PHONON–INDUCED SUPERCONDUCTIVITY IN THE PRESENCE OF THE CHARGE–DENSITY–WAVE PSEUDOGAP. STRONG–COUPLING DESCRIPTION*

ANNA CEBULA AND MARCIN MIERZEJEWSKI

Institute of Physics, Silesia University
Uniwersytecka 4, 40-007 Katowice, Poland
e-mail: ania@server.phys.us.edu.pl

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We have considered phonon-induced superconductivity in the presence of the pseudogap originating from Charge–Density–Wave (CDW) excitations within the two-dimensional lattice. Eliashberg formalism has been applied and the CDW effects have been taken into account with the help of the renormalization of propagators in the Dyson equation. The CDW gap has been incorporated in the semiphenomenological way, assuming the d-wave symmetry. We have evaluated the superconducting transition temperature T_c as a function of doping. The influence of the normal-state pseudogap on the isotope shift exponent has also been considered.

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In the underdoped high-temperature superconductors pseudogap behavior has widely been observed in experiments [1]. It seems that the pseudogap is intimately linked with superconductivity. The magnitude of the pseudogap decreases with increasing doping and scales with $T_{c\,\max}$ [2, 3]. In particular optical data indicate that the pseudogap is related to the superconducting gap [4]. Whether both of this quantities originate from the same or from different mechanisms seems to be an open problem [5]. The same refers to the actual origin and symmetry of the pseudogap. Here, if we stick to the concept of the same origin, d-wave symmetry can be accepted. This is due to the experimental evidence of the dominating role of this kind of the order parameter in the superconducting state [6]. Certainly, one of

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the possibilities is to consider antiferromagnetic spin fluctuations which lead to the charge–density–wave (CDW) and superconducting gaps (SC) having $d_{x^2-y^2}$-wave symmetry.

In the present paper we will consider phonon-induced superconductivity within the two-dimensional lattice. Eliashberg formalism will be applied and the CDW effects will be taken into account with the help of the renormalization of propagators in the Dyson equation. The CDW gap will be incorporated in the semiphenomenological way, assuming the d-wave symmetry [2, 7].

We consider the following model Hamiltonian

$$H = H_0 + H_{e-\mathrm{ph}} + H_{\mathrm{ph}}, \tag{1}$$

where

$$H_0 = \sum_{\boldsymbol{k}\sigma} \bar{\varepsilon}_{\boldsymbol{k}} c^+_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma}, \tag{2}$$

$$H_{e-\mathrm{ph}} = \sum_{\boldsymbol{k},\boldsymbol{q}} g_{\boldsymbol{k}\boldsymbol{k}+\boldsymbol{q}} c^+_{\boldsymbol{k}+\boldsymbol{q}\sigma} c_{\boldsymbol{k}\sigma} \left(b^+_{-\boldsymbol{q}} + b_{\boldsymbol{q}} \right), \tag{3}$$

$$H_{\mathrm{ph}} = \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} b^+_{\boldsymbol{q}} b_{\boldsymbol{q}}. \tag{4}$$

Here, H_0 represents non-interacting electrons in the second quantization form. $\bar{\varepsilon}_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}} - \mu$, where μ stands for the chemical potential and $\varepsilon_{\boldsymbol{k}} = -t\gamma(\boldsymbol{k})$ with $\gamma(\boldsymbol{k}) = 2(\cos k_x a + \cos k_y a)$ for the two-dimensional lattice. $H_{e-\mathrm{ph}}$ describes the electron–phonon interaction and H_{ph} stands for the Hamiltonian of non-interacting phonons. For the sake of simplicity phonons will be modelled by Einstein oscillator with frequency ω_0.

The self-energy can be derived from the matrix Dyson equation [10]:

$$\Sigma_{\boldsymbol{k}}(i\omega_l) = G^{-1}_{0\boldsymbol{k}}(i\omega_l) - G^{-1}_{\boldsymbol{k}}(i\omega_l), \tag{5}$$

where $G_{\boldsymbol{k}}(i\omega_l)$ stands for the Matsubara Green's function and $G_{0\boldsymbol{k}}(i\omega_l)$ denotes the unperturbed propagator; ω_l is the Matsubara frequency $\omega_l = (2l+1)\pi/\beta$, $\beta = (kT)^{-1}$. In order to account for the CDW state we renormalize the electron propagator in the Dyson equation. This corresponds to the substitution

$$G^{-1}_{0\boldsymbol{k}}(i\omega_l) = \begin{pmatrix} G^{-1}_{\mathrm{CDW}}(i\omega_l) & 0 \\ 0 & -G^{-1}_{\mathrm{CDW}}(-i\omega_l) \end{pmatrix}, \tag{6}$$

with

$$G^{-1}_{\mathrm{CDW}}(i\omega_l) = i\omega_l - \bar{\varepsilon}_{\boldsymbol{k}} - \frac{|\Delta_{\mathrm{CDW}}|^2}{4(i\omega_l - \bar{\varepsilon}_{\boldsymbol{k}+\boldsymbol{Q}})}, \tag{7}$$

