Vol. 30 (1999) ACTA PHYSICA POLONICA B No 11

THE HARD BREMSSTRAHLUNG IN $e^+e^- \to 4f$ WITH NON-ZERO FERMION MASSES* **

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(Received October 22, 1999)

An efficient method for calculating polarized matrix elements of the four fermion reactions $e^+e^- \to 4f$ and corresponding hard bremsstrahlung reactions with non-zero fermion masses is discussed. The numerical results for the total cross sections and some differential cross sections of $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ and $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ are given. The dependence on the fermion masses is illustrated by comparing the hard bremsstrahlung corrections to different semi-leptonic channels.

PACS numbers: 13.40.Ks, 02.70.Lq, 11.80.Cr

1. Introduction

The analysis of $W^\pm$-pair production at LEP2 and at future high energy e^+e^- linear colliders (NLC) requires the knowledge of the precise Standard Model (SM) predictions including radiative corrections. Actually, as the W bosons almost immediately decay, the precise predictions for the reactions

$$e^+e^- \to 4f, \tag{1}$$

where $4f$ denotes a possible four fermion final state, are necessary. A specific final state of (1) can be obtained not only by the production and decay of two virtual W bosons. It also receives a contribution from a single or no W boson exchange, which is sometimes referred to as a non doubly-resonant background. The lowest order SM results for reactions (1) including some classes of the radiative corrections such as the initial and final state radiation, Coulomb corrections, running of the fine structure constant, *etc.*, for all the

* Presented at the XXIII International School of Theoretical Physics "Recent Developments in Theory of Fundamental Interactions", Ustroń, Poland, September 15–22, 1999.

** Based on work in collaboration with Fred Jegerlehner.

possible four fermion final states have been already implemented in several Monte Carlo event generators and semi-analytic programs which have been thoroughly compared in Ref. [1].

A calculation of the complete $\mathcal{O}(\alpha)$ electroweak virtual corrections to reactions (1) is a formidable task which has not been completed yet for any of the possible final states. Such a complete calculation was performed for the production of the on shell W bosons in Ref. [2] and later extended to

$$e^+e^- \to W^+W^- \to 4f \tag{2}$$

in the double-pole approximation, see Ref. [7] for the most recent review and Ref. [6] for a simple analytic approximation in the high energy regime.

Concerning the real photon emission the situation looks much better. The hard photon bremsstrahlung to the on shell W-pair production was calculated in Refs. [3] and [4], and to the four-fermion reactions mediated by two resonant W bosons (2) in Ref. [8]. The complete lowest order calculation, including the non doubly-resonant background, of $e^+e^- \to e^-\bar{\nu}_e u\bar{d}\gamma$ was presented in Ref. [9] and calculations of

$$e^+e^- \to 4f\gamma \tag{3}$$

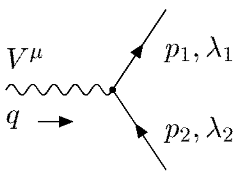
with the iterative algorithm `ALPHA` [10] for an arbitrary final state were reported in Ref. [11]. Results on bremsstrahlung for purely leptonic reactions have been published in Ref. [12] and recently predictions for all processes $e^+e^- \to 4f\gamma$ with massless fermions have been presented in Ref. [13].

Presently available $e^+e^- \to 4f$, $4f\gamma$ matrix–elements are precise enough for the analysis of LEP2 data, however, at the NLC, radiative corrections will get more significant. In particular the proper treatment of the collinear photons will be crucial. It requires to take into account the fermion masses appropriately. Therefore, an efficient method of calculating the hard photon bremsstrahlung for four fermion production in e^+e^- annihilation without neglecting the fermion masses has been proposed in Ref. [14]. With the non-zero fermion masses, the phase space integration can be performed to the very collinear limit, the cross sections can be calculated independently of angular cuts, the background contributions coming from undetected hard photons can be estimated, the pole arising when a photon exchanged in the t-channel approaches its mass shell can be correctly handled and the Higgs boson effects can be incorporated in a consistent way.

