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RARE MUON DECAYS*

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In this paper, we present a short review of searches for charged lepton flavour violation on the example of rare muon decays. We discuss the evaluation of electron spectrum for muon decay in orbit which is a background process for conversion experiments.

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1. Introduction

In the era of the LHC, precision experiments play a very important role in constraining New Physics (NP) models. Moreover, this constrains can also be used to improve theoretical prediction of expected signals in some NP models at the LHC [1]. Up to now, both in accelerator searches and in precision experiments, no clear signal of physics beyond Standard Model (SM) was observed. Today, the main discrepancy between experiment and SM is muon $g-2$, where the difference between theory and experiment is about 3–4 standard deviation [2–4]. The same kind of interactions which may contribute to dipole moments of charged leptons can induce flavour off-diagonal dipole moments, which lead to the Charged Lepton Flavour Violation (CLFV) process. In many models, CLFV and contributions to anomalous dipole moments can be related to the same NP scale and, therefore, both measurements of $g-2$ for charged leptons and searches for CLFV process are complementary sources of bounds on NP parameters and scale.

In this proceeding, we will shortly review the current status and main difficulties related to searches for exotic muon decays. For more comprehensive reviews, we refer any interested Reader to existing literature on this topic. For most recent reviews, see *e.g.* [5–7].

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2. $\mu \to e\gamma$

Searches for CLFV started just after the discovery of the muon, when it was conjectured that process $\mu \to e\gamma$ should be observed. Until now, no signal was observed. Recent measurement form the MEG experiment gives bound for the branching fraction of decay $\mu^+ \to e^+\gamma$ of [8]

$$\mathrm{Br}(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13} \tag{1}$$

which is still far from the SM value $\mathrm{Br}(\mu^+ \to e^+\gamma) \sim 10^{-54}$. However, many NP models give significant contribution to this process. Models such as the Supersymmetry, two Higgs doublet models, models with extra Z' particle *etc.* predict branching for $\mu \to e\gamma$ at a rate comparable with present experimental accuracy. Although the signal for this process is very clean — with back-to-back electron and photon both carrying energy equal to half of muon mass — except accidental experimental background, there is also a background from the Radiative Muon Decay (RMD). From an experimental point of view, it is important to have a good angular and energetic resolution to reduce the background [5]. MEG experiment is currently being upgraded [9] and is expected to reach accuracy improved by one order of magnitude with respect to present measurement, $\mathrm{Br}(\mu^+ \to e^+\gamma) < 6\times10^{-14}$.

3. Muon conversion

Another important CLFV process can occur with muons caught in orbit of an atom. Typically, when muon is stopped in some material, it loses its energy by emitting X-rays and, finally, is captured in $1S$ orbital state. This allows for direct searches for conversion which is a process in which muon is converted to electron, and the nucleus plays a role of spectator and source of electromagnetic field. In this section, we will describe muon conversion and the only physical background to this process — muon Decay In Orbit (DIO) *i.e.* normal decay mode ($\mu \to e\nu_\mu\bar{\nu}_e$) in the presence of electromagnetic field of nucleus.

The experiment measures the number of signal events normalized to all nuclear captures

$$R_{\mu e} = \frac{\Gamma\left[\mu + (A,Z) \to e + (A,Z)\right]}{\Gamma\left[\mu + (A,Z) \to \nu_\mu + (A,Z-1)\right]}\,. \tag{2}$$

The current best limit is obtained by the experiment SINDRUM II [12]

$$R_{\mu e} < 7 \times 10^{-13}\,, \tag{3}$$

however, the planned experiment Mu2e is expected to improve this limit by four orders of magnitude [13, 14].

The main reason that such accuracy can be obtained is that conversion experiments have an excellent signal-to-background ratio. The signal for the conversion is mono-energetic electron with the energy

$$E_{\rm conv} = m_\mu - E_{\rm bin} - E_{\rm rec}\,, \tag{4}$$

where $E_{\rm bin} = m\frac{(Z\alpha)^2}{2} + \mathcal{O}(Z\alpha)$ is binding energy of the muon, and $E_{\rm rec} = \frac{m_\mu^2}{2M}$ is recoil energy (M is mass of the nucleus).

