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Review

Testing Quantum Gravity in the Multi-Messenger Astronomy Era

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Abstract: Quantum gravity (QG) remains elusive despite almost century-long efforts to combine general relativity and quantum mechanics. All the approaches triggered and powered by purely theoretical considerations eventually failed with a prevailing feeling of a complete lack of guidance from the experimental side. Currently, however, this circumstance is beginning to change considerably. We have entered the era of multi-messenger astronomy. The electromagnetic window to the universe—so far the only one—has been tremendously enlarged in the energy range beyond gamma rays up to ultra-high-energy photons and has been complemented by other messengers: high-energy cosmic rays, cosmic neutrinos, and gravitational waves (GWs). This has created a unique environment in which to observationally constrain various phenomenological QG effects. In this paper, we focus on the LIV phenomenology manifested as energy-dependent time-of-flight delays and strong lensing time delays. We review results regarding time-of-flight delays obtained with GRBs. We also recall the idea of energy-dependent lensing time delays, which allow one to constrain LIV models independently of the intrinsic time delay. Lastly, we show how strongly a gravitationally lensed GW signal would place interesting constraints on the LIV.

Keywords: quantum gravity phenomenology; Lorentz Invariance Violation; strong gravitational lensing

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1. Introduction

As the best theory of gravity, general relativity (GR) is one of the pillars of contemporary physics. Yet, gravity is still understood (and probed) at only the classical level. All other fundamental interactions are either inherently of a quantum nature (weak and strong interactions) or have a well-defined quantum version (quantum electrodynamics). With this clear view that the fundamental laws of physics are of a quantum mechanical nature, it is reasonable to expect the same for gravity. However, despite century-long efforts to find quantum gravity (QG), it still eludes us. Planck units constructed from the fundamental constants of nature give some clues; one expects that QG reigns at distances as small as the Planck length $l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35}$ m and at energies as large as the Planck energy $E_{Pl} = \sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{19}$ GeV. This has always been disappointing from an experimental point of view. However, developments in string theories and other approaches to QG have ignited hopes that manifestations of QG could be probed at more accessible energy scales. One such robust prediction of many QG theories is the Lorentz Invariance Violation (LIV) manifested by an energy-dependent modification of the relativistic dispersion relation for particles and photons. Sources located at cosmological distances are particularly promising in this respect. Minute LIV effects could accumulate along the baseline offered by extragalactic distances to detectable levels.

However, the past decades resulted in incredible technological progress, pushing the exploration of the universe on the whole electromagnetic (EM) spectrum to the extremes.

Currently, we are probing the cosmos from the lowest radio frequencies (LOFAR [1] and future SKA [2]) through the microwave (CMB), infrared, optical-to-X-ray, and gamma-rays with plenty of sources monitored across this full range of frequencies. With the pioneering roles of the MAGIC [3], HESS [4], and VERITAS [5] Cherenkov observatories [6], extremely high-energy TeV photons are becoming detectable. In such a case, the Earth's atmosphere acts as a detector—a TeV photon initiates an atmospheric shower detectable by using a Cherenkov telescope. The future looks bright with the forthcoming CTA [7] and already-operating LHAASO observatories [8]. Moreover, IceCube is currently detecting high-energy neutrinos of cosmic origins [9,10], and in the future, we will have a KM3NeT detector [11] in the Mediterranean Sea. At last, gravitational wave (GW) astronomy has emerged with the first detection [12] of GW signals from coalescing black holes (BHs) by LIGO-Virgo. At present, the LIGO-Virgo-KAGRA collaboration is completing the third scientific run O3, and a rich catalog of the GW events registered so far has been released [13]. Luckily, a GW170817 event registered by ground-based interferometric detectors [14] was observed in gamma rays as a short gamma-ray burst (GRB) 2.7 s after its coalescence. Afterwards, a concerted campaign in the optical [15] and then radio waves allowed for a confirmation of the kilonova scenario underlying this event and led, among other things, to the first direct measurement of the Hubble constant from a standard siren [16]. Thus, multi-messenger astronomy acquired unprecedented opportunities.

With all this in mind, in the present paper, we review certain ideas about constraining LIV theories using extragalactic sources. In particular, we discuss the opportunities emerging from the potential detection of lensed high-energy and GW signals.

2. Modified Dispersion Relation and Time-Delay Technique

Various approaches to QG have predicted the existence of exotic non-standard effects, such as a violation of some of the essential principles underlying our understanding of nature (e.g., LIV); variability of fundamental constants; or non-zero mass of particles, such as photons or gravitons, which should be massless according to the widely accepted standard theory. As a consequence, this motivates one extremely popular phenomenological approach to constrain QG parameters (e.g., the QG energy scale $E_{\rm QG}$) via a modified dispersion relation (see a comprehensive review of this topic in, e.g., [17,18] and the references therein). Lacking any strong and convincing suggestions otherwise, we believe that $E_{\rm QG}$ should be close to the Planck energy scale $E_{Pl} \sim 10^{19}$ GeV), but of course, the $E_{\rm QG} < E_{Pl}$ case would be the most interesting and testable. The point is that at very high energies of the order of $E_{\rm QG}$, we expect that QG should replace the standard theory describing nature at the largest (general theory of relativity) and the smallest (quantum-field theory) scales, and thus, the dispersion relation for relativistic particles,

