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NOISE-ASSISTED CURRENTS IN A CYLINDER-LIKE SET OF MESOSCOPIC RINGS*

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We study magnetic fluxes and currents in a set of mesoscopic rings which form a cylinder. We investigate the noiseless system as well as the influence of equilibrium and non-equilibrium fluctuations on the properties of selfsustaining currents. Thermal equilibrium Nyquist noise does not destroy selfsustaining currents up to temperatures of the same order as the critical temperature for selfsustaining currents. For temperatures below the critical temperature, randomness in the distribution of parity of the coherent electrons can lead to disappearing of selfsustaining currents and inducing new metastable states. For temperatures above the critical temperature, it causes a creation of new metastable states with non-zero currents.

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1. Introduction

Quantum phenomena manifested at the mesoscopic level have attracted much experimental and theoretical attention. Phase coherence and persistent currents can be mentioned as examples. Persistent currents of the so called coherent electrons are a direct manifestation of the Aharonov–Bohm effect at the mesoscopic level. They were predicted as early as in 1938 [1] and have been observed experimentally only since 1990 [2]. In the paper we study the steady state magnetic fluxes and currents in mesoscopic rings under conditions when dissipation and fluctuations can play an important role [3]. Our system consists of a set of concentric one dimensional rings which form a cylinder. It is expected [4] that in such a system selfsustaining currents can occur in the absence of the external flux. In the ground state, at

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$T = 0$, only coherent electrons are present in the system and the persistent current flows without dissipation. The non-zero temperature $T > 0$ reduces the amplitude of the persistent current and some electrons become "normal" (*i.e.* non-coherent). Then coherent and normal electrons coexist.

In the system at temperature $T > 0$ there are various sources of noise and fluctuations. There are so-called universal conductance fluctuations [5] that arise from the random quantum interference between many electron paths which contribute to the conductance in the diffusive regime. These fluctuations decay algebraically with temperature and can be neglected at higher temperatures [5]. Inelastic transitions in the ring cause another kind of fluctuations. However, they do not destroy persistent currents but reduce their amplitude [6]. There is also a part of the current noise which is called shot noise [3], the spectral density of which is proportional to mean current. This noise can be reduced by increasing the size of rings [7]. Thermal motion of charge carriers in any conductor is a source of Nyquist noise [3]. This thermal equilibrium noise is universal and is present in any conductor. Moreover, this noise increases with temperature and induces fluctuations of current. We consider such conditions that universal conductance fluctuations and shot noise can be neglected. Let us notice that the system is characterized by parameters which qualitatively and quantitatively change the transport properties. As an example let us consider the parity of the coherent electron's number in the current channel. The change of the parity changes the response of the system for the applied magnetic flux from para- to diamagnetic and *vice versa.* In the paper we propose a method of dealing with this sensitivity. We consider the probability of an even number of coherent electrons in a single current channel to be either stochastic process (symmetric dichotomic process) or quenched noise (random variable). The role of Nyquist noise and other sources of fluctuations is the main subject of the paper.

2. The model

We consider a collection of rings, so called current channels, which form a cylinder with N_z channels in direction of the cylinder axis and N_r in the direction of the cylinder radius. We assume that the thickness of the cylinder wall is much smaller then the radius. The current in one ring, via mutual inductance, induces flux and current in other rings and so on. The effective interaction [8] between the ring currents, considered in the selfconsistent mean field approximation, results in the magnetic flux $\phi = LI_{\rm tot}$ felt by all electrons, where L is the cylinder inductance and $I_{\rm tot}$ is the total current in a cylinder. The inductance of a cylinder of the radius r and the height l_z reads [9]

$$L = \mu_0 \frac{\pi r^2}{l_z}\,, \tag{1}$$

where μ_0 is the permeability of the free space. At temperature $T > 0$, the current $I_{\rm coh}(\phi, T)$ of the coherent electrons in a set of $N = N_r \times N_z$ current channels forming the cylinder is either paramagnetic [4]

$$I_{\rm coh}(\phi, T) = I_e(\phi, T) = N I_0 \sum_{n=1}^{\infty} A_n(T) \sin\left(\frac{2n\pi\phi}{\phi_0}\right) \tag{2}$$

for an even number of coherent electrons in each single channel or diamagnetic

$$I_{\rm coh}(\phi, T) = I_0(\phi, T) = I_{\rm even}(\phi + \phi_0/2, T) \tag{3}$$

for an odd number of coherent electrons. The unit current

$$I_0 := heN_e/(2l_x^2 m_e)\,,$$

where l_x is the circumference of the cylinder, $k_{\rm F}$ is the Fermi momentum and N_e is the number of coherent electrons in a single current channel. The amplitude

$$A_n(T) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)} \cos(nk_{\rm F} l_x)\,. \tag{4}$$

The characteristic temperature T^* is given by the relation $k_{\rm B}T^* = \Delta_{\rm F}/2\pi^2$, where $k_{\rm B}$ is the Boltzmann constant and $\Delta_{\rm F}$ is the energy gap at the Fermi surface. For temperatures $T < T^*$ the coherent current flows in such a cylinder without dissipation but its amplitude (4) is reduced [10]. On the other hand, at temperature $T > 0$, normal electrons occur and their flow is dissipative. The motion of normal electrons is random, like the motion of electrons in a normal conductor and it generates random currents.

Since the current-flux characteristics for the coherent electrons is extraordinary sensitive to a change of parity of the coherent carriers number [10] we take into account the possible difference of parity in the rings and consider the current of coherent electrons as the average

$$I_{\rm coh}(\phi, T) = pI_e(\phi, T) + (1-p)I_0(\phi, T)\,, \tag{5}$$

where $p \in [0, 1]$ is the probability of the even number of coherent electrons in a given channel.