2. Method of calculation

The polarized matrix elements of any of reactions (1) and (3) can be calculated with the use of the method developed in Refs. [3] and [14]. The calculation is performed for a given set of external particle momenta in a fixed reference frame, *e.g.* the centre of mass system of the initial particles, where the initial momenta are parallel to the z axis. The Dirac γ^μ matrices are chosen in the Weyl representation and the fermion spinors in the helicity basis are used. Fermion masses are kept non-zero. For simplicity photon polarization vectors are chosen real. However, complex polarization vectors in the helicity basis could be used instead if one were interested in polarized cross sections. The reader is referred to Refs. [3] and [14] for the explicit definitions of the spinors and polarization vectors.

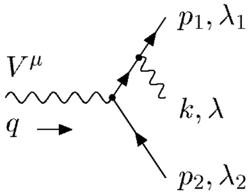
It is very useful to select parts of the Feynman graphs of reactions (1) and (3) which contain a single uncontracted Lorentz index and define generalized polarization vectors. Such a generalized polarization vector is for example the coupling of an internal gauge boson to the external fermions



$$\begin{aligned} &\to \varepsilon^\mu_V\,(p_1, p_2, \lambda_1, \lambda_2) \\ &= D^{\mu\nu}_V(q)\bar{\psi}_1(p_1,\lambda_1)\gamma_\nu\left(g^{(-)}_V P_- + g^{(+)}_V P_+\right)\psi_2(p_2,\lambda_2), \end{aligned} \tag{4}$$

where $\bar{\psi}_1(p_1,\lambda_1)$ and $\psi_2(p_2,\lambda_2)$ are spinors of a particle or an antiparticle of four-momentum p_i, mass m_i and helicity λ_i, $P_\pm$ are the chirality projectors and $g^{(\pm)}_V$ are the corresponding couplings, $D^{\mu\nu}_V(q)$ is the vector boson propagator and $q = \pm p_1 \pm p_2$ is the four-momentum transfer. The $+(-)$ sign corresponds to an outgoing (incoming) particle.

The photon emission from any of the external fermion legs of (4) can be taken into account by defining other generalized polarization vectors, *e.g.*

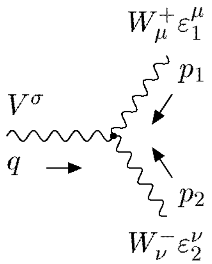


$$\begin{aligned} &\to \varepsilon^\mu_{\gamma V}\,(p_1, p_2, k, \lambda_1, \lambda_2, \lambda) \\ &= D^{\mu\nu}_V(q+k)\bar{\psi}_1(p_1,\lambda_1)g_{\gamma 1}\not{\varepsilon}(k,\lambda)\frac{\pm\not{p}_1 + \not{k} + m_1}{(\pm p_1 + k)^2 - m_1^2} \\ &\quad\times\gamma_\nu\left(g^{(-)}_V P_- + g^{(+)}_V P_+\right)\psi_2(p_2,\lambda_2), \end{aligned} \tag{5}$$

where the upper (lower) sign is assumed if ψ_1 represents an outgoing particle (an incoming antiparticle), $\varepsilon^\mu(k,\lambda)$ is the photon polarization vector and $g_{\gamma 1}$

is the photon coupling to ψ_1. The photon emission from the other fermion leg of (4) can be represented by a similar generalized polarization vector.

A contraction of the triple gauge boson coupling with two (generalized) polarization vectors can be considered as another generalized polarization vector, *e.g.*



$$\begin{aligned} \to \ \varepsilon_V^\sigma(1,2) &= D_V^{\sigma\rho}(q)\Gamma^{(WWV)}_{\mu\nu\rho}(p_1,p_2,q)\varepsilon_1^\mu\varepsilon_2^\nu \\ &= D_V^{\sigma\rho}(q) g_{WWV}\left[(p_1-p_2)_\rho\, \varepsilon_1\cdot\varepsilon_2 + (p_2-q)\cdot\varepsilon_1\, \varepsilon_{2\rho}\right. \\ &\qquad \left. +(q-p_1)\cdot\varepsilon_2\, \varepsilon_{1\rho}\right], \end{aligned} \tag{6}$$

where $V = \gamma, Z^0$ and the dependence on four momenta and polarizations of the generalized polarization vectors has been represented by subscripts 1 and 2.

