THE γ_5 AND DIMENSIONAL REGULARIZATION*

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The properties of the axial-vector current are investigated using dimensional regularization. The modified version of anti-commuting γ_5 in n dimensions is proposed. The VVA and AAA triangle diagrams are precisely calculated. The resulting amplitudes obey the naive vector Ward identities. In the axial vector Ward identities the Adler-Bell-Jackiw anomalies appear.

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1. *Introduction*

In proving the renormalizability [1] of gauge theory, it is crucial that the renormalized lagrangian is itself locally gauge (or BRS [2]) invariant after introducing a gauge fixing term. To preserve gauge invariance the most suitable method is to use dimensional regularization [3] which has also very good algebraic properties. In particular the method admits [4] commutativity, distributivity, associativity and change of integration variable. In the absence of axial coupling the method is straightforward but there is some confusion in the case of fermion loops with one or more factors of γ_5. There is no satisfactory generalization of γ_5 to arbitrary dimension. Natural generalization [5] gives γ_5 which in an even (odd) dimensional space anticommutes (commutes) with all the Dirac γ^μ matrices.

Attempts at resolving the problems of γ_5 in dimensional regularization may be divided into two categories. In the first approach the authors try to give some definition of the γ_5 in n-dimensions. In the second method the γ_5 is not defined, but all the necessary properties for calculating any of the Feynman diagrams are given. 't Hooft and Veltman [3] definition belongs to the first category. They define

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3. \tag{1.1}$$

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Then from the properties of the Dirac matrices γ^μ in n-dimensions

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g^\mu{}_\mu = n \tag{1.2}$$

the γ_5 has the characteristic peculiarity

$$\begin{aligned} \{\gamma_5, \gamma^\mu\} &= 0 \quad \text{for} \quad \mu = 0, 1, 2, 3, \\ [\gamma_5, \gamma^\mu] &= 0 \quad \text{for} \quad \mu = 4, 5, \ldots, n-1, \end{aligned} \tag{1.3}$$

and

$$\gamma_5^2 = 1.$$

Several subsequent attempts were made to introduce an appropriate generalization of γ_5 to n-dimensions [6]. All these approaches are essentially similar to that of 't Hooft and Veltman but the authors claim to have provided a more consistent formalism.

The Bardeen, Gastmans and Lautrup [7] method was the first approach where the authors do not define precisely γ_5 but give only its properties. Their γ_5 has the properties

$$\begin{aligned} \{\gamma_5, \gamma^\mu\} &= 0 \quad \text{for} \quad \mu = 0, 1, \ldots, n-1, \\ \gamma_5^2 &= 1. \end{aligned} \tag{1.4}$$

The latter authors considered diagrams with mixed loops containing both boson and fermion propagators (the fermion line is not closed). They found that the Ward identities are not satisfied if they use definition (1.1). However they found that the Ward identities are consistent with definition (1.4).

Chanowitz, Furman and Hinchliffe [8] have shown that using the definition (1.4) for single closed fermion loop diagrams with an even number of γ_5's all Ward identities are satisfied. The latter authors observed also that adopting the property (1.4) it is possible to express any trace of γ_5 with greater than four and even numbers of Dirac γ_μ matrices, by one arbitrary parameter b. Changing the value of this parameter it is possible to satisfy vector Ward identity, axial vector identity or none of them. The authors consider this b dependence as a reflection of the fact that the four-dimensional integration is not well--defined [9]. As the integral defining a triangle graph is linearly divergent, the value of the triangle graph is ambiguous and depends on labeling convention and the method of evaluation of the integral.

Gottlieb and Donohue [10] disagree with the previous result for a triangle graph with one γ_5 (VVA amplitude). They calculated once more the VVA amplitude without using any property for γ_5 and found that dimensional regularization yields a VVA graph which automatically satisfies the vector current conservation, leaving the anomaly in the axial vector divergence (ABJ anomaly [9, 11]).

Ovrut [12] also adopts the definition (1.4), but to remove the b dependence ambiguity from the paper [8] he does not anticommute γ_5 with any Dirac matrix inside the trace (if these contain more than six Dirac matrices). In this way he proved that for his kinds of diagrams Ward identities are valid independently of γ_5 definition in n-dimensions.

It is not convenient to fix positions of γ_5 inside a trace if the number of γ_5 is odd and greater than one. The ZZZ and ZZW vertices in the electroweak theory are of this type. We found the properties for γ_5 which allow us to calculate explicitly any kind of Feynman diagrams with any number of γ_5. Our prescription for γ_5 renders divergent Feynman integrals finite, and honors the Ward identities so it satisfies all necessary conditions which a regulator should have. These were checked explicitly for one loop triangle diagrams. The only problems appear sometimes with Bose symmetry, so after regularization we have to make symmetrization in these cases.

In the next Section our definition for γ_5 in n-dimensions is described. In Section 3 we show that Ward identities for all interested triangle diagrams are satisfied. In Section 4 we summarize our results.

2. *Properties of γ_5 in n-dimensions*

As we described in the Introduction the methods of using γ_5 in dimensional regularization (which are consistent with Ward identities) which we found in the literature allow us to calculate:

I. diagrams with mixed loops containing both boson and fermion propagators [7, 8],
II. diagrams with only internal fermion lines but with an even number of γ_5's [8],
III. diagrams with internal fermion lines with one γ_5 inside traces [10, 12, 13].

Taking the definition (1.4) it is possible in case I to anticommute γ_5's outside the internal part of Feynman integral where the normal n-dimensional Dirac algebra is used. In case II using (1.4), γ_5 matrices are eliminated inside the trace. In both cases canonical Ward identities are satisfied without any abnormal parts [7, 8, 10].