The current coming from the normal electrons can be induced by *e.g.* the change of the magnetic flux ϕ. From the Lenz's rule and the Ohm's law one infers that [11]

$$RI_{\rm nor}(\phi) = -\frac{d\phi}{dt}\,, \tag{6}$$

where R is the effective resistance of the system [6].

The relation between the magnetic flux and the current is given by

$$\phi = \phi_{\rm ext} + L(I_{\rm coh}(\phi, T) + I_{\rm nor}(\phi))\,, \tag{7}$$

i.e. it is a sum of the external flux $\phi_{\rm ext}$ and the flux coming from the total current.

Now, we assume that the only source of fluctuations is equilibrium noise induced by the resistance R. The correlation function of this source of fluctuations is assumed to be given by the Nyquist relation. If we take into account (5)–(7) and add the term describing current fluctuations then we obtain the equation (see the Appendix)

$$\frac{1}{R}\frac{d\phi}{dt} = -\frac{1}{L}(\phi - \phi_{\rm ext}) + I_{\rm coh}(\phi, T) + \sqrt{\frac{2k_{\rm B}T}{R}}\,\Gamma(t)\,, \tag{8}$$

where $\Gamma(t)$ is Gaussian white noise modeling Nyquist equilibrium current noise. This equation takes the form of a classical Langevin equation and is our basic evolution equation.

The dimensionless variables are introduced in the following way. In the Langevin equation (8), the basic quantity is the magnetic flux $\phi = \phi(t)$. The natural unit of the flux is the flux quantum $\phi_0 = h/e$. Accordingly, the flux is scaled as $x = \phi/\phi_0$. To identify the characteristic time τ_0, let us consider a particular case of (8), namely, when the persistent current and the external flux are zero. Then

$$\frac{d\phi}{dt} = -\frac{R}{L}\phi + \sqrt{2Rk_{\rm B}T}\,\Gamma(t)\,. \tag{9}$$

From this equation it follows that the mean value

$$\langle\phi(t)\rangle = \langle\phi(0)\rangle\,\exp(-t/\tau_0)\,, \tag{10}$$

where

$$\tau_0 = L/R \tag{11}$$

is the relaxation time of the averaged normal current. Therefore, time is scaled as $\tilde{t} = t/\tau_0$. In this case, Eq. (8) can be transformed into its dimensionless form

$$\dot{x} = -V'(x) + \sqrt{2D}\,\widetilde{\Gamma}(\tilde{t})\,, \tag{12}$$

where the dot denotes a derivative with respect to the rescaled time $\tilde{t}$ and the prime denotes a derivative with respect to x. The generalized potential

$$V(x) = V(x, \lambda, i_0, p, T) = \tfrac{1}{2}x^2 - \lambda x - i_0 F(x, p, T)\,, \tag{13}$$

where $\lambda = \phi_{\rm ext}/\phi_0$ is the rescaled external flux. The prefactor $i_0 = NLI_0/\phi_0$ is a coupling constant characterizing the interaction between ring currents (it is the rescaled amplitude of the flux created by the current — it leads to selfsustaining currents). The function

$$F(x) = F(x,p,T) = \int f(x,p,T)dx \tag{14}$$

characterizes the coherent electrons and

$$f(x,p,T) = pf_e(x,T) + (1-p)f_0(x,T)\,, \tag{15}$$

where

$$f_e(x,T) = \sum_{n=1}^{\infty} A_n(T)\sin(2n\pi x) \tag{16}$$

and

$$f_0(x,T) = f_e(x+\tfrac{1}{2},T)\,. \tag{17}$$

The dimensionless intensity D of rescaled Gaussian white noise $\widetilde{\Gamma}(\tilde{t}) \equiv \sqrt{\tau_0}\;\Gamma(\tau_0\tilde{t})$ is a ratio of thermal energy to the elementary energy stored up in the inductance,

$$D = \frac{1}{2}k_{\rm B}T/\varepsilon_0\,, \qquad \varepsilon_0 := \frac{\phi_0^2}{2L}\,. \tag{18}$$

Let us observe that the resistance R does not enter into the rescaled equation (12).

In order to evaluate the magnitudes of the parameters appearing in our equations let us notice that the rescaled coupling constant

$$i_0 = \frac{\mu_0 e^2 N}{8\pi m_e}\frac{N_e}{l_z}\,. \tag{19}$$

We assume that the cylinder has the radius $r = 3\times10^4$Å and the height $l_z = 100$Å. It consists of a set of $N \sim 50$ current channels [12] in a wall of width much smaller than the radius. If the number of electrons in each channel is $N_e \sim 2\times10^5$ then $i_0 \sim 1$. The energy gap at the Fermi surface $\Delta_{\rm F} = \hbar^2 N_e/(2m_e r^2)$ gives the rescaled noise amplitude

$$\frac{k_{\rm B}T^*}{2\varepsilon_0} = \frac{\mu_0 e^2}{16\pi^3 m_e}\frac{N_e}{l_z}\,. \tag{20}$$

For the above values of parameters the diffusion coefficient $D \sim 0.001T/T^*$. Below, unless stated otherwise, the parameters are fixed so that $i_0 = 1$, $D = 0.001T/T^*$ and the product $k_{\rm F}l_x = 0.1$ in the formula for the coherent current.

