Title: The Mathematics of Strange Objects

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Your Magnificence, Honourable Senate, Ladies and Gentlemen!

Let me stress that I am extremely happy and proud of the title bestowed on me today. Now, I would like to say a few words about myself and what I am working on. True, I would need a blackboard and mile-long formulas. So, instead, I will tell you a story. It is not very long. It covers about two and a half thousand years.

As we all know, mathematics, particularly geometry, deals with beautiful and regular objects, such as straight line, circle, ellipse, triangle, polygon, etc. It was so in the past, and to some degree it is so today. The role of those classic sets was taken over by differentiable manifolds, or – to describe them in the simplest way – smooth surfaces. But to many people it will be a surprise to discover that mathematics deals also with lines such as the border of the cloud or the shape of English shores. More than that, on the plane there are sets, interesting for a mathematician, which we cannot draw at all.

The question arises: how such changes came into being, what was their beginning? In fact, to understand modern mathematics, one has to go back to its history. And a strange thing can be noticed then, namely, the history of mathematics, and, more generally, the history of science is surprisingly closely twined with the history of art. This view was formulated in the most direct way by Naum Gabo, a sculptor and a theorist of art, one of the most active representatives of constructivism. He believed that *the connection between the knowledge and the art was never broken in all history of human culture.* Gabo illustrated such thesis with paradoxical examples, trying to show relations between Raphael's Madonnas and Copernicus's heliocentric theory.

Not all Gabo's arguments are convincing, but I do agree that similarities between the history of art and the history of maths are not just a coincidence. They have their own profound reasons. Let us begin with

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the observation that the powerful and well-organized state can afford to support both arts and sciences, in particular one of the oldest sciences – mathematics. The classical example is Egypt. Egyptian priests calculated the trajectories of stars and planets on the sky with great precision and using the technique we could today call "analogue." Egyptian temples inspire a sense of wonder not only with their size, but also with rare sense of shape.

But it was Greece, where the art and the science came closest. Greece, whose small states lacked the wealth and the power of pharaohs. So, what are the main elements of the phenomenon of Greek art? Well, the Greeks not only created masterpieces, but also set up canons for the art itself, in its many domains: in sculpture, architecture, theatre. For me personally, the symbol of Greek sculpture is Laocöon Group – father and sons being strangled by the serpents. The central figure of Laocöon expresses suffering, anger and terror, but, at the same time, it is absolutely beautiful, perfect in every aspect of the craft. Pliny considered Laocöon Group to be the highest achievement in the domain of plastic arts. There was no such art before the Greeks.

Masterpieces of Greek theatre can be seen on stage to this day. Its canon may be developed and revised, but still there are Greek roots in each great tragedy. Also, on almost any building of American university we can discover the traces of Greek architecture.

Similarly in mathematics. Greeks not only knew about many mathematical facts, not only enriched mathematics with so many discoveries, but – first and foremost – they understood the essence of the structure of mathematical theories, the structure of mathematics itself. Let me quote the words of Bourbaki[1]: "The essential originality of the Greeks consisted precisely of a conscious effort to order mathematical proofs in a sequence such that passing from one link to the next leaves no room for doubt and constrains universal assent." Not only did Greeks understand this, but they were able to carry this out. More than twenty-three centuries before our day a monumental work was created, that is, Euclid's *Elements*. It has shown the way, the method to deduce all known theorems in geometry from the few relatively simple axioms. It has set the canon of construction of mathematical theories – the canon that stands up to this day. There was no such mathematics before the Greeks.

What was the reason for such "big bang" of arts and of mathematics? The answer can be expressed in a simple way: both art and mathematics

[1]Nicolas Bourbaki, *Elements of the History of Mathematics*, trans. John Meldrum (*Eléments d'histoire des mathématiques*, Paris 1984). Springer-Verlag 1991.

are created by people, who are passionate and inventive, for whom the "old" is not enough. And ancient Greeks were just such people. The passion and the inventiveness are certainly the characteristics of any creator, and a man of science in particular, but the mathematician – compared to the researchers of other domains – is much less constricted by the external world, and this makes his work similar to an artist's. Just like an artist, he creates his own object of interest. In the words of Professor Tomasz Łuczak, "the art and the mathematics deal with reality existing as though alongside the world around us." It should be noted, though, that mathematical theories sooner or later prove useful for the description of our reality and coincide with it perfectly well. Why is it so, we do not know. But this is another matter.