Similarly the quartic gauge boson coupling contracted with three (generalized) polarization vectors is also a generalized polarization vector.

Corresponding scalar objects can be defined for the Higgs boson by replacing the vector boson couplings and propagator in Eqs. (4)–(6) by the Higgs boson couplings and propagator.

With the polarization vectors of Eqs. (4)–(6), an amplitude of any Feynman graph of $e^+e^- \to 4f(\gamma)$ can be written in a simple form without contractions of the Lorentz indices. In order to illustrate this consider the semi-leptonic process

$$e^+(p_1) + e^-(p_2) \to U(p_3) + \bar{D}(p_4) + l^-(p_5) + \bar{\nu}_l(p_6), \tag{7}$$

and the corresponding bremsstrahlung reaction

$$e^+(p_1) + e^-(p_2) \to U(p_3) + \bar{D}(p_4) + l^-(p_5) + \bar{\nu}_l(p_6) + \gamma(p_7). \tag{8}$$

In reactions (7) and (8), $U = u, c, t$; $D = d, s, b$; $l = \mu, \tau$ and the particle momenta have been indicated in the parenthesis. The Feynman diagrams of reaction (7) are shown in Fig. 1. The diagrams of reaction (8) can be obtain from those of Fig. 1 by attaching an external photon line to each charged particle and to the triple gauge boson vertex of diagrams (2), (3) in Fig. 1.

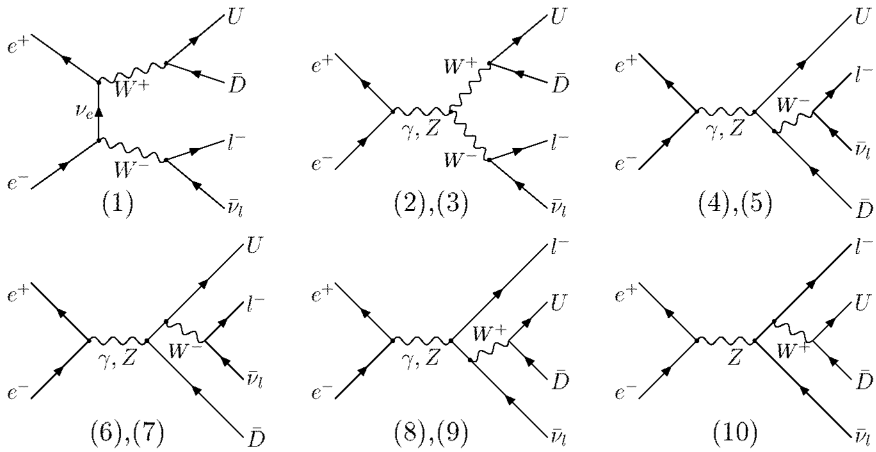


Fig. 1. The Feynman diagrams of reaction (7)

The necessary generalized polarization vectors are

$$
\begin{aligned}
\varepsilon^\mu_\gamma &\equiv \varepsilon^\mu_\gamma(p_1,p_2,\lambda_1,\lambda_2) = D^{\mu\nu}_\gamma(p_{12})\,\bar{v}_1(p_1,\lambda_1)\gamma_\nu g_{\gamma 1} u_2(p_2,\lambda_2),\\
\varepsilon^\mu_Z &\equiv \varepsilon^\mu_Z(p_1,p_2,\lambda_1,\lambda_2) = D^{\mu\nu}_Z(p_{12})\,\bar{v}_1(p_1,\lambda_1)\\
&\quad\times\gamma_\nu\left(g^{(-)}_{Z1}P_- + g^{(+)}_{Z1}P_+\right)u_2(p_2,\lambda_2),\\
\varepsilon^\mu_+ &\equiv \varepsilon^\mu_{W^+}(p_3,p_4,\lambda_3,\lambda_4) = D^{\mu\nu}_W(p_{34})\,\bar{u}_3(p_3,\lambda_3)\gamma_\nu g_W P_- v_4(p_4,\lambda_4),\\
\varepsilon^\mu_- &\equiv \varepsilon^\mu_{W^-}(p_5,p_6,\lambda_5,\lambda_6) = D^{\mu\nu}_W(p_{56})\,\bar{u}_5(p_5,\lambda_5)\gamma_\nu g_W P_- v_6(p_6,\lambda_6)\,, \quad (9)
\end{aligned}
$$