$$E^2 = m^2c^4 + p^2c^2, \tag{1}$$

where E, p, and m—the energy, momentum, and mass of a given particle— with c being the speed of light in a vacuum—should be replaced by some general and as-yet unknown function of the particle energy and momentum [18–20]

$$E^2 = m^2c^4 + p^2c^2 + f(E, p, m; E_{\rm QG}). \tag{2}$$

A Taylor expansion of this function in the currently accessible (and thus, rather low in comparison to $E_{\rm QG}$) energies provides tiny corrections to the standard case (1), thereby leading to the occurrence of extremely small exotic QG effects. Such effects should be almost null at low energies, supporting the strong experimental success of the standard theory. However, they should become stronger as the quantum-gravity energy scale is approached. The exact form of the deformation functions in Formula (2) depends on a particular model of quantum gravity (see a detailed discussion in [21,22]), but the common view is that such a modified dispersion relation may lead to changes in the travel time of signals of a different kind emitted from distant sources, opening the chance for quantum-

gravity testing within the so-called time-of-flight measurements. Such tests are promising when the particle is emitted from a source lying at cosmological distances. According to the common expectation that QG effects should accumulate on the path between a source and an observer, a tiny initial exotic signal should become enhanced to a detectable level [18]. Therefore, there is a high motivation to use particular types of high-energy astrophysical objects, such as active galactic nuclei (AGNs), GRBs, and double-compact object mergers (DCOs). The latter become extremely interesting in light of the recent successful detections provided by second-generation interferometric gravitational-wave detectors resulting in a still-increasing catalog of gravitational wave signals from coalescing compact binaries. The signals from these sources are either very regular (e.g., the inspiral phase of DCO) or have fine-scale (e.g., coalescence moment, GRB, or AGN) time structures, making the time-delay technique robust. Pulsars may also be used in this context, even if they are observed from only galactic distances according to the current pulsar catalogs. Besides radio emissions, they emit highly energetic signals of quasi-periodic natures, which is a valuable feature from the point of view of improving measurement accuracy to be sensitive to phenomena at the Planck scale (the expected non-standard effects should be picked up more easily against a regular pulse; see, e.g., the discussion in [23] and references therein). The time-of-flight technique may be applied for signals emitted in different energy channels (e.g., low and high energies; see [24,25] and references therein) and/or between different particle types (see, e.g., [26–29]). The latter suggests that all multi-messenger capabilities of present-day astronomy should be extremely useful for QG testing.

3. Astrophysical Tests of Quantum Gravity

A possible violation of the Lorentz Invariance would be a direct consequence of some mechanisms originating at the Planck scale E_{PL}, preventing this symmetry from holding exactly. Within this scenario, the standard dispersion relation of a relativistic particle would be modified, which phenomenologically may be written as the following power-law expansion:

$$E^2 = m^2c^4 + p^2c^2 + \epsilon E^2 \left(\frac{E}{\xi_n E_{\mathrm{QG}}} \right)^n, \tag{3}$$

where $\epsilon = \pm 1$ is the "sign" parameter separating the case of a superluminal ($\epsilon = +1$) motion of a particle from a subluminal one ($\epsilon = -1$), and ξ_n are dimensionless parameters associated with the size of an LIV within the nth correction term (e.g., $\xi_1 = 1$ for the first order $n = 1$ LIV correction to the standard case; see, e.g., [21,22]). The speed of a particle traveling radially from a source to an observer in the spatially flat expanding universe would therefore be described according to the following formula (see the details in [29]):

$$v \simeq c(1+z) \left[1 - \frac{1}{2} \frac{m^2c^4}{E_0^2} + \frac{1}{2}(n+1)\epsilon \left(\frac{E_0}{\xi_n E_{\mathrm{QG}}} \right)^n (1+z)^n \right]. \tag{4}$$

Here, E_0 is the observed energy of a particle under consideration. As a result, the particle's time of arrival (i.e., the comoving distance between the source and the observer, measured in light years) is

$$r(t) = \int_{t_e}^{t_0} v(t) dt \tag{5}$$

with t_e being the particle's emission time at the source.

Of course, the result would be slightly different than in the case of the standard Lorentz Invariant case. Rewriting the Formula (5) so that it becomes a function of redshift—observable in astrophysical observations—one obtains [29]

$$r(z) = \int_0^z \frac{v(z')dz'}{H(z')(1+z')}, \tag{6}$$

where $H(z) = H_0 h(z)$ is the expansion rate of the universe. The Hubble constant H_0 represents the current rate of this expansion; $h(z)$ is the dimensionless form of it, which depends on the cosmological model under consideration. For a widely accepted concordance ΛCDM scenario based on the flat Friedman–Robertson–Walker (FRW) geometry, $ds^2 = c^2dt^2 - a(t)^2\left[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$, with $a(t)$ being the scale factor parametrizing an expanding three-dimensional space; $H \equiv \frac{\dot{a}}{a}$, the dimensionless expansion rate is $h(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$, where Ω_m is the matter-density parameter representing the current value of the matter energy density in the universe as a fraction of critical density. Substituting (4) into (6), one obtains a particle time-of-flight formula in the presence of the LIV:

$$t = \int_0^z \left[1 - \frac{1}{2}\frac{m^2c^4}{E_0^2}\frac{1}{(1+z)^2} + \frac{1}{2}(n+1)\epsilon\left(\frac{E_0}{\xi_n E_{\mathrm{QG}}}\right)^n (1+z)^n\right]\frac{dz'}{H(z')}. \tag{7}$$