Case III was the most controversial one. Taking different definitions of γ_5 different results for the VVA amplitude (Fig. 1) were obtained (see e.g. Ref. [10, 12, 13]). Fixing the position of γ_5 inside the traces and using only normal n-dimensional Dirac algebra the first correct result for $\Pi_5^{\mu\nu\lambda}$ (Fig. 1) was got in Ref. [10]. It was found that the $\Pi_5^{\mu\nu\lambda}$

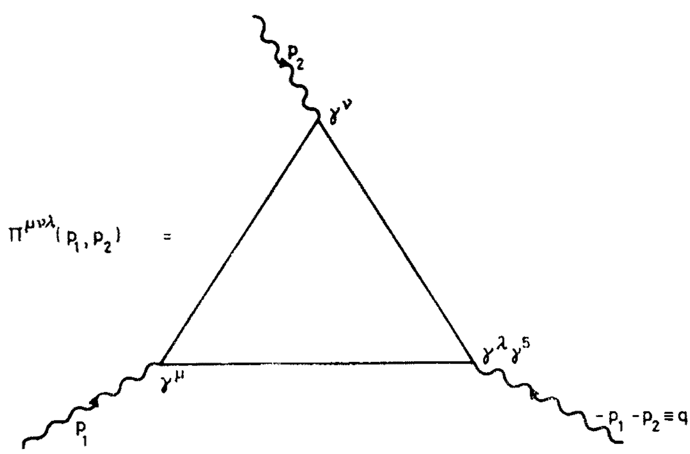

Fig. 1. The VVA diagram $\Pi_5^{\mu\nu\lambda}(p_1, p_2)$ with two vector and one axial vector couplings

satisfies vector Ward identities (2.1)

$$p_{1\mu}\Pi_5^{\mu\nu\lambda}(p_1, p_2) = 0, \quad p_{2\nu}\Pi_5^{\mu\nu\lambda}(p_1, p_2) = 0 \tag{2.1'}$$

but axial Ward identity has an abnormal part which does not depend on mass [9, 10]

$$-(p_1+p_2)_\lambda \Pi_5^{\mu\nu\lambda}(p_1, p_2) = 2m\Pi_5^{\mu\nu}(p_1, p_2), \tag{2.2}$$

where $\Pi_5^{\mu\nu}(p_1, p_2)$ is the diagram like $\Pi_5^{\mu\nu\lambda}(p_1, p_2)$ but after removing γ^λ matrix (for more details see Appendix and Fig. 3).

Without any other information about γ_5 there are problems, for example, in how to calculate a triangle diagram with one γ_5 in each of the vertices. We will give here the four conditions which are enough to calculate all Feynman diagram which are interesting from a physical point of view.

For convenience we describe all properties of γ_5 in n-dimensions, even the well-known ones.

A. Anticommuting property in n-dimensions

$$\begin{aligned} \{\gamma_5, \gamma^\mu\} &= 0, \quad \mu = 0, 1, \ldots, n-1, \\ \gamma_5^2 &= 1, \\ \mathrm{Tr}\,\gamma_5 &= 0. \end{aligned} \tag{2.3}$$

From this property it is easy to prove [8] that

$$(n-4)\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta) = 0, \tag{2.4}$$

and from this equation follows that $\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta)$, can be different from zero only for $n = 4$. So it is impossible to consider this trace as a smooth function of n e.g. it is not possible to have

$$\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta) = -4i\varepsilon^{\alpha\beta\gamma\delta}+(4-n)A_1^{\alpha\beta\gamma\delta}+(4-n)^2A_2^{\alpha\beta\gamma\delta}+\ldots, \tag{2.5}$$

where A_1 and A_2 are some unknown tensors. But if we consider finite Feynman diagrams, they do not have poles $(4-n)^{-k}$, and we can use the trace (2.5) because the parts with unknown tensors A_i vanish for $n \to 4$. All one fermion loop Feynman diagrams, with odd numbers of γ_5, are finite (see next Section). Using our prescription for γ_5 we are able to calculate those diagrams only for $n = 4$. If one tries to consider the diagrams in n-dimensions, it is necessary to reject (2.4), but then there is a contradiction with conditions (2.3)[1]. From conditions (2.3) it follows that

$$\mathrm{Tr}\,(\gamma_5\gamma^{\alpha_1}\gamma^{\alpha_2}\ldots,\gamma^{\alpha_{2i-1}}) = 0, \tag{2.6}$$

and

$$\mathrm{Tr}\,(\gamma_5\gamma^{\alpha_1}\gamma^{\alpha_2}) = 0. \tag{2.7}$$

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[1] The definition of γ_5 with which the formula (2.5) does not contradict (2.4) was given by Thompson and Yu [14]. But their definition is very difficult to apply. We have also checked that this definition does not agree with the Ward identities for even numbers of γ_5's [15].

(The equation (2.7) is easy to prove, taking into account the property of charge conjugation matrix C in n-dimensions: $C\gamma_\mu C^{-1} = -\gamma_\mu^{\mathrm{T}}$ and $C\gamma_5 C^{-1} = \gamma_5^{\mathrm{T}}$.)

B. We use normal Dirac algebra in n-dimensions

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \quad g^\mu{}_\mu = n,$$
$$\mathrm{Tr}\, 1 = 4. \tag{2.8}$$

C. For an even number of γ_5 inside a trace, using (2.3) we can eliminate them completely. In an odd number of γ_5 only one survives. Then we calculate

$$\mathrm{Tr}\,(\gamma_5\gamma^{\alpha_1}\gamma^{\alpha_2} \dots \gamma^{\alpha_{2i}}) \quad \text{for} \quad 2i = 6, 8, \dots \tag{2.9}$$

using normal Dirac algebra in n-dimensions. But if the trace (2.9) is contracted with an integral which is infinite then:

— we first calculate the trace using the appropriate form of γ_5 in 4-dimensions (see Appendix)

$$\gamma_5 = -\frac{i}{4!}\varepsilon_{\alpha\beta\gamma\delta}\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta, \tag{2. 0}$$

— and we intégrate over momentum.

Using (2.3), (2.8) and (2.10) we express the trace (2.9) as a combination of metric tensor elements $g^{\alpha_i\alpha_k}$ product with antisymmetric tensor $\varepsilon^{\alpha\beta\gamma\delta}$. If $g^{\alpha\beta}$ from a dimensionally regularized integral is contracted with $g^{\alpha_i\alpha_k}$ from a trace (2.9) we give to it the value n. If the trace (2.9) is contracted with the finite integral then all above restrictions are not necessary.

D. Using the properties A, B and C sometimes we get diagrams which do not satisfy Bose symmetry. To restore this symmetry we have to make appropriate symmetrization after dimensional regularization.

In the next Section we will show that all diagrams which we calculate using conditions A, B, C, and D are consistent with canonical Ward identities for anomaly free theory. Abnormal parts (which vanish after summation over all loop fermions) appear only in axial vector Ward identities.