where $p_{12} = p_1 + p_2$, $p_{34} = p_3 + p_4$ and $p_{56} = p_5 + p_6$; $g_{\gamma 1}, g^{(\pm)}_{Z1}$ and g_W are the SM couplings. The photon propagator $D^{\mu\nu}_\gamma(q)$ is taken in the Feynman gauge and the propagators of the massive gauge bosons $D^{\mu\nu}_V(q)$, $V = W, Z$, are defined in the unitary gauge. The constant widths Γ_W, Γ_Z are introduced through the complex mass parameters

$$M_V^2 = m_V^2 - i m_V \Gamma_V \quad (10)$$

in the propagators. The electroweak mixing parameter is kept real, though. This simple prescription violates the SU(2) gauge invariance. However, a comparison of the total cross sections of reactions (7), (8) with those of Ref. [13], which were obtained in different gauge and in the scheme where the complex masses of Eq. (10) are used both in propagators and couplings, shows an agreement within the Monte Carlo errors [14]. The agreement holds for energies up to 10 TeV which means that the unitarity cancellations are not spoiled to an extent that may have been relevant for present and future experiments. The electromagnetic gauge invariance of (8) is preserved for the non-zero fermion masses even if the widths Γ_W and Γ_Z are treated as two independent parameters. This has been checked analytically and confirmed by the numerical calculation.

With the generalized polarization vectors of Eq. (9), the amplitudes corresponding to the Feynman diagrams of Fig. 1 can be written as

$$
\begin{aligned}
M_1 &= \bar{v}_1 g_W \not{\varepsilon}_+ P_- \frac{\not{p}_2 - \not{p}_{56}}{(p_2 - p_{56})^2} g_W \not{\varepsilon}_- P_- u_2, \\
M_{2,3} &= g_{WWV} \left[(p_{12} + p_{56})\cdot\varepsilon_+ \; \varepsilon_V\cdot\varepsilon_- + (p_{34} - p_{56})\cdot\varepsilon_V \; \varepsilon_+\cdot\varepsilon_- \right. \\
&\quad \left. - (p_{12} + p_{34})\cdot\varepsilon_- \; \varepsilon_V\cdot\varepsilon_+ \right], \\
M_{4,5} &= \bar{u}_3 \not{\varepsilon}_V \left(g_{V3}^{(-)} P_- + g_{V3}^{(+)} P_+ \right) \frac{\not{p}_3 - \not{p}_{12} + m_3}{(p_3 - p_{12})^2 - m_3^2} g_W \not{\varepsilon}_- P_- v_3, \\
M_{6,7} &= \bar{u}_3 g_W \not{\varepsilon}_- P_- \frac{\not{p}_3 + \not{p}_{56} + m_4}{(p_3 + p_{56})^2 - m_4^2} \not{\varepsilon}_V \left(g_{V4}^{(-)} P_- + g_{V4}^{(+)} P_+ \right) v_3, \\
M_{8,9} &= \bar{u}_5 \not{\varepsilon}_V \left(g_{V5}^{(-)} P_- + g_{V5}^{(+)} P_+ \right) \frac{\not{p}_5 - \not{p}_{12} + m_5}{(p_5 - p_{12})^2 - m_5^2} g_W \not{\varepsilon}_+ P_- v_6, \\
M_{10} &= \bar{u}_5 g_W \not{\varepsilon}_+ P_- \frac{\not{p}_5 + \not{p}_{34}}{(p_5 + p_{34})^2} \not{\varepsilon}_Z g_{Z6}^{(-)} P_- v_6 \,,
\end{aligned} \tag{11}
$$

where the double subscripts on the left hand side correspond to $V = \gamma, Z^0$ on the right hand side. In the Weyl representation, the Dirac algebra of 4×4 matrices in Eqs. (9) and (11) can be easily reduced to the algebra of 2×2 matrices [14] and then they can be implemented in a `FORTRAN` program and computed numerically for any specific set of the particle momenta and polarizations. The Fortran 90 language standard which contains intrinsic functions for array manipulations is particularly suitable for this task. Note that amplitudes of the diagrams which differ only in contributions from the photon and Z propagators can be added, which reduces the number of amplitudes to be calculated.