This entails the possibility of LIV testing according to the following scheme. Taking as a benchmark the low-energy photons for which the LIV, if it really occurs, has a practically null effect, a tiny difference between the arrival times of such photons and a given high-energy particle (a photon, neutrino, or graviton) will occur as a consequence of the fact that high-energy particles in the presence of the LIV would travel a bit slower/faster (depending on ϵ) with respect to the low-energy Lorentz Invariant case. Despite the fact that the method based on the time-delay technique allows for robust tests of physics at the E_{QG} scale (the Lorentz symmetry is considered to be an exact one, and thus, there is no conventional process permitting, e.g., photons to travel with a speed different than c) it suffers, however, from several problems discussed in detail in the following subsections (see also [30] and the references therein).

3.1. Pair-Production Process in the Case of High-Energy Photons

High-energy photons of energies reaching the TeV energy range are expected to be produced by GRBs via the synchrotron self-Compton mechanism [31–33]. Indeed, such particles were already reported from blazars [6,34]—objects that are a special type of AGNs when the outflow of ionized matter in the form of relativistic jets is directed toward an observer [35]. Unfortunately, the use of high-energy photons in LIV testing may be highly influenced by a pair-production process via inverse Compton scattering. This is because the universe is filled with the low-energy photons of cosmic microwave background radiation—a remnant of the recombination epoch when electrons and protons combined to form neutral hydrogen atoms. This process has been associated with the emission of photons. Their energy was insufficient to ionize other atoms, so it gradually became redshifted with the expansion of the universe, reaching the current value of 2.7 K. As a consequence, high-energy photons traveling from the source are immersed in a bath of 2.7 K photons. Consequently, the universe becomes opaque for photons of energies above 10 TeV in an way analogous to the Greisen–Zatsepin–Kuzmin (GZK) threshold for high-energy cosmic-ray particles. The remedy for this tricky situation is to use other types of particles instead of photons: high-energy neutrinos [22] or gravitons [30,36]. Neutrinos with ultra-high energies (100–10^4 TeV) should be produced within the prompt emission of gamma-ray bursts by accelerated non-thermal protons in photohadronic interactions [22,37]. Considering distant source with redshift z emitting at the same time two signals: 100TeV neutrinos and low energy photons e.g. in the optical range, X-rays or even gamma-rays, then the time delay between these two particles types in the presence of LIV would be:

$$\Delta t_\nu = \int_0^z \left[\frac{1}{2}\frac{m_\nu^2c^4}{E_{\nu,0}^2}\frac{1}{(1+z)^2} - \frac{1}{2}(n+1)\epsilon\left(\frac{E_{\nu,0}}{\xi_n E_{\mathrm{QG}}}\right)^n (1+z)^n\right]\frac{dz'}{H(z')}. \tag{8}$$

Such a time delay may range from about 0.6 days for sources at $z = 3$ (i.e., the median redshift of GRBs) to almost 1.4 days for GRBs at $z = 6$, assuming a Λ CDM model [29].

This idea seems to be promising, especially in light of the first detection by the IceCube Neutrino Observatory of the 290 TeV neutrino signal, which originated at the blazar TXS 0506 + 056 (IceCube-170922A) [9,10]. Recently, the PeV-energy neutrino event detected by IceCube has been used to obtain a lower bound on the first order effective QG energy scale of $E_{QG} > 1.09 \times 10^{17}$ GeV [38] based on time-of-flight difference between the neutrino signal and EM one from a giant flare of the blazar PKS B1424-418 (i.e., assuming physical association of this neutrino event with the blazar). However, no high-energy GRB-associated neutrinos have been detected so far. In the near future, the next-generation neutrino observatories, such as the KM3NeT detector [11] or Baikal Gigaton Volume Detector [39] (the first phase, GVD-I, was completed in 2021), will help to break this impasse.