3. *Triangle diagrams and Ward identities*

We will not calculate Green's functions where γ_5's can be eliminated. In this case we need only conditions A and B (Eqs (2.3) and (2.8)). These diagrams were calculated and Ward identities were checked [8, 10]. Application of the dimensional regularization to triangle diagrams with three vertices was more controversial. Let us consider the sum of diagrams given in Fig. 2 where vertices are denoted by Γ_1, Γ_2, Γ_3

$$\Pi^{\Gamma_1\Gamma_2\Gamma_3}(p_1, p_2) = -i^3 \int \frac{d^n k}{(2\pi)^n}\, \mathrm{Tr}\left[\frac{1}{\hat{k}+\hat{p}_2-m}\Gamma_2\frac{1}{\hat{k}-m}\Gamma_1\frac{1}{\hat{k}-\hat{p}_1-m}\Gamma_3\right]$$
$$-i^3 \int \frac{d^n k}{(2\pi)^n}\, \mathrm{Tr}\left[\frac{1}{\hat{k}+\hat{p}_1-m}\Gamma_1\frac{1}{\hat{k}-m}\Gamma_2\frac{1}{\hat{k}-\hat{p}_2-m}\Gamma_3\right]. \tag{3.1}$$

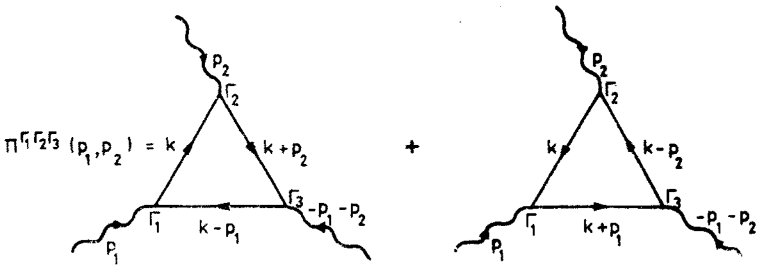

Fig. 2. The general triangle diagram $\Pi^{\Gamma_1\Gamma_2\Gamma_3}(p_1, p_2)$ with couplings Γ_1, Γ_2 and Γ_3 in the vertices. Normal and crossed diagrams are shown

Each Γ_i can be given by

$$\Gamma_i = 1, \gamma_5, \gamma^\mu \quad \text{and} \quad \gamma^\mu\gamma_5. \tag{3.2}$$

Altogether there are 64 diagrams but only 6, which are given in Fig. 3, are not trivial (part of the diagrams can be got from these 6 after changing of variables). All graphs in Fig. 3 occur in the electroweak theory in the couplings between Z, photon and Higgs: ZZZ, ZZA, AAZ, ZZH, AAH and ZAH. To find canonical Ward identities let us define four kinds of current $\Gamma_i(x)$

$$\begin{aligned} S(x) &= \bar{\psi}(x)\psi(x), \\ P(x) &= \bar{\psi}(x)\gamma_5\psi(x), \\ V^\mu(x) &= \bar{\psi}(x)\gamma^\mu\psi(x), \\ A^\mu(x) &= \bar{\psi}(x)\gamma^\mu\gamma_5\psi(x). \end{aligned} \tag{3.3}$$

Each 1PI Green's function in Fig. 3 can be described by

$$\begin{aligned} &(2\pi)^4\delta^{(4)}(p_1+p_2+p_3)\Pi^{\Gamma_1\Gamma_2\Gamma_3}(p_1, p_2) \\ &= \int d^4xd^4yd^4z e^{ip_1x}e^{ip_2y}e^{ip_3z}\langle 0|\mathrm{T}(\Gamma_1(x)\Gamma_2(y)\Gamma_3(z))\,|0\rangle. \end{aligned} \tag{3.4}$$

Formulae (3.4) can be treated more generally than Green's function in Fig. 3. We get diagrams of one loop approximation in Fig. 3 taking $\psi(x)$ and $\bar{\psi}(x)$ in the currents (3.3) as a free field. Generally $\Pi^{\Gamma_1\Gamma_2\Gamma_3}(p_1, p_2)$ are full 1PI Green's functions with fermion loop outside and all possible internal lines depending on an interaction Lagrangian. Let us assume that, in theory, one fulfilled

$$\partial_\mu V^\mu(x) = 0, \tag{3.5}$$

and

$$\partial_\mu A^\mu(x) = 2miP(x). \tag{3.6}$$

From the equal time anticommuting relations for ψ fields we get

$$[V^0(x), \Gamma_i(y)]\delta(x_0 - y_0) = 0, \tag{3.7}$$

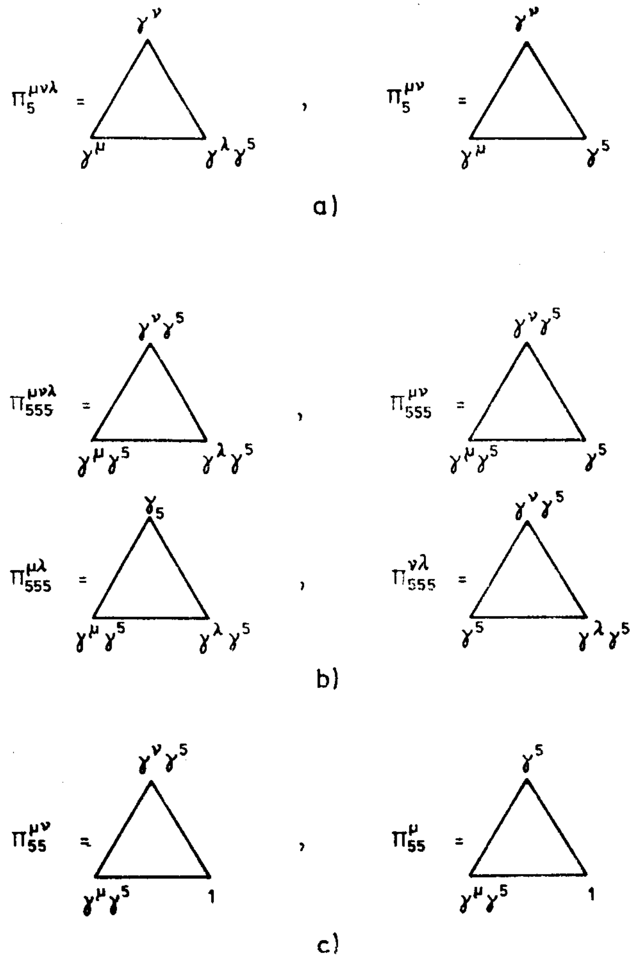

Fig. 3. a) The VVA diagram $\Pi_5^{\mu\nu\lambda}$ and the VVP diagram $\Pi_5^{\mu\nu}$ which are connected by the axial vector Ward identity; b) the AAA diagram $\Pi_{555}^{\mu\nu\lambda}$ and the AAP diagrams $\Pi_{555}^{\mu\nu}$, $\Pi_{555}^{\mu\lambda}$ and $\Pi_{555}^{\nu\lambda}$. All diagrams are connected by Ward identities; c) the AAS diagram $\Pi_{55}^{\mu\nu}$ and the APS diagram which satisfied the Ward identity

for any $\Gamma_i(x)$ from (3.3) and

$$\begin{aligned}
[A^0(x), S(y)]\delta(x_0-y_0) &= -2P(x)\delta^{(4)}(x-y), \\
[A^0(x), P(y)]\delta(x_0-y_0) &= -2S(x)\delta^{(4)}(x-y), \\
[A^0(x), V^\mu(y)]\delta(x_0-y_0) &= 0, \\
[A^0(x), A^\mu(y)]\delta(x_0-y_0) &= 0.
\end{aligned} \tag{3.8}$$