The only physical background for conversion searches is muon DIO. Unlike the case of free muon decay where the electron maximal energy is $\frac{m_\mu}{2}$ due to energy and momentum conservation, maximal energy for the case of muon DIO is $E_{\rm conv}$. Therefore, the tail of the electron spectrum from muon DIO overlaps with the region where the conversion signal is expected to be found. Planned precision of the experiment requires the evaluation of this background with very high accuracy.

3.1. DIO

Most recent numerical calculations, which took into account finite size of the nucleus and recoil effect, were done in [10, 11]. These analyses were based on numerical solutions of the Dirac equation. In this section, we will rather concentrate on analytical expression which can be obtained for the electron spectrum. Therefore, we will neglect the corrections coming from the finite size of nucleus. We will also neglect the recoil effect which can be easily reintroduced at the leading order. Electron spectrum can be divided into two regions in which two different expansions can be performed.

The first region is defined by the condition that momentum transfer q between the muon, the electron and the nucleus is much larger than $mZ\alpha$. In this region, an expansion of the electron spectrum into powers of $Z\alpha$ can be safely performed. This region is of special importance for the conversion experiments because in this energy range conversion signal is expected to appear. The main difficulty in this region is a slow convergence of a perturbation series. To illustrate this effect, we expand the spectrum around $E_e = E_{\rm conv}$. For simplicity, we will now consider muon decaying to electron and some neutral scalar. This scalar can be interpreted, for example, as a Majoron particle [15]. In this case, the spectrum has the following expansion

$$\frac{m_\mu}{\Gamma_0}\frac{d\Gamma}{dE_e} \approx (Z\alpha)^5 (E_e - E_{\rm conv})^3 f(Z\alpha) \tag{5}$$

with Γ_0 equal to the free decay rate and the function $f(Z\alpha)$ can be expanded as follows

$$f(Z\alpha) \approx \frac{512}{3\pi} - 160\, Z\alpha + \frac{6064 + 473\pi^2 - 2944\log(2) - 1536\log(Z\alpha)}{9\pi}(Z\alpha)^2\,.$$

The subsequent coefficients of this series are large and in order to get reliable results, we have to expand the spectrum to high order in $Z\alpha$.

We will present here a simple derivation of the lowest order expansion coefficient of the function $f(Z\alpha)$. The decay rate is given by

$$\Gamma = \frac{1}{2m_\mu}\int \frac{d^3p}{(2\pi)^3 2E_e}\frac{d^3k}{(2\pi)^3 2E_J}(2\pi)\delta(E_\mu - E_e - E_j)\mathcal{J}\mathcal{J}^\dagger \tag{6}$$

with

$$\mathcal{J} = \int d^3r\left[\overline{u}'\left(1 - \frac{i}{2m_\mu}\gamma^0\vec{\gamma}\cdot\vec{\nabla}\right)u\psi_{\text{non}-r}e^{-i\left(\vec{p}+\vec{k}\right)\cdot\vec{r}} + \overline{\psi}^{(1)}e^{-i\vec{k}\cdot\vec{r}}u\psi_{\text{non}-r}\right]. \tag{7}$$

By $\vec{p}$, we denote the electron momentum and $\vec{k}$ is the Majoron momentum. We used the electron wave function expanded in terms of the external field as a plain wave plus the first order correction $\psi' = \frac{1}{\sqrt{2E}}\left(u'e^{i\vec{p}\cdot\vec{r}} + \psi^{(1)}\right)$, and the muon $1S$ wave function with the first order relativistic correction $\psi = \left(1 - \frac{i}{2m_\mu}\gamma^0\vec{\gamma}\cdot\vec{\nabla}\right)u\psi_{\text{non}-r}$. u and u' are spinors respectively for muon and electron. Explicit expression for these wave functions can be found in any standard textbook on relativistic quantum mechanics, see *e.g.* [16]. Integrating by parts in the first term, we get

$$\mathcal{J} = \int d^3r\left[\overline{u}'\left(1 + \frac{1}{2m_\mu}\gamma^0\vec{\gamma}\cdot\left(\vec{p}+\vec{k}\right)\right)u\psi_{\text{non}-r}e^{-i\left(\vec{p}+\vec{k}\right)\cdot\vec{r}} + \overline{\psi}^{(1)}e^{-i\vec{k}\cdot\vec{r}}u\psi_{\text{non}-r}\right]. \tag{8}$$