Because the use of GRB neutrinos for LIV testing may not be feasible in the near future, it is reasonable to focus on using other messenger types. Since the first GW signal (i.e., GW150914) detected by LIGO interferometers [12] GW astronomy became a new branch of science with increasing capabilities, and the GW catalogs obtained during the subsequent LIGO observing runs O1–O3 included an increasing number of GW signals observed from DCO (BH-BH, NS-NS or mixed, and BH-NS) coalescences [40]. The relativistic dispersion relation for gravitons modified by the existence of the non-zero LIV-effect Equation (2) will lead to the energy (frequency)-dependent speeds of GW signals [36,41]. It contradicts general relativity predictions according to which GW speed is constant and equal to the speed of light in vacuum c. In the case of the LIV, a high-frequency GW signal corresponding to high-energy gravitons (high-frequency modes) will be affected by QG effects. In contrast, low-frequency GW signals, related to gravitons with lower energies, should not be influenced by LIV corrections as they are almost zero at the low-energy range. This will change the characteristic chirping time pattern of the GW signal in a characteristic way, squeezing or tightening it depending on the sign parameter ϵ in Equation (2). In other words, a low-frequency GW signal that is emitted at an earlier stage of inspiral will travel at c, whereas a high-frequency one emitted later will travel with an energy-dependent velocity; see [42]). This method is very similar to the one proposed to constrain the graviton mass postulated within massive-gravity scenarios (i.e., a certain class of QG theories based on modifications of the theory of gravity; see, e.g., [43–49]). The test can be done through a detailed numerical comparison of the measured GW signal-chirp structure with those calculated for a given DCO merger [50–53]. Using GW150914, Ellis et al. [36] made the first attempt to constrain the QG energy scale with GWs. Assuming that a low-frequency GW signal travels with the speed of light and a high-frequency GW signal is affected by the LIV, such that the latter travels with a speed slightly different from c, they found that $E_{QG} \gtrsim 100$ keV, which is many orders of magnitude less than the expectation that $E_{QG} \sim 10^{19}$ GeV. Even if this limit is weak, it indicates the direction in which LIV tests in the GW window may go. Other GW LIV tests were based on comparisons between the speeds of EM and GW signals, both of which are affected by the LIV and are emitted from the same source [36]. One possible category of such a source may be short GRBs producing GW signals resulting from NS-NS coalescences. This scenario was confirmed several years ago by LIGO-Virgo detectors when a GW signal from a binary neutron-star merger was observed along with its EM counterpart at different wavelengths [54–56]. This event pushed multi-messenger astronomy to the next level, not just from the perspective of possible QG probes. Detailed information concerning LIV tests in the gravity sector can be found in the rich and comprehensive review [30] and the references therein.

3.2. Background Cosmology Impact on the Time-Delay Technique

One of the reasons for searches for QG is the existence of two significant problems that cannot be explained within the known theories (i.e., general relativity and particle physics); the problems are known widely as dark energy [57–62] and dark matter [63–71]. The existence of a dark matter sector besides baryonic (interacting electromagnetically, and hence visible) matter is strongly suggested by the flat rotation curves of stars in spiral galaxies [63,64] and stabilities of galaxy clusters [65–67]. Moreover, it is strongly

supported by the reconstruction of mass distribution in galaxies with gravitational lensing data [68–70] as well as studies on structure formation in the universe [71]. However, many independent and precise cosmological observations, such as the Hubble diagrams for type Ia supernovae [57], the cosmic microwave background (CMB) anisotropy power spectrum combined with baryon acoustic oscillations data [58–60], and the distance ratio measurements for carefully selected samples of strong gravitational lenses [61,62], revealed the present stage of the accelerating expansion of the universe. This can be explained by postulating the existence of a hypothetical fluid filling the universe uniformly and isotropically, described phenomenologically as a cosmological constant Λ. The standard ΛCDM cosmological model of a spatially flat universe with cold dark matter [57–60,62] is the simplest and most useful model taking into account the dark matter and dark energy phenomena within a single framework. Even if it is treated as a concordance one, ΛCDM cannot be treated as a complete theory, thereby motivating the emergence of a great number of alternative cosmological scenarios trying to explain the dark matter and dark energy phenomena.

From the Formula (7) for the time-of-flight of a relativistic particle in the presence of the LIV, it is clear that it depends on background cosmology through $H(z)$. To see how strong the influence of a given cosmological scenario is with respect to a QG energy-scale estimation, we calculated (using Equation (8)) the time delay between 100 TeV GRB neutrinos and a photon signal registered in the low-energy optical band as a function of the GRB redshift $0 \leq z \leq 6$ in five different cosmological models representative for different dark-energy scenarios: ΛCDM (taken as a fiducial model), the quintessence model with a constant and varying in-time equation of state, generalized Chaplygin gas [72], and the brane world scenario. As a result, LIV-induced time delays may differ noticeably when calculated for different cosmologies. The most prominent is a mismatch between ΛCDM and quintessence with the varying equation of state—it ranges from 1.25 h (in the LIV case when a dispersion relation is modified up to the first-order term; $n = 1$) to 6 h (in an $n = 2$ LIV) for a source located at $z = 3$. The respective values for a more distant source (at $z = 6$) are almost 4 h ($n = 1$) and 27.5 h ($n = 2$). This effect does not affect the usability of the time-delay method for LIV testing but does introduce uncertainty at a level from 35% to 70% for inferred bounds on the QG energy scale (actually on $\xi_n E_{\mathrm{QG}}$), respectively, in $n = 1$ and $n = 2$ cases. Details concerning the cosmological models used in our analysis as well as a thorough description of our calculation procedure can be found in [29].