3. Analysis

In this section the properties of system described by Eq. (12) are analyzed. We consider in details two special cases. In the noiseless case, we neglect the influence of Nyquist noise. It is a justified approximation for very small intensity of noise. Formally, it can be neglected only when temperature $T = 0$ (see Eq. (8)) and, consequently, we should put $T = 0$ in $I_{\rm coh}(\phi, T)$. However, first we want to analyze the deterministic system which corresponds to the case $\widetilde{\Gamma}(\tilde{t}) = 0$ in (12) and next to investigate influence of Nyquist noise. As follows from (7), the total current is linearly related to the magnetic flux ϕ (or the rescaled flux x). In a consequence, the properties and behavior of the current are identical to the properties and behavior of the magnetic flux. Therefore, below we use equivalently these two characteristics of the system.

3.1. Selfsustaining currents

First, let us consider the deterministic case of the Langevin stochastic equation (12) formally neglecting the Nyquist noise term $\widetilde{\Gamma}(\tilde{t})$, *i.e.*,

$$\dot{x} = -V'(x)\,. \tag{21}$$

The stationary solutions $x_{\rm s}$ of (21), for which $\dot{x}_{\rm s} = 0$, correspond to extrema of the generalized potential (13),

$$V'(x_{\rm s}) = x_{\rm s} - \lambda - i_0 f(x_{\rm s}, p, T) = 0\,. \tag{22}$$

The solutions $x_{\rm s}$ of the gradient differential equation (21) are stable provided they correspond to a minimum of the generalized potential (13) and they are unstable in the case of a maximum [13]. In the following we investigate properties of solutions $x_{\rm s}$ with respect to four independent parameters: the temperature T, the coupling constant i_0 which characterizes the mean-field interaction between rings, the probability p of the occurrence of the channel with an even number of coherent electrons and the external flux λ.

3.1.1. T and i_0-dependence

The dependence of the potential (13) on the temperature for $\lambda = 0$, $i_0 = 1$ and the probability $p = 1/2$ is shown in Fig. 1. In high temperatures, only one stable solution, corresponding to zero stationary flux $x_{\rm s} = 0$ and zero current, exists. If temperature decreases, a bifurcation occurs — the potential becomes bistable and two non-zero symmetric minima appear at $x_{\rm s} = \pm x_{\rm m}$. They correspond to two stable stationary solutions. Physically, it means that below some critical temperature $T_{\rm c}$ the *spontaneous flux* [14]

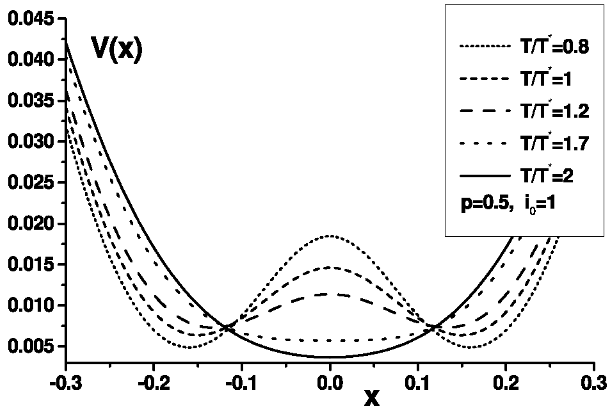

Fig. 1. The dimensionless generalized potential $V(x)$ is shown as a function of the dimensionless magnetic flux x for two values of the scaled temperature T/T^*. The scaled amplitude $i_0 = 1$ and scaled external magnetic flux $\lambda = 0$.

appears and non-zero stationary current flows in the system. This critical temperature T_c is defined by the condition that $V''(x_s = 0) = 0$. The corresponding diagram is shown in Fig. 2. The phenomenon is analogous to the *continuous phase transition* in macroscopic systems, and appears here as a result of the interaction of ring currents. The central maximum $x_s = x_M = 0$, corresponds to the unstable stationary solution of (21). More generally, one can notice that the stationary solutions occur where the linear part $x - \lambda$ of (22) is equal to its periodic part $i_0 f(x, p, T)$. In the limit of $i_0 \to 0$ (very small, or no interaction of ring currents), regardless T, the

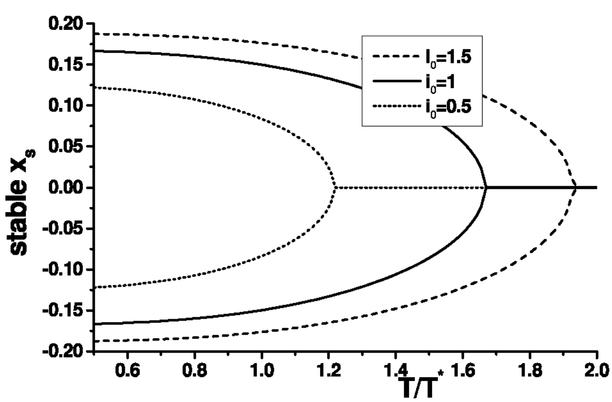

Fig. 2. Bifurcation of the stable stationary magnetic flux x_s with respect to temperature for a fixed external magnetic flux $\lambda = 0$.

only stationary solution of (22) is the external flux $x_s = \lambda$. For intermediate i_0 (typical interaction of mesoscopic rings) two stable non-zero stationary states can exist below T_c and this number of solutions is preserved in the limit $T \to 0$. As one can infer from (13)–(17), decreasing temperature enhances the periodic part of (22) but only to a maximal value defined by $T \to 0$. Further enhancement of the periodic part is possible only by increasing the coupling constant i_0. As a result of that, the critical temperature T_c increases with i_0. Therefore, if i_0 is sufficiently large (very strong interaction of rings), even more stationary states can occur. The number of stationary states below T_c and for $p = 1/2$ can, in general, be equal to $4k - 1$, $(k = 1, 2...)$ but only $2k$ of them of stable states. Lowering the temperature below T_c results then in a cascade of bifurcations. The first bifurcation takes place at $T = T_c$. With further lowering the temperature at $T = T_{c_1} < T_c$ two additional pairs of stationary solutions appear and so on. There is one metastable and one unstable solution in every pair. The metastable solutions correspond to the so called *flux trapped* in the cylinder. Notice that in the limit $T \to 0$ and typical $i_0 > 0$ there are always spontaneous flux solutions whereas the flux trapped solutions can be obtained only for sufficiently large i_0.