The matrix element of the bremsstrahlung reaction (8) is calculated in the same way. Three representatives of the 71 Feynman diagrams which contribute to reaction (8) if the Higgs exchange is neglected are depicted in Fig. 2. The corresponding amplitudes read

$$
\begin{aligned}
M_1^\gamma &= \bar{v}_1 g_{\gamma 1} \not{\varepsilon}_7 \frac{\not{p}_7 - \not{p}_1 + m_1}{(p_7 - p_1)^2 - m_1^2} g_W \not{\varepsilon}_+ P_- \frac{\not{p}_2 - \not{p}_{56}}{(p_2 - p_{56})^2} g_W \not{\varepsilon}_- P_- u_2, \\
M_{2,3}^\gamma &= = g_{\gamma VWW} \left(\varepsilon_V\cdot\varepsilon_+ \; \varepsilon_7\cdot\varepsilon_- + \varepsilon_V\cdot\varepsilon_- \; \varepsilon_7\cdot\varepsilon_+ - 2\varepsilon_V\cdot\varepsilon_7 \; \varepsilon_+\cdot\varepsilon_- \right), \\
M_{4,5}^\gamma &= \bar{u}_3 \not{\varepsilon}_{\gamma V} \left(g_{V3}^{(-)} P_- + g_{V3}^{(+)} P_+ \right) \frac{-\not{p}_4 - \not{p}_{56} + m_3}{(-p_4 - p_{56})^2 - m_3^2} g_W \not{\varepsilon}_- P_- v_3 \,,
\end{aligned} \tag{12}
$$

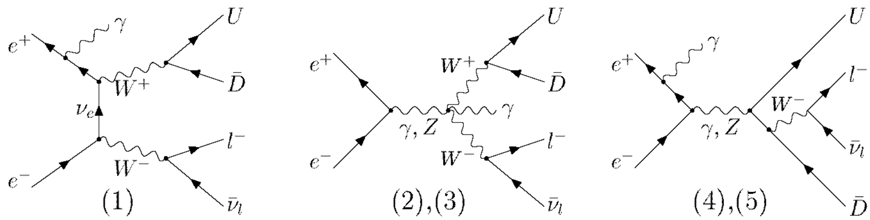


Fig. 2. The Feynman diagrams of reaction (7)

where $\varepsilon_7^\mu \equiv \varepsilon^\mu(p_7, \lambda_7)$ is the photon polarization vector and the generalized polarization vector $\varepsilon_{\gamma V}$ is defined according to Eq. (5).

In the soft photon limit, $|\mathbf{p}_7| < \omega$, the matrix element of reaction (8) factorizes

$$\begin{aligned} M^\gamma\big|_{|\mathbf{p}_7|<\omega} &= -\left(g_{\gamma 1}\frac{p_1^\mu}{p_1\cdot p_7} - g_{\gamma 2}\frac{p_2^\mu}{p_2\cdot p_7} + g_{\gamma 3}\frac{p_3^\mu}{p_3\cdot p_7}\right. \\ &\qquad \left. -g_{\gamma 4}\frac{p_4^\mu}{p_4\cdot p_7} + g_{\gamma 5}\frac{p_5^\mu}{p_5\cdot p_7}\right)\varepsilon_\mu(p_7, \lambda_7) M_0, \end{aligned} \tag{13}$$

where $g_{\gamma i}$, $i = 1, \ldots, 5$ are the photon-fermion couplings.
The spin averaged matrix element squared is then computed numerically.