From (3.5)–(3.8) we get for 1PI Green's function in Fig. 3 the next Ward identities: Green's functions in Fig. 3a:

$$p_{1\mu}\Pi_5^{\mu\nu\lambda}(p_1, p_2) = 0, \quad p_{2\nu}\Pi_5^{\mu\nu\lambda}(p_1, p_2) = 0, \tag{3.9}$$

$$-(p_1, p_2)_\lambda \Pi_5^{\mu\nu\lambda}(p_1, p_2) = 2m\Pi_5^{\mu\nu}(p_1, p_2), \tag{3.10}$$

Green's functions in Fig. 3b:

$$p_{1\mu}\Pi^{\mu\nu\lambda}_{555}(p_1, p_2) = 2m\Pi^{\nu\lambda}_{555}(p_1, p_2),$$

$$p_{2\nu}\Pi^{\mu0\lambda}_{555}(p_1, p_2) = 2m\Pi^{\mu\lambda}_{555}(p_1, p_2),$$

$$-(p_1+p_2)_\lambda\Pi^{\mu\nu\lambda}_{555}(p_1, p_2) = 2m\Pi^{\mu\nu}_{555}(p_1, p_2), \tag{3.11}$$

Green's functions in Fig. 3c:

$$p_{1\mu}\Pi^{\mu\nu}_{55}(p_1, p_2) = 2m\Pi^{\nu}_{55}(p_1, p_2)+2i\Omega^{\nu}_{55}(p_2, -p_2),$$

$$p_{2\nu}\Pi^{\mu\nu}_{55}(p_1, p_2) = 2m\Pi^{\mu}_{55}(p_1, p_2)+2i\Omega^{\mu}_{55}(p_1, -p_1), \tag{3.12}$$

where $\Omega^{\nu}_{55}(p_2, -p_2)$ and $\Omega^{\mu}_{55}(p_1, -p_1)$ are two point Green's functions defined by

$$\delta^{(4)}(p+r)\Omega^{\varrho}_{55}(p, r) = \int d^4x d^4y e^{-ixp}e^{-iry}\langle 0|\, \mathrm{T}\,(A^{\varrho}(x)P(y))\, |0\rangle. \tag{3.13}$$

From (3.13) we have

$$2i\Omega^{\varrho}_{55}(p, -p) = -\frac{m}{2\pi^2}\left(c_{\mathrm{UV}}-\int_0^1 dx \ln D_2(x)\right)p^{\varrho},$$

where

$$D_2(x) = m^2-p^2x(1-x), \quad C_{\mathrm{UV}} = \frac{2}{\varepsilon}-\gamma+\ln 4\pi. \tag{3.14}$$

Using our γ_5 definition from Section 2, we calculate in the Appendix all necessary Green's functions.
From (Appendix A.8) there is

$$p_{1\mu}\Pi^{\mu\nu\lambda}_{5}(p_1, p_2) = \mathrm{Tr}\,(\gamma_5\gamma^{\alpha}\gamma^{\beta}\gamma^{\nu}\gamma^{\lambda})\,(p_{1\alpha}p_{2\beta})\,[-B_2+B_5p_1^2+B_6p_1p_2],$$

and

$$p_{2\nu}\Pi^{\mu\nu\lambda}_{5}(p_1, p_2) = \mathrm{Tr}\,(\gamma_5\gamma^{\alpha}\gamma^{\beta}\gamma^{\mu}\gamma^{\lambda})\,(p_{1\alpha}p_{2\beta})\,[-B_1+B_3p_1p_2+B_4p_2^2], \tag{3.15}$$

and from (Appendix A.9) we see that vector Ward identities (3.9) for triangle diagram $\Pi^{\mu\nu\lambda}_5(p_1, p_2)$ with one γ_5 are satisfied. For the axial current we have

$$-(p_1+p_2)_\lambda\Pi^{\mu\nu\lambda}_{5}(p_1, p_2) = \mathrm{Tr}\,(\gamma_5\gamma^{\alpha}\gamma^{\beta}\gamma^{\mu}\gamma^{\nu})\,(p_{1\alpha}p_{2\beta})\,[-B_1+B_2]$$

$$= 2m\Pi^{\mu\nu}_{5}(p_1, p_2)+\frac{1}{8\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^{\alpha}\gamma^{\beta}\gamma^{\mu}\gamma^{\nu})\,(p_{1\alpha}p_{2\beta}), \tag{3.16}$$

where $\Pi^{\mu\nu}_5(p_1, p_2)$ is given by formula (Appendix A.13), so axial vector Ward identity (3.10) is not fulfilled. We obtained the mass independent anomaly part (the ABJ anomaly [9, 11]) and as we know, this term has the ability to destroy unitarity and renormalizability

[16]. In an anomaly free theory, because of summation over internal fermions, anomaly part vanishes and unitarity and renormalizability are restored [16].

It is worth mentioning that our three boson amplitude obeys Bose symmetry for the vector current vertices, and we have

$$\Pi_5^{\mu\nu\lambda}(p_1, p_2) = \Pi_5^{\nu\mu\lambda}(p_2, p_1). \tag{3.17}$$

The problem with Bose symmetry appears in the diagram with three γ_5's, $\Pi_{555}^{\mu\nu\lambda}$ (Appendix A.10). As we chose p_1 and p_2 as two independent four momenta, our diagram possesses a symmetry as in (3.17).

$$\Pi_{555}^{\mu\nu\lambda}(p_1, p_2, q) = \Pi_{555}^{\nu\mu\lambda}(p_2, p_1, q). \tag{3.18}$$

In the other vertices ($\mu \leftrightarrow \lambda$) and ($\nu \leftrightarrow \lambda$) the diagram has no proper symmetry

$$\Pi_{555}^{\mu\nu\lambda}(p_1, p_2, q) \neq \Pi_{555}^{\lambda\nu\mu}(q, p_2, p_1),$$

and

$$\Pi_{555}^{\mu\nu\lambda}(p_1, p_2, q) \neq \Pi_{555}^{\mu\lambda\nu}(p_1, q, p_2). \tag{3.19}$$