3.1.2. The p-dependence

In the following part of this section the temperature is set below T_c. If the probability $p = 1$ we have an even number of coherent electrons and paramagnetic current in each channel. The potential possesses two minima corresponding to spontaneous fluxes (Fig. 3). Decreasing p, the

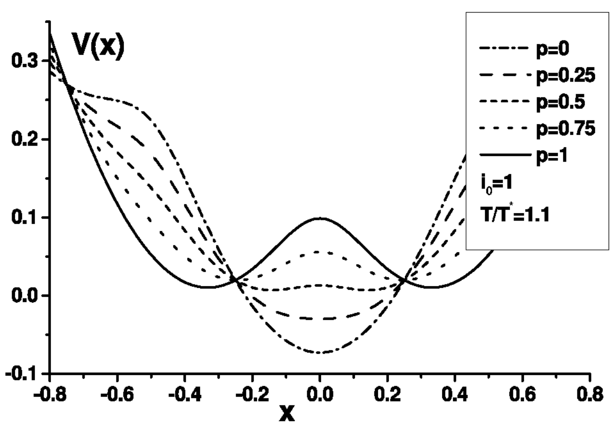

Fig. 3. The dimensionless generalized potential $V(x)$ is shown as a function of the dimensionless magnetic flux x for characteristic values of the probability p. The amplitude $i_0 = 1$ and $\lambda = 0$.

probability $1-p$ of finding odd channels with diamagnetic currents increases and the spontaneous flux solutions x_{s} decrease to coalesce finally into a single absolutely stable solution at $x_{\mathrm{s}}=0$. The ratio p at which the coalescence occurs decreases with decreasing temperature. Now, for sufficiently large i_0, five stationary states exist. Note that apart from the stable fluxless solution $x_{\mathrm{s}}=0$ there are two metastable solutions at $\frac{1}{2}<|x_{\mathrm{s}}|<1$ and two unstable solutions. The metastable solutions correspond to the flux trapped in the cylinder. In realistic devices they are hardly accessible due to the value of the necessary parameters. The value of p has a important impact on the properties of persistent currents. The case when p is a fixed deterministic quantity is studied in [15].

3.1.3. The λ-dependence

There are three different types of the generalized potential. First is a symmetric double well potential which appears for $\lambda=k/2$ with integral k. The stable solutions x_{s} are then always around the external flux $0<|x_{\mathrm{s}}-\lambda|<\frac{1}{2}$. For the values of λ close but not equal to $k/2$ the solutions remain in that range but the double-well potential becomes asymmetric — one of the stable solutions becomes metastable. For the values of external flux far from half integer values $k/2$ one obtains the potential with only a single stable solution. All the mentioned types of potentials are accessible for $0\leq\lambda<1/2$ indicating a kind of the 'structural periodicity' with respect to the external flux. An interesting feature of the $x-\lambda$ characteristic is the occurrence of the hysteresis loop (Fig. 4). With increasing λ, at its certain value, the system undergoes discontinuous jump of x. Decreasing then the value of λ, the opposite jump of x occurs at lower λ producing a hysteresis

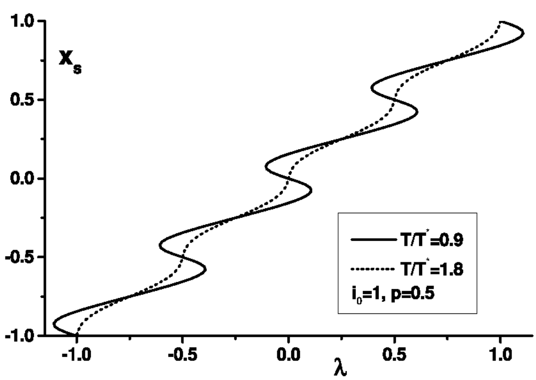

Fig. 4. The hysteretic behavior of the stationary flux with respect to the external flux. The part of the graph with negative slope corresponds to unstable x_{s}. The amplitude $i_0=1$.

loop. It is a hallmark of the *first order phase transition.* The transition can occur only below the critical temperature T_c. Due to the 'structural periodicity' the hysteresis loop is repeated with the period $\lambda = 1/2$ what results in the formation of a family of loops.

3.2. Noise-assisted selfsustaining currents

In this section we discuss the influence of both equilibrium and non-equilibrium perturbations on the properties of the flux in the mesoscopic cylinders. First we discuss the thermal noise and later the system for which the probability of the given parity of coherent electrons in the channel is a random process.