As I said, I do not entirely share Gabo's views; I do not suggest that the history of art and history of mathematics are parallel. The lots of such sophisticated, such different worlds cannot be identical. However, twenty-two centuries after the creation of *Elements*, that is, at the beginning of the twentieth century, mathematicians observed the facts showing a clear similarity to the experiments of artists.

Let us recount, what did the beginning of the twentieth century bring to art. Broadly speaking, it brought the fundamental question about the essence of art. What is it, what the subject of its expression should be? By all means, the problem is rather old, but it was stormed with greater intensity than ever before. Never before have so many answers, so many mutually opposing artistic programs been formulated. Cubism, futurism, expressionism, dadaism, constructivism or pop-art – to recall only a few best known names from more than a dozen described in art history textbooks. Have they resolved the problem of art and what should it be? No, clearly not, but they have shown the vast and diverse possibilities. Some of them brought us great creations and masterpieces, while others were almost forgotten.

What is their relation to mathematics? Certainly not that the objects in the cubist paintings were constructed from relatively simple geometrical figures. The point is that, at the beginning of the twentieth century, mathematicians – just like artists – started to pose questions about the basic notions, never questioned before. They asked, for example, what is a line, and what is a dimension of a set? Obviously, straight line, ellipse, parabola or hyperbola are lines. A graph of any sufficiently regular function would be called a line, as well. But each spectator of a catastrophic, earthquake movie certainly noticed that just before the shock itself, the seismometer stylus draws a more and more condensed graph, practically completely covering some part of the paper surface. Is that also a line?

The question of dimension is ostensibly simple too. It is clear that all mentioned lines should have the dimension 1. But what dimension should we attribute to the graph of the function modeling the motion of a crazed seismometer?

To a considerable degree, these difficulties were resolved. Resolved too well, even, we can say sarcastically. In fact, several definitions of a line were formulated, and more than a dozen definitions of dimension. Not all of them were equivalent, not all of them became the starting points to further research. But together they have shown us the wealth of possibilities hidden in such fundamental concepts. It was demonstrated that there exist lines having very unusual properties. For example, we can construct a line branching in each point. There also exists a line so rich, so intricately woven that it contains the (homeomorphic) image of any other line. The real master of construction of such sets was Wacław Sierpiński.

However, this is just the beginning of the story. Sets (lines) constructed by Sierpiński have some other strange properties. Strangest of them all is probably the fact that their (Hausdorff) dimension is not an integer. Thus, they were called fractals – from the Latin *fractus*, meaning "broken," "fractured," "not whole." Today, fractal theory became one of the most rapidly developing branches of mathematics. This fact has at least two reasons of theoretical nature. First, limit sets, attractors of many important dynamic systems, are fractals. Second, mathematical analysis can be developed on fractals, just like it is on linear spaces. Yet there is also a third reason, possibly crucial: fractals can be easily constructed with computers; moreover, fractals can approximate any arbitrary picture. The cover of first edition of the popular monograph *Fractals Everywhere* by Michael Barnsley was illustrated with the painting of a small Peruvian girl with deep, beautiful eyes, wearing ornamental Indian hat. This is also a fractal, or rather a union of a number of fractals. In this way, fractals became the medium of information, or – to say it in a professional way – one of compression techniques. They are now inseparably connected with computer graphics. Through fractals, mathematics and art today are closer than ever before.

For a layman, the symbol of modern art is Pablo Picasso. He was born in 1881 and lived over 90 years. Wacław Sierpiński was born one year later and lived almost equally long time. In 1914 Picasso paints his famous piece *Figure*, consisting of geometrical lines and blocks. In 1915 Sierpiński constructs the most important of all fractals – his famous triangle. This coincidence of dates looks like a joke of history. But it is not a joke that –

thanks to fractal techniques – mathematics can be useful for art in a different, more sophisticated way than calculating proportions of Greek and Romans buildings.

I would like to summarize my lecture as follows: modern art, as well as modern mathematics, and fractal theory in particular, have proven that both human creative possibilities and research fields are vastly wider than we thought a hundred years ago. Let us hope that in a hundred years students of our students will be able to say the same.