3.3. Statistical Approach to the Time-Delay Technique

A technique based on searching for LIV-induced energy-dependent time-of-flight delays between the relativistic particles from a given single astrophysical source is seriously restricted by unknown intrinsic time lags (i.e., time delays originating in the source). It is quite evident that we cannot expect that such particles were emitted simultaneously. As a remedy, one may apply a time-delay analysis to an ensemble of such sources and formulate an intrinsic time-lag problem statistically in terms of linear regression [73]. The idea is the following. The observed time delay Δt_{obs} can be split into two parts:

$$\Delta t_{obs} = \Delta t_{LIV} + \Delta t_{\mathrm{intrinsic}}, \tag{9}$$

where the first part is induced by the presence of the LIV and the second one is linked to intrinsic delays caused in the source [73]. The modified dispersion relation (3) leads to a change in the time-of-flight Equation (7), which for photons reduces to

$$t_\gamma = \int_0^z \left[1 + \frac{E_{\gamma,0}}{E_{\mathrm{QG}}}(1+z')\right] \frac{dz'}{H(z')}, \tag{10}$$

when the $n = 1$ ($\xi_1 = 1$) LIV case and $\epsilon = -1$ are taken into account. Thus, the time delay between high-energy (i.e., with energies of the order of GeV and better) and low-energy

(e.g., measured in the optical band) photons from a given cosmological source is of a simple form [25,73,74]:

$$\Delta t_\gamma = \frac{\Delta E_{\gamma,0}}{E_{\rm QG}} \int_0^z \frac{(1+z')dz'}{H(z')}, \tag{11}$$

where $\Delta E_{\gamma,0}$ is simply the energy difference between observed EM channels. With this formula (i.e., Equation (11)), the observed time-delay relation given by Equation (9) becomes [25,73]

$$\Delta t_{obs} = a_{LIV}(1+z)K(z) + b(1+z), \tag{12}$$

with the cosmological time-dilation factor $1+z$ taken into account. Formula (12) allows for using a simple statistical-regression procedure based on the relation $\Delta t_{obs}/(1+z) = a_{LIV}K(z) + b$, where the slope contains the information concerning the LIV effects

$$a_{LIV} = \frac{\Delta E_{\gamma,0}}{H_0 E_{\rm QG}}, \tag{13}$$

and intercept b informs one about intrinsic time lags. The $K(z)$ in Formula (12) is a function of redshift, and it depends on the cosmological model assumed:

$$K = \frac{1}{1+z} \int_0^z \frac{(1+z')dz'}{h(z')}. \tag{14}$$

Using this technique on a sample of 35 GRBs for which data was taken from BATSE, HETE, and Swift experiments, and by seeking time lags in GRB light curves registered in different energy channels, Ellis et al. in [73] were able to obtain a statistically robust lower bound on the QG energy scale of $E_{\rm QG} > 0.9 \times 10^{16}$ GeV (with the assumption of the ΛCDM model), thereby showing the power of this idea (see, e.g., [75–77] for a more recent analysis). The lower bound on the QG energy sale was recently improved to $E_{\rm QG} \sim 10^{17}$ GeV with high-energy GRB photons registered by the Fermi telescope [75,78,79]. Bearing in mind that the $K(z)$ function relies on the cosmological expansion rate, in [25], we performed a linear fitting procedure on time delays between different energy bands for a selected sample of GRBs (to be comparable, the same sample as used in [73]) versus the $K(z)$ function calculated in five cosmological models used in our method discussed in Section 3.2. We showed that an effect similar to that in [73] is present in all the cosmological models taken into account, suggesting that it cannot be an artifact of the assumed ΛCDM cosmology. Recently, Amelino-Camelia et al. [79] used nine GRB neutrino candidates with energies falling into the 60–500 TeV window, which were extracted over a four-year operation time from the IceCube dataset collected between June 2010 and May 2014. The selection procedure, in this case, was a highly nontrivial task. First, the neutrino interaction probability with matter is small such that neutrino detectors can catch only at least one neutrino from the whole GRB. Second, when contemplating any LIV processes disturbing the particle propagation in a vacuum, one has to take into account the time delays between the detection moments of GRB emission in the neutrino and EM window (this can be of the order of a few hours up to a couple of days for 100 TeV neutrinos, depending on the cosmological scenario; see Sections 3.1 and 3.2). Thus, a strong linear trend revealed during the analysis may suggest the presence of LIV-disturbing neutrino propagation but may also be an artifact of incorrect selection criteria in the sample, while a significant portion of the GRB neutrino candidates used here are actually background neutrinos that were accidentally correlated with the GRB signal (see a detailed discussion in [79]).