3.2.1. Nyquist noise

Noise and fluctuations are ubiquitous in real systems and idealization of the noiseless systems is sometimes not justified. In the following, we will focus on the system (12) subjected to Nyquist noise. From the mathematical point of view, the Langevin equation (12) defines a Markov diffusion process. Its probability density $p(x, \tilde{t})$ obeys the Fokker–Planck equation in the form [16]

$$\frac{\partial}{\partial \tilde{t}} p(x, \tilde{t}) = \frac{\partial}{\partial x} V'(x) p(x, \tilde{t}) + D \frac{\partial^2}{\partial x^2} p(x\,, \tilde{t}) \tag{23}$$

with the natural boundary condition $\lim_{|x|\to\infty} p(x, \tilde{t}) = 0$. The stationary solution $p_s(x)$ is asymptotically stable [17] and takes the form

$$p_s(x) = N_0 \mathrm{e}^{-V(x)/D} \tag{24}$$

with a normalization constant

$$N_0^{-1} = \int_{-\infty}^{\infty} \mathrm{e}^{-V(x)/D}\, dx\,. \tag{25}$$

Let us first consider the case of absence of the external flux, $\lambda = 0$. If in the noiseless case the system possesses only one stationary solution $x_s = 0$, the probability density (24) has maximum at $x = 0$ and the mean value of the flux $\langle x \rangle = 0$. If in the noiseless case the system possesses three stationary states, the probability density (24) has three extremal points: two symmetric maxima which correspond to the spontaneous fluxes and one minimum at $x = 0$ which corresponds to the unstable stationary state (see Fig. 5). Because the potential is reflection-symmetric, $V(x) = V(-x)$, the mean

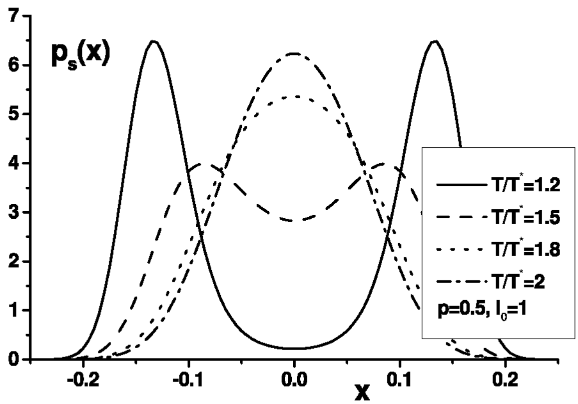

Fig. 5. The stationary probability density for the system subject to Nyquist noise. The amplitude $i_0 = 1$ and $D = 0.001T/T^*$.

value of any odd function of the flux is zero. In particular, the mean value of the flux $\langle x \rangle = 0$ and the mean value of the current is zero as well. From this point of view, properties of stationary states are trivial and non-zero fluxes and currents are impossible. However, in some situations the statistical moments are not good characteristics of the system because much information is lost when an integration is performed calculating the statistical moments [18]. The relevant quantity is a stationary probability distribution which contains much more information about the system. Is any reasonable method to determine the critical value of temperature $T_{\rm c}$ in this case? One possibility is to define the phase transition in the following way [18,19]: the phase transition point is a value of the relevant parameter γ of the system at which the profile of the stationary distribution function changes drastically (*e.g.* if a number of maxima of the distribution function changes) or if a certain most probable point x_0 begins to change to an unstable state. In some cases, it is indeed a good 'order parameter' of the system. For example, from the measurements of the laser experiment (see *e.g.* [20]), one can obtain the stationary probability distribution of the laser intensity and one can observe a phase transition according to the above definition. In the case considered here, for sufficiently low temperatures, thermal fluctuations are small and one expects the experimental results to be accumulated around the *most probable values* of the stationary probability distribution. It follows from (24) that the most probable values of the flux correspond exactly to the stationary states (22) of the system (21). In this sense, the properties of the system are the same as discussed in the previous subsection. We want to emphasize that it is correct for low temperatures because then the residence time in a stable state is long. For higher temperature T, thermal fluctuations become larger. In turn, fluctuations of the magnetic flux around the most probable value become larger and larger and the residence time in a stable state becomes shorter. One can guess that the spontaneous current should vanish at temperature T_0 which is lower than the critical temperature T_c in the noiseless case. This is because of influence of Nyquist fluctuations. The argumentation is the following. If the potential is multistable then one can introduce characteristic time scales of the system. The first characteristic time $\tau_d = 1/V''(x_s)$ describes decay within the attractor $x_s = \pm x_m$ of the potential $V(x)$. The second characteristic time is the escape time τ_e from the well around $\pm x_m$. This time is related to the mean first passage time from the minimum of the potential to the maximum. If these time scales are well separated, *i.e.* if $\tau_e >> \tau_d$ then the description based on the most probable value seems to be correct. Otherwise, this description fails and we should characterize the system by averaged values of relevant variables. In the noiseless case, for $i_0 = 1$ and $D = 0.001T/T^*$, from Eq.(13) we estimated the critical temperature $T_c \approx 1.66T^*$. We observed that roughly for temperatures $T < 0.9T_c$, the characteristic time τ_d is more than one order of magnitude less than τ_e. Both time scales are well separated and selfsustaining currents are long-living states. In this sense, they are not destroyed by Nyquist noise.

The stationary flux variance or mean-squared deviation $\sigma = \langle x^2\rangle - \langle x\rangle^2 = \langle x^2\rangle$ is a non-monotonic function of temperature (Fig. 6): For $T = 0$ the variance $\sigma = x_s^2$, where x_s is a stationary solution of (21). As the temperature increases, σ diminishes attaining a minimal value at some temperature T_1. The temperature T_1 seems to be always larger than T_c what has been confirmed by numerical studies. A further increase of temperature leads to an increase of the variance. In the high temperature limit, the dependence is linear as for the Gaussian distribution. Indeed, below the critical temperature, the distribution (24) possessing two peaks is clearly non-Gaussian. However, for higher temperatures the probability density is one-peaked. For this case, the kurtosis

$$\mathrm{Kurt} = \frac{\langle x^4\rangle}{3\langle x^2\rangle^2} - 1 \tag{26}$$

measures the relative flatness of the distribution (24) to the Gaussian distribution. The kurtosis is negative and it means that the distribution (24) is flat. It approaches zero in the high temperature limit and then the distribution (24) approaches the Gaussian distribution.