To restore Bose symmetry let us define

$$\Omega_{555}^{\mu\nu\lambda}(p_1, p_2, q) = \tfrac{1}{6}\,[\Pi_{555}^{\mu\nu\lambda}(p_1, p_2, q)+\Pi_{555}^{\nu\mu\lambda}(p_2, p_1, q)$$

$$+\Pi_{555}^{\lambda\nu\mu}(q, p_2, p_1)+\Pi_{555}^{\mu\lambda\nu}(p_1, q, p_2)+\Pi_{555}^{\nu\lambda\mu}(p_2, q, p_1)+\Pi_{555}^{\lambda\mu\nu}(q, p_1, p_2)], \tag{3.20}$$

which by definition has proper symmetry in each pair of vertices ($\mu \leftrightarrow \nu$, $\mu \leftrightarrow \lambda$, and $\nu \leftrightarrow \lambda$). From formulae (Appendix (A.10), (A.11) and (A.12)) one gets

$$\begin{aligned}\Omega_{555}^{\mu\nu\lambda}(p_1, p_2, q) &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda)\,[C_1 p_{1\alpha}+C_2 p_{2\alpha}]\\ &+ \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu)\,(p_{1\alpha}p_{2\beta})\,[C_3 p_1^\lambda + C_4 p_2^\lambda]\\ &+ \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[C_5 p_1^\nu + C_6 p_2^\nu]\\ &+ \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[C_7 p_1^\mu + C_8 p_2^\mu],\end{aligned} \tag{3.21}$$

where

$$C_1 = \frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\,\frac{1}{D_3(x, y)}\,[3p_1^2 y(1-y)(2y-1)+p_2^2 x(x-1+5y-6xy)$$
$$+p_1p_2 y(y-1-4x(1-3y))],$$

$$C_2 = -\frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\,\frac{1}{D_3(x, y)}\,[p_1^2 y(y-1+5x-6xy)+3p_2^2 x(1-x)(2x-1)$$
$$+p_1p_2 x(x-1-4y(1-3x))],$$

$$C_3 = \frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{y(1+x-y)}{D_3(x,y)},$$

$$C_4 = \frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{x(1-x+y)}{D_3(x,y)},$$

$$C_5 = \frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{y(y-1+2x)}{D_3(x,y)},$$

$$C_6 = \frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{x(2-2x-y)}{D_3(x,y)},$$

$$C_7 = -\frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{y(2-2y-x)}{D_3(x,y)},$$

$$C_8 = -\frac{1}{12\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{x(x-1+2y)}{D_3(x,y)}. \tag{3.22}$$

Now, we can check Ward identity (3.11)

$$\begin{aligned} p_{1\mu}\Omega^{\mu\nu\lambda}_{555}(p_1, p_2, q) &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[-C_2+C_7p_1^2+C_8p_1p_2] \\ &= 2m\Pi^{\nu\lambda}_{555}(p_1, p_2)+\frac{1}{24\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta}), \end{aligned} \tag{3.23}$$

where $\Pi^{\nu\lambda}_{555}$ is given by (Appendix (A.18)).

Similarly

$$\begin{aligned} p_{2\nu}\Omega^{\mu\nu\lambda}_{555}(p_1, p_2, q) &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[-C_1+C_5p_1p_2+C_6p_1^2] \\ &= 2m\Pi^{\mu\lambda}_{555}(p_1, p_2)-\frac{1}{24\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta}), \end{aligned} \tag{3.24}$$

$$\begin{aligned} &-(p_1+p_2)_\lambda\Omega^{\mu\nu\lambda}_{555}(p_1, p_2, q) \\ &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu)\,(p_{1\alpha}p_{2\beta})\,[-C_1+C_2-C_3(p_1^2+p_1p_2)-C_4(p_2^2+p_1p_2)] \\ &= 2m\Pi^{\mu\nu}_{555}(p_1, p_2)+\frac{1}{24\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu)\,(p_{1\alpha}p_{2\beta}), \end{aligned} \tag{3.25}$$

where $\Pi^{\mu\lambda}_{555}(p_1, p_2)$ and $\Pi^{\mu\nu}_{555}(p_1, p_2)$ are given by (Appendix (A.16)) and (A.14), respectively.

Using our prescriptions for γ_5 we got the triangle diagram with three axial vector couplings $\Pi^{\mu\nu\lambda}_{555}$ which does not satisfy Bose symmetry. After symmetrization our new trianle diagrams $\Omega^{\mu\nu\lambda}_{555}(p_1, p_2, q)$ have proper Bose symmetry and satisfy abnormal Ward identities (3.23), (3.24) and (3.25). The abnormal terms do not depend on mass, so after summation over internal fermions they vanish in anomaly free theory.

In the end, using (Appendix (A.19), (A.20), (A.21)), and (3.14) one finds that Ward identities (3.12) are satisfied. The proof of the infinite part is trivial, but in order to check that the finite parts obey relation (3.12) we need some algebra.

4. Conclusions

We have been considering the method of using dimensional regularization for theories with γ_5 couplings. We proposed a modified version of anticommuting γ_5 in n-dimensions. It was checked before in the literature that totally anticommuting γ_5 is consistent with Ward identities for diagrams with an even numbers of γ_5's and for diagrams with mixed loops containing both boson and fermion propagators.

In the case of odd numbers of γ_5 couplings we calculated precisely the triangle diagrams with one and three γ_5 matrices. They are useful in the ZZZ, ZZA and ZAA couplings in the electroweak theory. The resulting amplitudes obey the naive vector Ward identities. In the axial vector Ward identities Adler-Bell-Jackiw anomalies appear which vanish however after summation over internal fermions if a theory is anomaly free.