4. Gravitational Lensing for Quantum-Gravity Testing

When the light emitted from a distant astrophysical source (a quasar, in most cases) is deflected by the presence of a massive object (a galaxy), then a strong gravitational lensing occurs. As a consequence, we observe multiple images of a source that are magnified and

distorted [80]. Galaxy-lensing studies suggested that massive elliptical galaxies are the most likely population of lenses [69], motivating the use of the so-called singular isothermal sphere (SIS) model of 3D mass distribution in the lensing galaxy:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}, \tag{15}$$

where σ_v^2 is the one-dimensional velocity dispersion of stars in the lensing galaxy and G is the gravitational constant. With this model, we observe two images: one magnified and one demagnified (i.e., μ_+ and μ_-) formed on opposite sides of the lens. The angular separation between images is $\theta_\pm = \beta \pm \theta_E$, with β being the angular position of the source with respect to the optical axis (unobservable), and θ_E being the so-called Einstein radius given in the SIS model by using the following simple formula (e.g., [80,81]):

$$\theta_E = 4\pi \frac{\sigma_v^2}{c^2} \frac{D_{ls}}{D_s}, \tag{16}$$

with D_s and D_{ls} being, respectively, the angular diameter distance from the observer to the source and between the lens and the source. The Einstein radius θ_E carries information concerning image separation, or better, sets the scale of image separation. The lensing magnifications are $\mu_\pm = 1 \pm \frac{1}{y}$, where $y = \beta/\theta_E$. Another important observable is the lensing time delay between images. It originates due to two effects: a Shapiro delay and a geometric time delay. The first one is caused by the gravitational potential of the lens (slowing down the clocks in gravitational potential), and the second one is related to the fact that the light rays from different images travel along paths of different lengths [80,81]. The lensing time delay in the SIS model is given using [81]:

$$\Delta t_{SIS} = \frac{2(1+z_l)}{c} \frac{D_l D_s}{D_{ls}} \theta_E \beta, \tag{17}$$

where z_l is the lens redshift and D_l is the angular diameter distance to the lens. With Equation (16) and remembering that $D(z) = r(z)/(1+z)$, where $r(z)$ is the comoving distance, this formula may be written in a slightly different manner as $\Delta t_{SIS} = \frac{8\pi\sigma_v^2}{c^3} r_l \beta$, revealing the direct dependence of the lensing time delay on the comoving distance from the observer to the lens r_l. In order to use this formula in practice, one must face the problem that β is unobservable directly but can be assessed via modeling the strong lensing system, which demands very good quality imaging combined with spectroscopy (to take advantage of stellar kinematics). Therefore, let us rewrite it using the already-mentioned quantity y, making use of the formula for θ_E, and finally, expressing y by using the flux ratio of images $\alpha = \left|\frac{\mu_+}{\mu_-}\right|$:

$$\Delta t_{SIS} = \frac{32\pi^2}{H_0} \left(\frac{\sigma_v}{c}\right)^4 \frac{\alpha+1}{\alpha-1} \frac{\tilde{r}_l \tilde{r}_{ls}}{\tilde{r}_s} \tag{18}$$

where $\tilde{r}(z) = \frac{H_0}{c} r(z)$ is the dimensionless comoving distance.

This opens a chance to test LIV effects with the lensing time-delay method for the following idea (for more details, see [24]). Namely, the possible non-zero LIV effects might be manifested as small corrections to lensing time delays that are energy dependent. The best way to calculate this is through corrections in the formulae for comoving distances. Physically, this construction looks a bit artificial because the true comoving distance of the object is just $r(z)$ irrespective of the LIV. However, the reasoning is closely related

to the time-of flight-difference $\Delta t = \Delta r_{\rm LIV}(z)/c$, where $\Delta r_{\rm LIV}(z) = |r_{\rm LIV}(z) - r(z)|$. Using Equations (6) and (7) from Section 3, one can easily see that

$$r_{\rm LIV}(z) = r(z) + \frac{1}{2}(n+1)\epsilon\left(\frac{E_0}{\xi_n E_{\rm QG}}\right)^n \int_0^z \frac{c(1+z)^n dz}{H(z)}. \tag{19}$$

Introducing the notation $I_n(z_1, z_2) = \int_{z_1}^{z_2} \frac{(1+z')^n dz'}{h(z')}$, where $h(z)$ is the dimensionless expansion rate, one may write: $r_{\rm LIV}(z) = r(z) - \Delta r_{\rm LIV}(z)$, where $r(z) = \frac{c}{H_0} I_0(0, z)$ is the standard comoving distance of the source at the redshift z and the LIV correction is

$$\Delta r_{\rm LIV}(z) = \frac{c}{H_0}\frac{n+1}{2}\epsilon\left(\frac{E}{\xi_n E_{\rm QG}}\right)^n I_n(0, z). \tag{20}$$

Furthermore, it would be useful to simplify expressions by introducing the small parameter ε:

$$r_{\rm LIV}(z) = r(z)[1 - \varepsilon_{\rm LIV}(z)], \tag{21}$$

where

$$\varepsilon_{\rm LIV} = \frac{n+1}{2}\epsilon\left(\frac{E}{E_{\rm QG}}\right)^n \frac{I_n(0, z)}{I_0(0, z)}. \tag{22}$$