The behavior of the second moment $\langle x^2\rangle$ has a simple explanation in terms of the average energy stored in the magnetic field, *i.e.*

$$\langle E\rangle = \langle \phi^2\rangle/2L = \varepsilon_0\langle x^2\rangle\,, \tag{27}$$

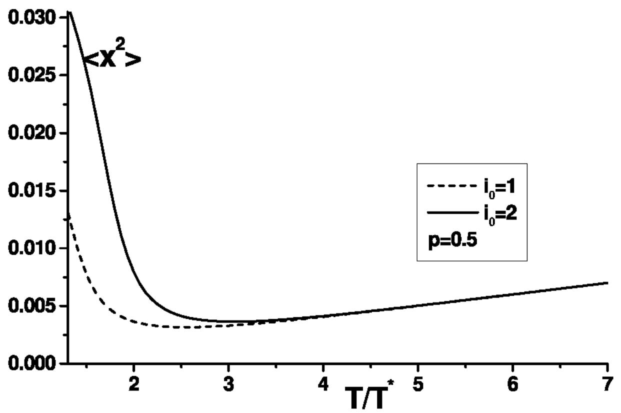

Fig. 6. Averaged magnetic energy $\langle E\rangle/\varepsilon_0$ given by Eq. (27) *vs* scaled temperature for two values of i_0, fixed $\lambda = 0$ and $D = 0.001T/T^*$.

where ε_0 is given in (18). For low temperatures, fluctuations are small and the main contribution to the energy comes from the deterministic part $\phi^2/2L$. Because the magnetic flux ϕ decreases as temperature increases (*cf.* Fig. 2), hence $\langle E\rangle$ decreases as well. On the other hand, for high temperature the stationary probability density approaches the Gaussian distribution and in consequence the main contribution to the average magnetic energy comes from thermal energy, $\langle E\rangle \propto kT$ which obviously increases when T grows. The competition between these two mechanisms leads to the minimal value of $\langle E\rangle$ for a certain value of temperature T_1. At this temperature, fluctuations of the current are minimal.

The influence of the external field on the properties of the stationary density (24) may be deduced from Fig. 3. Finally, let us consider the limit of a very weak coupling between the ring currents corresponding to a very small value of i_0. The selfsustaining stable solutions are non accessible. The solutions of Eq. (21) correspond then to the persistent currents driven by the external field. The stationary density forms a family of one peak curves with the most probable values given by λ. We conclude that even in the weak coupling limit the presence of Nyquist noise does not destroy the persistent currents.

3.2.2. Random parity of coherent electrons

The value of p is not a fixed parameter for the systems at temperatures $T > 0$ when the energy gap at the Fermi surface becomes smaller. There are then coherent electrons which can become normal and vice versa, there are normal electrons which may become coherent. In such a case the probability p itself is a random function of time. Further we limit our discussion to the transitions $p = 0 \leftrightarrow p = 1$. They correspond to the change from even to odd number of coherent electrons in every current channel in the cylinder. The transitions satisfy the following assumptions: first, the change even$\leftrightarrow$odd occurs simultaneously and immediately (*i.e.* it takes no time) in every channel and second, the number of electrons changing their "fluid" is small enough to keep i_0 fixed in the transition. The dynamics of flux in the absence of the external field can be modeled by the following equation

$$\dot{x} = -x + f_+(x) + f_-(x)\xi(\tilde{t}) + \sqrt{2D}\widetilde{\Gamma}(\tilde{t})\,, \tag{28}$$

where $f_\pm(x) := i_0(f_e(x) \pm f_0(x))/2$ and $\xi(\tilde{t}) = \{-1, 1\}$ is a zero-mean, exponentially correlated dichotomic process of the correlation time τ [21]. If $\xi(\tilde{t}) = 1$ then there is an even number of coherent electrons and if $\xi(\tilde{t}) = -1$ then there is an odd number of coherent electrons.

With the random process Eq. (28), which is clearly non-Markovian, we associate the two dimensional process $\{x(\tilde{t}), \xi(\tilde{t})\}$, which is Markovian. The probability densities $p_+(x,\tilde{t}) := p(x(\tilde{t}), \xi(\tilde{t}) = 1)$ and $p_-(x,\tilde{t}) := p(x(\tilde{t}), \xi(\tilde{t}) = -1)$ satisfy the master equation [21]

$$\begin{aligned}
\frac{\partial}{\partial \tilde{t}} p_+(x,\tilde{t}) &= -\frac{\partial}{\partial x}\left[-x + f_+(x) + f_-(x)\right] p_+(x,\tilde{t}) \\
&\quad -\frac{1}{2\tau}\left[p_+(x,\tilde{t}) - p_-(x,\tilde{t})\right] + D\frac{\partial^2}{\partial x^2} p_+(x,\tilde{t})\,, \\
\frac{\partial}{\partial \tilde{t}} p_-(x,\tilde{t}) &= -\frac{\partial}{\partial x}\left[-x + f_+(x) - f_-(x)\right] p_-(x,\tilde{t}) \\
&\quad -\frac{1}{2\tau}\left[p_-(x,\tilde{t}) - p_+(x,\tilde{t})\right] + D\frac{\partial^2}{\partial x^2} p_-(x,\tilde{t})\,.
\end{aligned} \tag{29}$$