where $\boldsymbol{Q} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$. Here, we have made use of the Balseiro–Falicov form of the quasiparticle energies [11]. We restrict ourselves to the mean-field form of Δ_{CDW} [2,3,7] and assume,

$$\Delta_{\mathrm{CDW}} = E_g(\cos k_x a - \cos k_y a). \tag{8}$$

The usual ansatz for $\Sigma_{\boldsymbol{k}}(i\omega_l)$ is of the form

$$\Sigma_{\boldsymbol{k}}(i\omega_l) = [1 - Z_{\boldsymbol{k}}(i\omega_l)]\, i\omega_l \tau_0 + \phi_{\boldsymbol{k}}(i\omega_l)\tau_1 + \chi_{\boldsymbol{k}}(i\omega_l)\tau_3. \tag{9}$$

$\tau_0, \ldots, \tau_3$ stand for the Pauli matrices. $Z_{\boldsymbol{k}}(i\omega_l)$ denotes the wave function renormalization factor which can be considered in the local approximation. Here, $\chi_{\boldsymbol{k}}(i\omega_l)$ is a small quantity and can be neglected in numerical calculations. For the case of local and nearest-neighbor Cooper pairs we can express the momentum dependence of the superconducting order parameter by

$$\phi_{\boldsymbol{k}}(i\omega_l) = \phi_0(i\omega_l) + \gamma(\boldsymbol{k})\phi_\gamma(i\omega_l) + \eta(\boldsymbol{k})\phi_\eta(i\omega_l). \tag{10}$$

Here, $\phi_0(i\omega_l)$, $\phi_\gamma(i\omega_l)$, $\phi_\eta(i\omega_l)$ denotes the s-wave, extended s-wave and d-wave amplitudes, respectively; $\eta(\boldsymbol{k}) = 2(\cos k_x a - \cos k_y a)$.

In the case of d-wave superconductivity the transition temperature is determined by the Eliashberg equations

$$Z(i\omega_l) = 1 + \frac{1}{\beta\omega_l} \sum_{\omega_n} \frac{\lambda\, \nu^2}{(l-n)^2 + \nu^2}\, d_Z(i\omega_n), \tag{11}$$

$$\phi_\eta(i\omega_l) = \frac{1}{4\beta} \sum_{\omega_n} \frac{\lambda_\gamma\, \nu^2}{(l-n)^2 + \nu^2}\, d_{\eta^2}(i\omega_n)\phi_\eta(i\omega_n), \tag{12}$$

where we have denoted

$$d_Z(i\omega_n) = \frac{1}{N} \sum_{\boldsymbol{k}} \omega_n d_{\boldsymbol{k}}(i\omega_n) \left[Z(i\omega_n)(\omega_n^2 + \bar{\varepsilon}^2_{\boldsymbol{k}+\boldsymbol{Q}} + |\Delta_{\mathrm{CDW}}|^2)\right], \tag{13}$$

$$d_{\eta^2}(i\omega_n) = \frac{1}{N} \sum_{\boldsymbol{k}} \eta^2(\boldsymbol{k}) d_{\boldsymbol{k}}(i\omega_n)(\omega_n^2 + \bar{\varepsilon}^2_{\boldsymbol{k}+\boldsymbol{Q}}) \tag{14}$$

with

$$\begin{aligned} d^{-1}_{\boldsymbol{k}}(i\omega_n) = {} & (\bar{\varepsilon}^2_{\boldsymbol{k}} + \omega_n^2 Z^2(i\omega_n))(\omega_n^2 + \bar{\varepsilon}^2_{\boldsymbol{k}+\boldsymbol{Q}}) - 2\bar{\varepsilon}_{\boldsymbol{k}}\bar{\varepsilon}_{\boldsymbol{k}+\boldsymbol{Q}}|\Delta_{\mathrm{CDW}}|^2 \\ & + |\Delta_{\mathrm{CDW}}|^4 + \omega_n^2 |\Delta_{\mathrm{CDW}}|^2 Z(i\omega_n). \end{aligned} \tag{15}$$

In addition one has the equation for the chemical potential

$$\frac{n}{2} = \frac{1}{N} \sum_{\boldsymbol{k}} \sum_{i=1}^{2} (-1)^{i+1} \frac{\omega_{i\boldsymbol{k}} - \bar{\varepsilon}_{\boldsymbol{k}+\boldsymbol{Q}}}{\omega_{1\boldsymbol{k}} - \omega_{2\boldsymbol{k}}} f(\omega_{i\boldsymbol{k}}), \tag{16}$$

where

$$\omega_{1,2\boldsymbol{k}} = \frac{1}{2}\left(\bar{\varepsilon}_{\boldsymbol{k}} + \bar{\varepsilon}_{\boldsymbol{k}+\boldsymbol{Q}} \pm \sqrt{(\bar{\varepsilon}_{\boldsymbol{k}} - \bar{\varepsilon}_{\boldsymbol{k}+\boldsymbol{Q}})^2 + 4|\Delta_{\mathrm{CDW}}|^2}\right). \tag{17}$$

Note that when accounting for nearest-neighbor Cooper pairs, one obtains two different electron–phonon coupling functions λ and λ_γ [12]. Generally, λ and λ_γ depend on the occupation number [13], however, in order to simplify our model calculations, we consider them as a parameters. For numerical purposes we have used Kresin's method of introducing an average phonon frequency $\langle\Omega\rangle$ [14]

$$\nu = \frac{\langle\Omega\rangle}{2\pi k T_{\mathrm{c}}} \tag{18}$$

which corresponds to the frequency ω_0 of an Einstein oscillator ($\omega_0 = 0.1t$ has been used throughout this paper).