Assume that we observe a source at a cosmological distance emitting low-energy and high-energy photons that undergo gravitational lensing by a foreground galaxy—the lens. Then, we would observe the time-delay Equation (17) between the two images. According to standard physics, this delay would be the same in each energy channel. If the LIV effects are present, we would again observe time delays. However, there would be a combined effect of lensing-delay Equation (17) and energy-dependent time-of-flight delay Equation (7). In order to calculate the lensing time delay $\Delta t_{\rm SIS,LIV}$ in the presence of LIV, one should take Equation (18) with comoving distances modified according to (21) and retain terms of the first order in $\varepsilon_{\rm LIV}(z)$. Assuming the LIV effects of the first order $n = 1$ case and restricting to the "subluminal" ($\epsilon = -1$) motion of high-energy photons, the difference in time delays would be

$$\Delta t_{\rm SIS,LIV} - \Delta t_{\rm SIS} = \frac{32\pi^2}{H_0}\left(\frac{\sigma_v}{c}\right)^4\left(\frac{\alpha+1}{\alpha-1}\right)\left(\frac{\Delta E_0}{E_{\rm QG}}\right) J_1(z_l, z_s), \tag{23}$$

where ΔE_0 is again the energy difference between observed EM channels and

$$J_n(z_l, z_s) \equiv \left[\frac{I_n(0, z_l)}{I_0(0, z_l)} + \frac{I_n(z_l, z_s)}{I_0(z_l, z_s)} - \frac{I_n(0, z_s)}{I_0(0, z_s)}\right]. \tag{24}$$

The method described in this section is purely phenomenological and thus is independent of any particular LIV QG model. The next advantage is that it is free from any intrinsic time lags raised at the source; being the same for different images, they will be canceled when calculating the time-delay difference. This makes the method highly attractive, even if it is less restrictive than those based on the time-of-flight measurements described in Section 3. The lesser restrictive power is due to the fact that the distance dependence here involves ratios of distances in the system; thus, the cumulation process of the LIV effects is not so straightforward (see also the discussion in [30]).

The LIV tests based on the time-delay differences of the lensed EM signals may be extended to other particle types (e.g., gravitons). The propagation of GWs and EM waves in the geometric-optics regime is analogous: both photons and gravitons travel along null geodesics. Hence, there is a chance of a strong lensing of GW signals. The general expectation is that such a phenomenon would result within the SIS model in two differently magnified (which translates into the difference between the amplitudes of each lensed signal) time-delayed waveforms with the same chirping structure [82–85] (see also [86]). Such an expectation is true for chirp signals and most of the realistic duration of the lensing

time delay. If this delay is very short, signals from images might interfere. Surely, this is the case for continuous sources, including the inspiral phase well before coalescence. For a more comprehensive discussion of these issues, see [87] and the references therein.

Following the idea of using EM time-delay differences as a possible LIV test discussed in this section, one may now think of taking advantage of simultaneous detections of lensed coalescence signals in both the EM and GW domains. Indeed, the idea presented above was used in [26,27] to propose the method of constraining the speed of gravity and hence the graviton mass (see also [88]). Recently, Ref. [89] presented a new method of measuring graviton mass (or equivalently measuring the speed of gravity) based on the diffraction patterns of lensed gravitational waves. Such a pattern may be produced, for example, in lensing by point-mass lenses, such as intermediate-mass black holes (see, e.g., [87]). If gravitons have mass, the dispersion relation and speed of gravitational waves will be affected in a frequency-dependent manner: $v_g(f) = 1 - \frac{m_g^2}{8\pi^2 f^2}$ (in units $G = c = \hbar = 1$), which would leave imprints in the diffraction pattern if the waves are lensed. More precisely, the GW wave-strain in the frequency domain is $\tilde{h}(f) = \tilde{A}(f)e^{i\Psi(f)}$. In the case of the massive graviton, the dispersive GWs propagating in space–time will acquire a dephasing due to a difference in the propagation speeds among different frequencies: $\Psi_{disp} = -\frac{\pi D(z)}{\lambda_g^2}\frac{1}{(1+z)f}$, with $D(z)$ being the comoving distance and λ_g being Compton's wavelength of graviton. Eventually, $\tilde{h}_{disp}(f) = \tilde{h}(f)e^{i\Psi_{disp}(f)}$. In the wave-optics regime, the GW signal undergoing gravitational lensing is (in the frequency domain) multiplied by the amplification factor $F(f; M_{lens}, y)$ given by the Kirchhoff integral (see, e.g., [87]). For point mass and SIS lenses, the amplification factor can be calculated analytically. Finally, we have $\tilde{h}_L(f; m_g) = F(f; M_{lens}, y)\tilde{h}(f)e^{i\Psi_{disp}(f)}$, and as demonstrated in [89], lensing modifies the waveform morphologies of dispersive GWs, making the morphology changes sensitive to the graviton mass. This can be further improved through an increase in the signal-to-noise ratio due to lensing.