Now, we calculate all required Fourier transformations and we get

$$\mathcal{J} = \frac{8\pi(Z\alpha m_\mu)^{\frac{5}{2}}}{\left(\vec{k}+\vec{p}\right)^2\sqrt{\pi}}\overline{u'}Au \tag{9}$$

with

$$A = \left(1 + \frac{1}{2m}\gamma^0\vec{\gamma}\cdot\left(\vec{p}+\vec{k}\right)\right)\frac{1}{\left(\vec{k}+\vec{p}\right)^2} + \frac{2\gamma^0E - \vec{\gamma}\cdot\left(\vec{k}+\vec{p}\right)}{2m_\mu\left(\vec{k}^2 - \vec{p}^2\right)}\gamma^0. \tag{10}$$

At this stage of calculations, we already expanded muon wave function, keeping only the lowest order term in $Z\alpha$. Next, we average over the spin states of muon and sum over the spins of electron, finally, we integrate over phase space and expand around $E_e = m_\mu$ such that we obtain

$$\frac{m_\mu}{\Gamma_0}\frac{d\Gamma}{dE_e} = \frac{512(m_\mu - E_e)^3(Z\alpha)^5}{3\pi} + \mathcal{O}\left((E_e - m_\mu)^4\right). \tag{11}$$

This simple and instructive derivation shows us that effects of electromagnetic field both for the electron and the muon have to be taken into account with the same accuracy. This is characteristic feature of the spectrum expansion in this region.

The other region is defined by a condition that $q \sim mZ\alpha$. In this region, the dominant effect is smearing of the spectrum due to the motion of a muon in the atom. We will not describe in any more details evaluation of the electron spectrum in this energy range as this region is not interesting from the point of view of the experiments searching for the muon conversion.

4. Conclusions and outlook

In this short note, we presented a brief summary of searches for the exotic muon decays. Processes of that kind are clear signal of physics beyond the SM. Also the ratio of signal-to-background is very good which allows for precise measurements. But increasing accuracy of the experiments requires also better determination of the background. In searches of $\mu \to e\gamma$, spectrum of both electron and photon from RMD has to be precisely evaluated. For the case of the conversion experiments, a spectrum of electrons produced in DIO has to be well understood and evaluated with good precision in the endpoint region.

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REFERENCES

[1] J. Chakrabortty, J. Gluza, R. Sevillano, R. Szafron, *J. High Energy Phys.* **1207**, 038 (2012).

[2] F. Jegerlehner, R. Szafron, *Eur. Phys. J.* **C71**, 1632 (2011).

[3] R. Szafron, F. Jegerlehner, *PoS* **RADCOR2011**, 035 (2011).

[4] M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, *Eur. Phys. J.* **C73**, 2453 (2013).

[5] R.H. Bernstein, P.S. Cooper, *Phys. Rep.* **532**, 27 (2013).

[6] A. de Gouvea, P. Vogel, *Prog. Part. Nucl. Phys.* **71**, 75 (2013).

[7] W.J. Marciano, T. Mori, J.M. Roney, *Ann. Rev. Nucl. Part. Sci.* **58**, 315 (2008).

[8] J. Adam *et al.* [MEG Collaboration], *Phys. Rev. Lett.* **110**, 201801 (2013).

[9] A. Baldin *et al.* [MEG Collaboration], `arXiv:1301.7225 [physics.ins-det]`.

[10] A. Czarnecki, X. Garcia i Tormo, W.J. Marciano, *Hyperfine Interact.* **210**, 19 (2012).

[11] A. Czarnecki, X. Garcia i Tormo, W.J. Marciano, *Phys. Rev.* **D84**, 013006 (2011).

[12] W.H. Bertl *et al.* [SINDRUM II Collaboration], *Eur. Phys. J.* **C47**, 337 (2006).

[13] D. Glenzinski [Mu2e Collaboration], *AIP Conf. Proc.* **1222**, 383 (2010).

[14] F. Cervelli [Mu2e Collaboration], *J. Phys. Conf. Ser.* **335**, 012073 (2011).

[15] X. Garcia i Tormo, D. Bryman, A. Czarnecki, M. Dowling, *Phys. Rev.* **D84**, 113010 (2011).

[16] V.B. Berestetsky, E.M. Lifshitz, L.P. Pitaevsky, *Quantum Electrodynamics*, Course of Theoretical Physics, vol. 4 Oxford, UK: Pergamon, 1982.