, $\mathcal{G}_{\mathrm{F}}$, $\Theta_W$, oblique parameters $(S, T, U)$. Working under the assumption that the right-handed symmetry breaking scale is larger than the electroweak ones, *i.e.*, $v_R \gg \kappa_{1,2}, v_L$, we have noted that only five operators belonging to the $\phi^2 X^2$ class are relevant. We have invoked symmetry breaking in a cascade and computed the spectrum and modifications in the renormalized vertices due to these effective operators. Employing that, we have calculated the corrections to our chosen set of observables, up to $1/\Lambda^2$.

Our further aim is to perform a fitting to adjudge the allowed ranges of Wilson coefficients associated with the considered relevant effective operators.

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