APPENDIX

For convenience we give some details in how we use our definition from Sect. 2. Any diagram in Fig. 3 can be got from (3.1) which we rewrite introducing Feynman parameters

$$\begin{aligned}
\Pi^{\Gamma_1\Gamma_2\Gamma_3}(p_1, p_2) = -2i \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{(2\pi)^n} \frac{1}{[-k^2-2kP+D]} \\
\times \{Q_{1\alpha}k_\beta Q_{2\gamma}[\mathrm{Tr}\,(\gamma^\alpha\Gamma_1\gamma^\beta\Gamma_2\gamma^\gamma\Gamma_3) - \mathrm{Tr}\,(\gamma^\gamma\Gamma_2\gamma^\beta\Gamma_1\gamma^\alpha\Gamma_3)] \\
+ mQ_{1\alpha}k_\beta[\mathrm{Tr}\,(\gamma^\alpha\Gamma_1\gamma^\beta\Gamma_2\Gamma_3) + \mathrm{Tr}\,(\Gamma_2\gamma^\beta\Gamma_1\gamma^\alpha\Gamma_3)] \\
+ mQ_{1\alpha}Q_{2\beta}[\mathrm{Tr}\,(\gamma^\alpha\Gamma_1\Gamma_2\gamma^\beta\Gamma_3) + \mathrm{Tr}\,(\gamma^\beta\Gamma_2\Gamma_1\gamma^\alpha\Gamma_3)] \\
+ mk_\alpha Q_{2\beta}[\mathrm{Tr}\,(\Gamma_1\gamma^\alpha\Gamma_2\gamma^\beta\Gamma_3) + \mathrm{Tr}\,(\gamma^\beta\Gamma_2\gamma^\alpha\Gamma_1\Gamma_3)] \\
+ m^2 Q_{1\alpha}[\mathrm{Tr}\,(\gamma^\alpha\Gamma_1\Gamma_2\Gamma_3) - \mathrm{Tr}\,(\Gamma_2\Gamma_1\gamma^\alpha\Gamma_3)] \\
+ m^2 k_\alpha[\mathrm{Tr}\,(\Gamma_1\gamma^\alpha\Gamma_2\Gamma_3) - \mathrm{Tr}\,(\Gamma_2\gamma^\alpha\Gamma_1\Gamma_3)] \\
+ m^2 Q_{2\alpha}[\mathrm{Tr}\,(\Gamma_1\Gamma_2\gamma^\alpha\Gamma_3) - \mathrm{Tr}\,(\gamma^\alpha\Gamma_2\Gamma_1\Gamma_3)] \\
+ m^3[\mathrm{Tr}\,(\Gamma_1\Gamma_2\Gamma_3) + \mathrm{Tr}\,(\Gamma_2\Gamma_1\Gamma_3)]\},
\end{aligned} \tag{A.1}$$

where

$$Q_1 = k+p_1, \quad Q_2 = k-p_2, \quad P = yp_1-xp_2,$$

and

$$D = m^2-p_1^2y-p_2^2x.$$

To calculate $\Pi_5^{\mu\nu\lambda}$ from Fig. 3a we put to (A.1) $\Gamma_1 = \gamma^\mu$, $\Gamma_2 = \gamma^\nu$ and $\Gamma_3 = \gamma^\lambda\gamma_5$. Using property (2.3) and (2.8) we easily get

$$\Pi_5^{\mu\nu\lambda}(p_1, p_2) = -2i\int_0^1 dx \int_0^{1-x} dy \int \frac{d^nk}{(2\pi)^n} \frac{1}{[-k^2-2kP+D]^3}$$
$$\times\{Q_{1\alpha}k_\beta Q_{2\gamma}(\mathrm{Tr}\,(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5)-\mathrm{Tr}\,(\gamma^\gamma\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma_5)]$$
$$+2m^2(k+p_1-p_2)_\alpha\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda)\}. \tag{A.2}$$

To calculate the traces in formula (A.2), we shall use the property C in Sect. 2. Then generally, one can obtain:

$$\begin{aligned}\mathrm{Tr}\,(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5) = -4i[&g^{\gamma\lambda}\varepsilon^{\alpha\mu\beta\nu}-g^{\nu\lambda}\varepsilon^{\alpha\mu\beta\gamma}\\ &+g^{\nu\gamma}\varepsilon^{\alpha\mu\beta\lambda}+g^{\beta\lambda}\varepsilon^{\alpha\mu\nu\gamma}-g^{\beta\gamma}\varepsilon^{\alpha\mu\nu\lambda}\\ &+g^{\beta\nu}\varepsilon^{\alpha\mu\gamma\lambda}+g^{\mu\gamma}\varepsilon^{\alpha\beta\nu\lambda}-g^{\mu\lambda}\varepsilon^{\alpha\beta\nu\gamma}\\ &+g^{\mu\beta}\varepsilon^{\alpha\nu\gamma\lambda}-g^{\mu\nu}\varepsilon^{\alpha\beta\gamma\lambda}+g^{\alpha\lambda}\varepsilon^{\mu\beta\nu\gamma}\\ +g^{\alpha\nu}\varepsilon^{\mu\beta\gamma\lambda}-g^{\alpha\gamma}&\varepsilon^{\mu\beta\nu\lambda}-g^{\alpha\beta}\varepsilon^{\mu\nu\gamma\lambda}+g^{\alpha\mu}\varepsilon^{\beta\nu\gamma\lambda}],\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}\mathrm{Tr}\,(\gamma^\gamma\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma_5) = -4i[&g^{\gamma\beta}\varepsilon^{\alpha\mu\nu\lambda}-g^{\gamma\nu}\varepsilon^{\alpha\mu\beta\lambda}\\ &+g^{\gamma\alpha}\varepsilon^{\mu\beta\nu\lambda}-g^{\gamma\mu}\varepsilon^{\alpha\beta\nu\lambda}+g^{\gamma\lambda}\varepsilon^{\alpha\mu\beta\nu}\\ &+g^{\nu\mu}\varepsilon^{\alpha\beta\gamma\lambda}-g^{\nu\beta}\varepsilon^{\alpha\mu\gamma\lambda}-g^{\nu\alpha}\varepsilon^{\mu\beta\gamma\lambda}\\ &+g^{\beta\alpha}\varepsilon^{\mu\nu\gamma\lambda}-g^{\nu\lambda}\varepsilon^{\alpha\mu\beta\gamma}-g^{\beta\mu}\varepsilon^{\alpha\nu\gamma\lambda}\\ +g^{\beta\lambda}\varepsilon^{\alpha\mu\nu\gamma}+g^{\alpha\lambda}&\varepsilon^{\mu\beta\nu\gamma}-g^{\mu\alpha}\varepsilon^{\beta\nu\gamma\lambda}-g^{\mu\lambda}\varepsilon^{\alpha\beta\nu\gamma}].\end{aligned} \tag{A.4}$$