The phase space integration is done with the Monte Carlo routine `VEGAS` [15]. The number of integrations is reduced from 8 to 7 for reaction (7) and from 11 to 10 for the bremsstrahlung reaction (8) by integrating out the dependence on the azimuthal angle related to the rotational symmetry with respect to the beam axis. In order to account for a number of peaks in the matrix element a few different phase space parametrizations of reaction (8) are used which are then combined in a single multichannel Monte Carlo. In the soft photon limit, the integrals over the photon phase space can be performed analytically due to the factorization of Eq. (13). The reader is referred to Ref. [14] for the details of the phase space integration.

3. Numerical results

In this section, some numerical results for reactions (7) and (8) will be given. The relevant physical parameters are the gauge boson masses and widths $m_W = 80.23$ GeV, $\Gamma_W = 2.085$ GeV, $m_Z = 91.1888$ GeV, $\Gamma_Z = 2.4974$ GeV, the fermion masses: $m_e = 0.51099906$ MeV, $m_\mu = 105.658389$ MeV, $m_\tau = 1777.05$ MeV, $m_u = 5$ MeV, m_d=10 MeV, m_s=170 MeV, m_c=1.3 GeV. The SM couplings are parametrized by $\alpha_W = 1/128.07$ and by the electroweak mixing parameter $\sin^2\theta_W = 0.22591$. The couplings of the bremsstrahlung photon are parametrized by $\alpha = 1/137.0359895$ which means in practice that we multiply the matrix element squared by the ratio α/α_W.

The matrix elements of reactions (7) and (8) have been checked against `MADGRAPH` [16] and the electromagnetic gauge invariance of (8) has been verified both analytically and numerically. The phase space integrals have been checked against their asymptotic limits obtained analytically. The soft photon cut off independence of the splitting of the bremsstrahlung cross section into the soft and hard photon part has been verified. Finally, the total cross section of $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ has been compared against Refs. [17] and [13] and the total cross section of the corresponding bremsstrahlung reaction in the presence of the canonical cuts has been compared with Ref. [13].

The energy dependence of the total cross sections of reactions $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ and $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$, is shown in Fig. 3. The hard bremsstrahlung cross section has been calculated with the photon energy cut $E_\gamma = 1$ GeV.

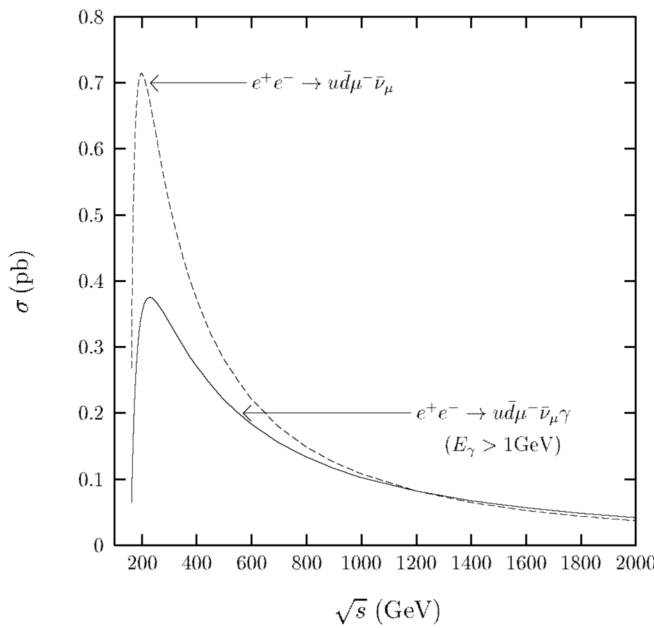


Fig. 3. The energy dependence of the total cross sections

In order to illustrate the dependence on the fermion masses the cross sections of different final states of reaction (8) for a few centre of mass energies are listed in Table I. The cross sections, for the energy cut $E_\gamma > 0.1$ GeV, change by about -9% at 189 GeV and by about -6% at 2 TeV.