The stationary state is described by the stationary reduced probability density $p(x) = \lim_{\tilde{t}\to\infty} p(x,\tilde{t}) = \lim_{\tilde{t}\to\infty}[p_+(x,\tilde{t}) + p_-(x,\tilde{t})]$. An analytical form of the stationary solution of (29) is known when $D = 0$, *i.e.* when temperature $T = 0$. If $T > 0$ then we should consequently assume that $D > 0$. In this case, an analytical formula for $p(x)$ can be derived for the limiting case $\tau \to \infty$ (adiabatic noise). In a general case, one should numerically solve Eq. (29) with zero left hand sides. We have applied the Finite Element Method [22]. The results are presented in Figs. 7 and 8. For temperature below the critical temperature (Fig. 7), dichotomic noise of a short correlation time does not influence the system: there are two stable and symmetric states of non-zero selfsustaining currents (the double-peaked density for the case $\tau = 0.0259294$ in Fig. 7). If the correlation time τ increases then the states of non-zero currents disappear. The state of zero current is stable and two new metastable states of non-zero currents occur (the triple-peaked density for the case $\tau = 0.239503$). For temperature above the critical temperature (Fig. 8), dichotomic noise of a short correlation time does not influence

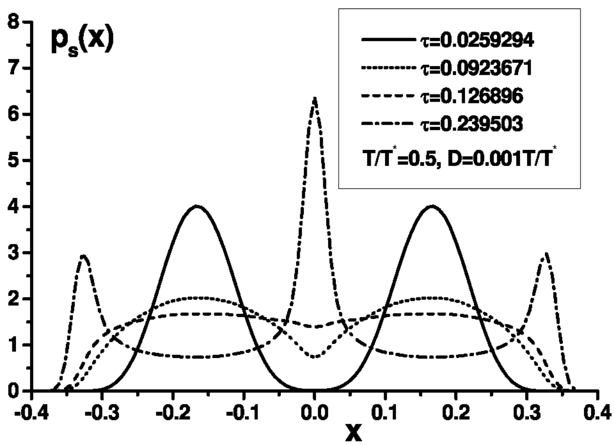

Fig. 7. The stationary probability density for the system with dichotomic fluctuations of p for several values of their correlation time τ. The amplitude $i_0 = 1$ and $T = 0.5T_c$.

the system: the single-peaked density for the case $\tau = 0.0672336$ in Fig. 8 corresponds to zero-current case. For long correlation times, dichotomic noise can induce new metastable states which correspond to non-zero currents (the triple-peaked density for the case $\tau = 3.0392$). In both cases, the noise-induced metastable states are located in the neighborhood of zeros of the 'diffusion function'

$$D(x) = f_-^2(x) - (f_+(x) - x)^2 . \tag{30}$$

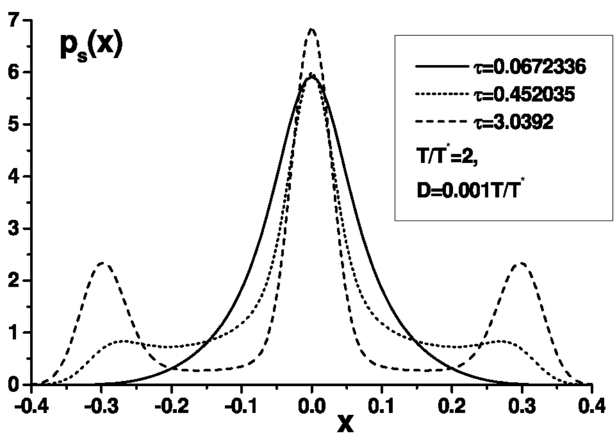

Fig. 8. The stationary probability density for the system with dichotomic fluctuations of p. The amplitude $i_0 = 1$ and $T = 2T_c$.

The model Eq. (28) is clearly a simplification of the realistic one which should incorporate both the Nyquist noise and the possibility of independent transitions in each channel. Such an independent transitions can be described by a vector stochastic dichotomic process $\overrightarrow{\xi} := (\xi_1, \xi_2, \ldots, \xi_N)$ where N is the number of channels in the cylinder and ξ_i are independent of each other dichotomic processes described above. Further we assume that $D \neq 0$ and the configuration of $\overrightarrow{\xi}$ is quenched *i.e.* the probability of the even number of coherent electrons in a single current channel is a random variable uniformly distributed on the interval $[0, 1]$. The stationary probability density of the flux is now expressed as

$$p_{\rm s}(x) = \int_0^1 p(x|z)dz\,, \tag{31}$$

where the conditional probability distribution

$$p(x|z) = N_0(z)\exp(-V(x,z)/D) \tag{32}$$

with $V(x,z) := -x + i_0 z f_e(x,T) + i_0(1-z)f_0(x,T)$ and the normalization constant $N_0(z)$. The stationary probability density (31) is plotted in Fig. (9) for several values of the temperature. Its profile is very different if compared with the case $p = 1/2$. First, one should note that the density is not very sensitive for the changes of temperature and second the maxima for non-zero selfsustaining fluxes are dominated by the maximum appearing at zero. It means that even relatively small thermal fluctuations destroy the selfsustaining currents in the system.

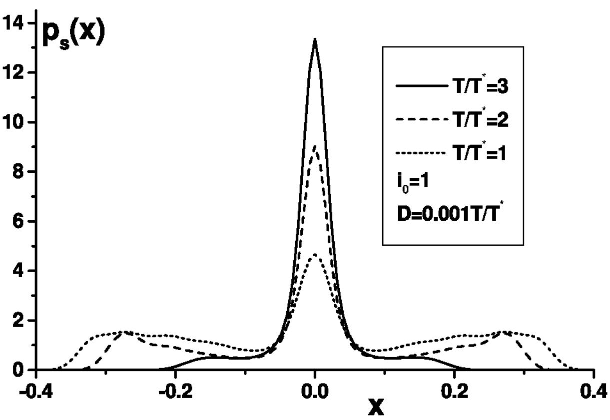

Fig. 9. The stationary probability density for the system of randomly distributed p for several values of T/T^*. The amplitude $i_0 = 1$ and $D = 0.001T/T^*$.