In figure 1 we present results for the superconducting transition temperature in the case of d-wave symmetry. It is clear that the doping dependence of T_{c} in the presence of the CDW pseudogap better reflects the experimental data.

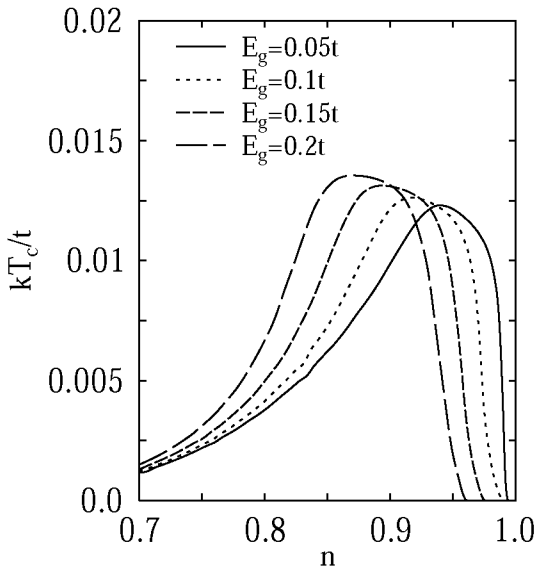

Fig. 1. The superconducting transition temperature as a function of band filling for d-wave symmetry. We have assumed $\lambda = 3$ and $\lambda_\gamma = 1.5$ for electron–phonon coupling.

An intriguing experimental result, which contributes to the discussion on the possible role of phonons in high-temperature superconductivity, is the variation of the isotope shift exponent α as a function of doping [8, 9]. We have evaluated the isotope shift exponent under assumption that kernels in the Eliashberg equations depend on M only through the average phonon frequency $\omega_0 \sim M^{-1/2}$. Then

$$\alpha = \frac{\omega_0}{2T_c}\frac{dT_c}{d\omega_0}. \tag{19}$$

Figure 2 shows the dependence of the isotope shift exponent on the magnitude of the pseudogap for different values of the occupation number. This results can be compared to values presented in figure 3. Here, we show α as a function of n for different values of E_g. One can say that the inclusion of the pseudogap can contribute to smaller values of α, than the BCS value of $\frac{1}{2}$. This feature remains in qualitative agreement with experimental results [8].

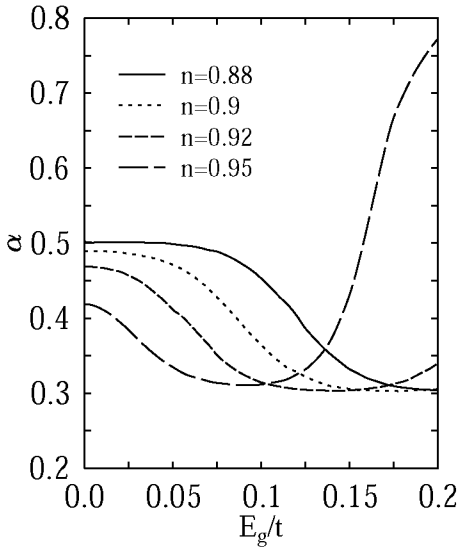

Fig. 2. The isotope shift exponent α as a function of the amplitude of the CDW order parameter E_g for different values of the occupation number.

The above results suggest that the occurrence of the pseudogap originating from charge–density–waves can play non-negligible role when considering properties of high-temperature superconductors with electron–phonon interaction as a driving mechanism.

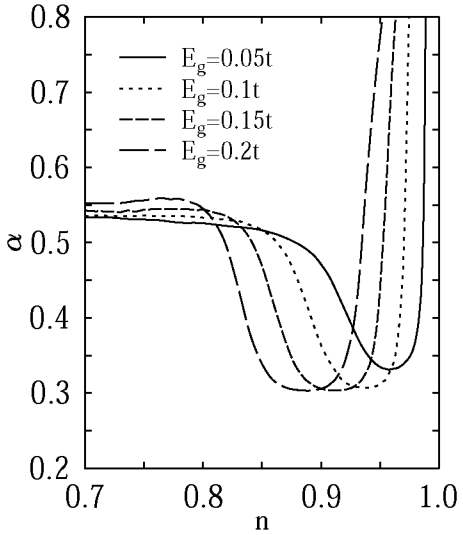

Fig. 3. The isotope shift exponent α as a function of band filling for different values of E_g.

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