From (A.2) we see that we have to contract the traces (A.3) and (A.4) with the terms $k_\alpha k_\beta k_\gamma$, $k_\alpha k_\beta p_{2\gamma}$, $p_{1\alpha}k_\beta k_\gamma$ and $p_{1\alpha}k_\beta p_{2\gamma}$. The fourth term $p_{1\alpha}k_\beta p_{2\gamma}$ gives a finite integral, so we can calculate it in any way. We obtain:

$$k_\alpha k_\beta k_\gamma[\mathrm{Tr}\,(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5)-\mathrm{Tr}\,(\gamma^\gamma\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma_5)]$$
$$= -2k^2k_\alpha\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda), \tag{A.5}$$

$$p_{1\alpha}k_\beta k_\gamma[\mathrm{Tr}\,(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5)-\mathrm{Tr}\,(\gamma^\gamma\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma_5)]$$
$$= -4k^\nu(p_{1\alpha}k_\beta)\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)-2k^2p_{1\alpha}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda) \tag{A.6}$$

and

$$k_\alpha k_\beta p_{2\gamma}[\mathrm{Tr}\,(\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\gamma\gamma^\lambda\gamma_5)-\mathrm{Tr}\,(\gamma^\gamma\gamma^\nu\gamma^\beta\gamma^\mu\gamma^\alpha\gamma^\lambda\gamma_5)]$$
$$= -4k^\mu(k_\alpha p_{2\beta})\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)-2k^2p_{2\alpha}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda). \tag{A.7}$$

For the product $p_{1\alpha}k_\beta p_{2\gamma}$ we at first make integration and then contraction with the trace tensor. Putting now all parts together we get:

$$\begin{aligned}\Pi_5^{\mu\nu\lambda}(p_1, p_2) &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda)\,[B_1p_{1\alpha}+B_2p_{2\alpha}]\\ &+\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[B_3p_1^\nu+B_4p_2^\nu]\\ &+\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[B_5p_1^\mu+B_6p_2^\mu],\end{aligned} \tag{A.8}$$

where

$$B_1 = \frac{1}{8\pi^2}\int_0^1 dx\int_0^{1-x} dy\left\{\ln D_3(x, y)\,(3y-1)+\frac{1}{D_3(x, y)}\,[p_1^2y(1-y)\,(1-2y)\right.$$
$$\left.+4p_1p_2xy(1-y)+p_2^2x(1+(1-y)(1-2x))]\right\},$$

$$B_2 = \frac{1}{8\pi^2}\int_0^1 dx\int_0^{1-x} dy\left\{\ln D_3(x, y)\,(.-3x)+\frac{1}{D_3(x, y)}\,[p_1^2y((1-x)\,(2y-1)-1)\right.$$
$$\left.-4p_1p_2xy(1-x)+p_2^2x(1-x)\,(2x-1)]\right\},$$

$$B_3 = \frac{1}{4\pi^2}\int_0^1 dx\int_0^{1-x} dy\,\frac{xy}{D_3(x, y)},$$

$$B_4 = \frac{1}{4\pi^2}\int_0^1 dx\int_0^{1-x} dy\,\frac{x(1-x)}{D_3(x, y)},$$

$$B_5 = -\frac{1}{4\pi^2}\int_0^1 dx\int_0^{1-x} dy\,\frac{y(1-y)}{D_3(x, y)},$$

$$B_6 = -\frac{1}{4\pi^2}\int_0^1 dx\int_0^{1-x} dy\,\frac{xy}{D_3(x, y)},$$

where

$$D_3(x, y) = m^2 - p_1^2 y(1-y) - p_2^2 x(1-x) - 2p_1p_2xy.$$

Using a similar procedure as in Ref. [10] it is easy to get

$$B_1 = B_3p_1p_2 + B_4p_2^2,$$

and

$$B_2 = B_5p_1^2 + B_6p_1p_2. \tag{A.9}$$

Thus our result for $\Pi_5^{\mu\nu\lambda}$ is the same as in Ref. [10] in spite of different γ_5 definition. Putting now $\Gamma_1 = \gamma^\mu\gamma^5$, $\Gamma_2 = \gamma^\nu\gamma_5$, $\Gamma_3 = \gamma^\lambda\gamma_5$ into (A.1) we get

$$\begin{aligned}\Pi_{555}^{\mu\nu\lambda}(p_1, p_2) &= \mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\lambda)\,[A_1p_{1\alpha} + A_2p_{2\alpha}]\\ &+\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[B_3p_1^\nu + B_4p_2^\nu]\\ &+\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\lambda)\,(p_{1\alpha}p_{2\beta})\,[B_5p_1^\mu + B_6p_2^\mu],\end{aligned} \tag{A.10}$$

where B_i, $i = 3, 4, 5, 6$, are the same as in (A.8) and

$$A_1 = B_1 - \frac{m^2}{4\pi^2}\int\limits_0^1 dx \int\limits_0^{1-x} dy\,\frac{1-2y}{D_3(x, y)}, \tag{A.11}$$

and

$$A_2 = B_2 - \frac{m^2}{4\pi^2}\int\limits_0^1 dx \int\limits_0^{1-x} dy\,\frac{2x-1}{D_3(x, y)}. \tag{A.12}$$

The other diagrams in Fig. 3 are simpler. Putting $\Gamma_1 = \gamma^\mu$, $\Gamma_2 = \gamma^\nu$, $\Gamma_3 = \gamma_5$ into (A.1) we get $\Pi_5^{\mu\nu}(p_1, p_2)$:

$$\Pi_5^{\mu\nu}(p_1, p_2) = -\frac{m}{8\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu)\,(p_{1\alpha}p_{2\beta})\int\limits_0^1 dx \int\limits_0^{1-x} dy\,\frac{1}{D_3(x, y)}. \tag{A.13}$$

In the same way putting $\Gamma_1 = \gamma^\mu\gamma_5$, $\Gamma_2 = \gamma^\nu\gamma_5$, $\Gamma_3 = \gamma_5$ into (A.1) we obtain

$$\Pi_{555}^{\mu\nu}(p_1, p_2) = -\frac{m}{\pi^2}\,\mathrm{Tr}\,(\gamma_5\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu)\,(p_{1\alpha}p_{2\beta})\int\limits_0^1 dx \int\limits_0^{1-x} dy\,\frac{2(x+y)-1}{D_3(x, y)}. \tag{A.14}$$

After changing the variables in (A.14) we easily get the other diagrams in Fig. 3b. To obtain $\Pi_{555}^{\mu\lambda}$ we change

$$\mu \to \lambda, \quad \nu \to \mu, \quad p_1 \to q = -p_1 - p_2, \quad p_2 \to p_1, \quad x \to 1-x-y, \quad y \to x, \tag{A.15}$$

and then

$$\Pi^{\mu\lambda}_{555}(p_1, p_2) = \frac{m}{8\pi^2} \operatorname{Tr}(\gamma_5\gamma^\mu\gamma^\lambda\gamma^\alpha\gamma^\beta)(p_{1\alpha}p_{2\beta}) \int_0^1 dx \int_0^{1-x} dy \, \frac{1-2y}{D_3(x, y)}. \tag{A.16}$$