4. Summary

Persistent and selfsustaining currents are beautiful manifestation of quantum coherence in mesoscopic systems. The natural question is how do they behave in the presence of randomness and fluctuations. Assuming the two fluid model for mesoscopic system we have investigated the influence of Nyquist noise and non-equilibrium fluctuations of one of the parameters (parity of the coherent electrons number). Our discussion is limited to stationary states of the magnetic flux and current although the proposed model of the flux dynamics can be, in principle, applied to study time dependent problems. The general conclusion is that Nyquist noise preserves the selfsustaning currents, *i.e.* for some parameters there are states of the long-living non-zero flux and current. In the case of fixed p the properties of the stationary flux are determined by the generalized potential $V(x)$. Assuming non-equilibrium (dichotomic) fluctuations of the number of coherent electrons in the channel we conclude that noise of sufficiently large correlation time can *induce* non-zero flux states determined by maxima of the probability density at $x \neq 0$. In the case of p being the uniformly distributed random variable the long-living currents are observable at low temperatures. In that sense the quenched randomness of p destroys selfsustaining currents much more than equilibrium fluctuations.

Appendix

For the paper to be self-contained, we remind one of the form of the fluctuation–dissipation theorem and the Nyquist relation exploited in our basic Eq. (8). The Brownian motion of a particle of mass m in a fluid of temperature T is described by a Langevin equation [16]. According to the fluctuation-dissipation theorem [16], its form for the velocity $v = v(t)$ reads

$$m\dot{v} + \gamma v = \sqrt{2\gamma k_{\rm B} T}\, \Gamma(t)\,, \tag{33}$$

where a dot denotes a derivative with respect to time, γ is the friction coefficient, $k_{\rm B}$ is the Boltzmann constant and $\Gamma(t)$ is the zero-mean and Dirac δ-correlated Gaussian stochastic process (white noise),

$$\langle \Gamma(t) \rangle = 0, \quad \langle \Gamma(t)\Gamma(s) \rangle = \delta(t-s)\,. \tag{34}$$

Mutatis mutandis, the Langevin equation for the current $I = I(t)$ in the RL circuit takes the form [23]

$$L\dot{I} + RI = \sqrt{2Rk_{\rm B}T}\, \Gamma(t). \tag{35}$$

It is one of the form of the Nyquist relation. In the case when

$$\phi = LI \tag{36}$$

it can be rewritten as

$$\frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi = \sqrt{\frac{2k_{\mathrm{B}}T}{R}}\Gamma(t)\,, \tag{37}$$

which justifies the prefactor of the noise term in Eq. (8).

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REFERENCES

[1] F. Hund, *Ann. Phys. (Leipzig)* **32**, 102 (1938); M. Buttiker, Y. Imry, R. Landauer, *Phys. Lett.* **96A**, 365 (1983).

[2] L.P. Levy, G. Dolan, J. Dunsmuir, H. Bouchiat, *Phys. Rev. Lett.* **64**, 2074 (1990).

[3] Sh. Kogan, *Electronic Noise and Fluctuations in Solids*, Cambridge University Press, Cambridge 1996.

[4] M. Szopa, E. Zipper, *Solid State Commun.* **77**, 739 (1991); D. Wohlleben *et al.*, *Mod. Phys. Lett.* **B6**, 1481 (1992).

[5] T. Dittrich, P. Hänggi, G.-L. Ingold, B. Kramer, G. Schön, W. Zwerger, *Quantum Transport and Dissipation*, Wiley-VCH, Weinheim 1998.

[6] L. Landauer, M. Buttiker, *Phys. Rev. Lett.* **54**, 2049 (1985).

[7] Y. Imry, *Introduction to Mesoscopic Physics*, Oxford University Press, New York 1997.

[8] M. Lisowski, E. Zipper, M. Stebelski, *Phys. Rev.* **B59**, 8305 (1999).

[9] F. Bloch, *Phys. Rev.* **137**, 787 (1965).

[10] H.F. Cheung, Y. Gefen, E.K. Riedel, W.H. Shih, *Phys. Rev.* **B37**, 6050 (1988).

[11] J.D. Jackson, *Classical Electrodynamics*, PWN, Warszawa 1987.

[12] E.K. Riedel, H. Cheung, *Phys. Scr. (T)* **25**, 357 (1989).

[13] J. Hale, H. Koçak, *Dynamics and Bifurcations*, Springer, New York 1991.

[14] M. Szopa, E. Zipper, *Int. J. Mod. Phys.* **B9**, 161 (1995).

[15] J. Dajka, J. Luczka, M. Szopa, in *Symmetry and Structural Properties of Condensed Matter*, eds. T. Lulek *et al.*, World Scientific, Singapore 2002 (in press).

[16] H. Risken, *The Fokker–Planck Equation*, Springer, Berlin 1984.

[17] A. Lasota, M.C. Mackey, *Probabilistic Properties of Deterministic Systems*, Cambridge University Press, Cambridge 1985.

[18] W. Hosthemke, R. Lefever, *Noise-Induced Transitions*, Springer, Berlin 1984.

[19] M. Suzuki, *Adv. Chem. Phys.* **46**, 195 (1981).

[20] P. Lett, E.C. Gage, T.H. Chyba, *Phys. Rev.* **A35**, 746 (1987).

[21] C. Van den Broeck, P. Hänngi, *Phys. Rev.* **A30**, 2730 (1984).

[22] O.C. Zienkiewicz, R.L. Taylor, *Finite Element Method*, Butterworth-Heinemann, London 2000.

[23] C.W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences*, Springer, Berlin 1998.