Concerning the LIV theories, the imprint of the modified dispersion relation $E^2 = p^2c^2 + m_g^2c^4 + \mathbb{A}p^nc^n$ on the propagation of a GW signal was studied in [42]. It was shown that besides the dephasing $\Psi_{disp}(f)$ due to a non-zero graviton mass, GWs acquire an additional change of phase due to the LIV: $\Psi_{LIV}(f) = -\frac{1}{1-n}\frac{\pi D_n(z)}{f^{1-n}\lambda_{\mathbb{A}}^{2-n}}$ for $n \neq 1$ and $\Psi_{LIV}(f) = \frac{\pi D_1(z)}{\lambda_{\mathbb{A}}}\ln\left(\frac{f}{f_{coal}}\right)$ for $n = 1$. The following notations were applied in the above mentioned formulas: $\lambda_{\mathbb{A}} = h\mathbb{A}^{1/(n-2)}$ and $D_n(z) = \frac{(1+z)^{1-n}}{H_0}I_{n-2}(z)$, with $I_n(z)$ introduced earlier in this section. One may expect that following the idea proposed by [89] would result in a promising method to detect or constrain the LIV effects with strongly lensed GW signals. A respective study is under consideration.

5. Discussion and Future Prospects

With the lack of any solid guidance on both the theoretical and experimental sides, all up-to-date attempts to build the QG theory can still be regarded as wandering in a fog. The only robust landmark is the prediction common to several QG frameworks that QG may reveal itself in a low-energy regime in the form of some non-standard effects, such as the LIV. This creates an opportunity for QG testing with a phenomenological approach based on the deformed dispersion relation of relativistic particles, as was discussed in Section 2. Such a technique may be applied to high-energy astrophysical sources, especially when all its multi-messenger capabilities are considered. In this context, searches for tiny differences induced by energy-dependent LIV corrections in the time-of-flight of photons from distant astrophysical sources may not only be provided by the ultra-high energies beyond gamma rays but also be supported by other particle types, such as high-energy cosmic rays, neutrinos, or gravitons (GW signals), as was discussed in detail in Section 3 (see also a comprehensive review of this topic in [30] and the references therein). In particular, the idea of using gravitationally lensed time delays via a careful analysis

between lensed EM signals observed in different energy bands or between lensed GWs and EM waves may be promising for LIV testing independent of the unknown intrinsic time lags raised in the source (moreover, the influence of a background cosmology is not very strong in this method; see Section 4). The latter idea is especially interesting in light of the successful operation of LIGO-Virgo [90–92] (joined recently by KAGRA [93,94]) detectors and is particularly promising in light of the future observational runs of a new generation of ground-based GW interferometers, such as the Einstein telescope (ET; see [95]), and space-borne missions, such as the laser interferometer space antenna (LISA; [96]) and the DECihertz Interferometer Gravitational Wave Observatory (DECIGO; [97–99] and its smaller-scale version B-DECIGO [100,101]). This will boost GW astronomy to a higher level; the greatly improved sensitivities of these planned new detectors will allow the detectors to probe volumes many times larger than those accessible to LIGO-Virgo observations. For example, the ET detector's range for an initial configuration is predicted to be $r_0 = 1527$ Mpc, which may be extended to $r_0 = 1918$ Mpc when an advanced 'xylophone' configuration will be finalized [95]. The DECIGO noise spectrum suggests a detector reach of $r_0 = 6709$ Mpc, which will allow for probing of an about 64 times larger volume than the ET [102]. Consequently, we expect impressively rich catalogs of GW signals from coalescing double-compact objects (i.e., BH-BH, BH-NS, and NS-NS). For example, the expected yearly rates at which such sources would be detected in the GW window are of the order of 10^4–10^6 for the ET [83], 10^2–10^6 for DECIGO and 10^3–10^5 for B-DECIGO [85], depending on the type of observed double-compact object system, population synthesis scenario, and galaxy metallicity evolution. The predictions concerning gravitationally lensed GW signals, which will potentially be registered by these planned detectors, are such that the ET would be able to detect 50–100 lensed GW signals yearly [83] and DECIGO/B-DECIGO would register about 50 lensed GW events per year [85] (mainly binary black hole systems, especially in the DECIGO/B-DECIGO case when the background from unresolved double-compact objects influences the detection ability of the lensed NS-NS and BH-NS coalescences; see [85] for more information concerning this question). From the perspective of LIV testing via the time-delay technique with lensed EM and GW signals, of particular significance is the fact that the ET and especially LISA or DECIGO/B-DECIGO will have increased capabilities for the registration of GW signals at frequencies lower than about 1 Hz. Thanks to this, a double-compact object would be observable in the inspiral phase for a long time (i.e., for weeks up to years; see [101]) before the coalescences. Thus, monitoring such a system in the inspiral phase allows for being prepared to register it later in the merger stage not only as a chirping GW waveform but also as a GRB in the EM window (binary neutron star mergers are expected to produce transient EM-signal observables as short GRBs). In light of the first LIGO-Virgo observational evidence for the coalescing binary neutron star system GW170817 [14], registered later on at different wavelengths as its EM counterpart [15], this proposal seems to be realistic, especially when the ET will be able to observe a few such events yearly [82–84,103]. In addition, the time delay between lensed GW signals is predicted to range from a few seconds for ground-based detectors up to a few months for GW interferometers in space [104], thereby supporting the utility of LIV tests with gravitational lensing in the GW domain.

Let us conclude that the future prospects of multi-messenger astronomy for constraining QG phenomenologies look interesting. In particular, the opening of the GW window and extending the EM domain to the TeV range are creating unprecedented opportunities.

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