To get $\Pi^{\nu\lambda}_{555}$ we substitute

$$\mu \to \nu, \quad \nu \to \lambda, \quad p_1 \to p_2, \quad p_2 \to q = -p_1-p_2, \quad x \to y, \quad y \to 1-x-y, \tag{A.17}$$

so we have

$$\Pi^{\nu\lambda}_{555}(p_1, p_2) = -\frac{m}{8\pi^2} \operatorname{Tr}(\gamma_5\gamma^\nu\gamma^\lambda\gamma^\alpha\gamma^\beta)(p_{1\alpha}p_{2\beta}) \int_0^1 dx \int_0^{1-x} dy \, \frac{1-2x}{D_3(x, y)}. \tag{A.18}$$

Putting $\Gamma_1 = \gamma^\mu\gamma_5$, $\Gamma_2 = \gamma^\nu\gamma_5$ and $\Gamma_3 = 1$ we obtain $\Pi^{\mu\nu}_{55}$ from Fig. 3c.

$$\Pi^{\mu\nu}_{55}(p_1, p_2) = g^{\mu\nu}\frac{m}{\pi^2}\left(C_{\mathrm{UV}} - 1 - 2\int_0^1 dx \int_0^{1-x} dy \ln D_3(x, y)\right.$$

$$+ \frac{m}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy \, \frac{1}{D_3(x, y)}\Big\{p_1^\mu p_1^\nu 2y(2y-1) + p_2^\mu p_2^\nu 2x(1-x) + p_1^\mu p_2^\nu(1-4xy)$$

$$\left. + p_2^\mu p_1^\nu(2x+2y-4xy-1) + g^{\mu\nu}(p_1^2 y(1-2y) + p_2^2 x(1-2x) + p_1p_2(4xy-2x-2y+1))\Big\}\right), \tag{A.19}$$

where

$$C_{\mathrm{UV}} = \frac{2}{4-n} - \gamma_{\mathrm{E}} + \ln 4\pi \text{ and } \gamma_{\mathrm{E}} \text{ is Euler constant.}$$

The second diagram in Fig. 3c is obtained by putting $\Gamma_1 = \gamma^\mu\gamma_5$, $\Gamma_2 = \gamma_5$, and $\Gamma_3 = 1$ into (A.1)

$$\Pi^{\mu}_{55}(p_1, p_2) = D_1 p_1^\mu + D_2 p_2^\mu,$$

where

$$D_1 = \frac{1}{4\pi^2} C_{\mathrm{UV}} + \frac{1}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy (3y-2) \ln D_3(x, y)$$

$$- \frac{1}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy \, \frac{1}{D_3(x, y)} [p_1^2 y(1-y)(2y-1)$$

$$+ p_2^2 x(y+2(1-y)(x-1)) + 2p_1p_2 y(1-y)(1-2x)], \tag{A.20}$$

and

$$D_2 = \frac{1}{2\pi^2}(C_{UV}-1) - \frac{1}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy(1+3x)\ln D_3(x, y)$$

$$+\frac{1}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy \frac{1}{D_3(x, y)}[p_1^2 xy(1-2y)$$

$$+p_2^2 x(1-x)(2x- \quad)+2p_1 p_2 xy(2x-1)]. \tag{A.21}$$

REFERENCES

[1] G.'t Hooft, *Nucl. Phys.* **B35**, 167 (1971); B. W. Lee, J. Zinn-Justin, *Phys. Rev.* **D5**, 3121 (1972); **D5**, 3137 (1972); **D8**, 4654 E (1973); **D5**, 3155 (1972); **D7**, 1049 (1973).

[2] C. Becchi, A. Rouet, R. Stora, *Commun. Math. Phys.* **42**, 127 (1975); *Phys. Lett.* **52B**, 344 (1974); *Ann. Phys.* (USA) **98**, 287 (1976).

[3] G. t' Hooft, M. Veltman, *Nucl. Phys.* **B44**, 189 (1972); C. G. Bollini, J. J. Giambiagi, A. Gonzalez Dominguez, *Nuovo Cimento* **31**, 551 (1964); J. Ashmore, *Nuovo Cimento Lett.* **4**, 289 (1972).

[4] H. C. Lee, Yang-Mills Theories in Axial and Light-Cone Gauges, Analytic Regularization and Ward Identities, Chalk River Nuclear Laboratories preprint AECL-8630, December 1984.

[5] P. Cvitanović, *Group Theory*, Nordita Notes, January 1984.

[6] D. Akyeampong, R. Delbourgo, *Nuovo Cimento* **17A**, 578 (1973); *Nuovo Cimento* **19A**, 219 (1974); P. Breitenlohner, Maison, *Commun. Math. Phys.* **52**, 11 (1977); T. Marinucci, M. Tonin, *Nuovo Cimento* **31A**, 381 (1976); G. Costa, T. Marinucci, M. Tonin, J. Julve, *Nuovo Cimento* **38A**, 373 (1977).

[7] W. A. Bardeen, R. Gastmans, B. Lautrup, *Nucl. Phys.* **B46**, 319 (1972).

[8] M. Chanowitz, M. Furman, I. Hinchliffe, *Nucl. Phys.* **B195**, 225 (1979).

[9] S. L. Adler, *Phys. Rev.* **177**, 2426 (1969).

[10] S. Gottlieb, J. T. Donohue, *Phys. Rev.* **D20**, 3378 (1979).

[11] J. S. Bell, R. Jackiw, *Nuovo Cimento* **60A**, 47 (1969).

[12] B. A. Ovrut, *Nucl. Phys.* **B213**, 241 (1983).

[13] P. H. Frampton, *Phys. Rev.* **D20**, 3372 (1979); K. Ishikawa, K. A. Milton, *Phys. Rev.* **D21**, 1700 (1980); S. Gottlieb, J. T. Donohue, *Phys. Rev.* **D21**, 1705 (1980).

[14] G. Thompson, H. L. Yu, *Phys. Lett.* **151B**, 119 (1985).

[15] M. I. Abdelhafiz, K. Kołodziej, M. Zrałek, Comments on new γ_5 definition in n-dimensions, TPJU — 2/86, preprint.

[16] C. Bouchiat, J. Iliopoulos, Ph. Meyer, *Phys. Lett.* **38B**, 519 (1972); D. J. Gross, R. Jackiw, *Phys. Rev.* **D6**, 477 (1972).