TABLE I

Mass dependence of the total cross sections of (8) in fb for $E_\gamma > 0.1$ GeV

$E_{\rm cm}$ GeV	$\sigma_\gamma(u\bar{d}\mu^-\bar{\nu}_\mu)$	$\sigma_\gamma(c\bar{s}\mu^-\bar{\nu}_\mu)$	$\sigma_\gamma(u\bar{d}\tau^-\bar{\nu}_\tau)$
189.0	573.4(4)	525.2(4)	522.6(4)
360.0	448.5(4)	418.4(4)	414.1(4)
500.0	322.8(4)	302.0(4)	298.1(3)
2000.0	56.48(27)	53.19(25)	52.67(13)

The photon spectra at $\sqrt{s} = 189$ GeV and $\sqrt{s} = 500$ GeV for reaction $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ are shown in Fig. 4. It is seen that the spectra are relatively soft, with a substantial fraction of events having photon energies of $O(\Gamma_W)$. A bump of the 189 GeV spectrum at $E_\gamma \sim 25$ GeV reflects the W-pair production threshold.

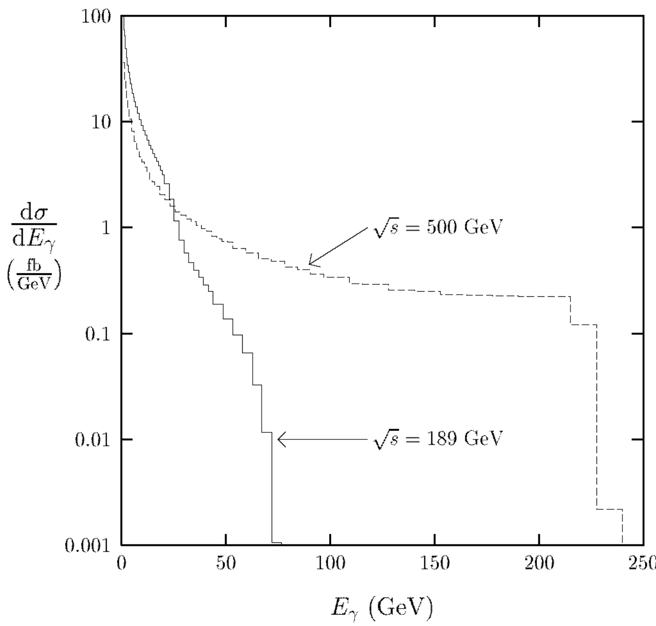


Fig. 4. The photon spectra at $\sqrt{s} = 189$ GeV and $\sqrt{s} = 500$ GeV

Finally the invariant mass distributions of the $u\bar{d}$ quark pair $d\sigma/dm_{34}^2$ of reactions $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ at $\sqrt{s} = 189$ GeV are plotted as a function of the invariant mass m_{34} in Fig. 5.

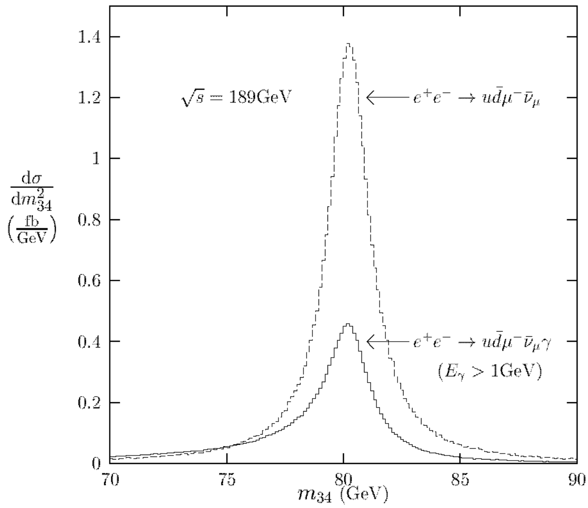


Fig. 5. The differential cross section $d\sigma/dm_{34}^2$ at $\sqrt{s} = 189$ GeV versus the invariant mass of the $u\bar{d}$ pair m_{34}.

4. Conclusions and outlook

An efficient method for calculating photon radiation cross sections for massive fermions has been discussed. It allows for a correct treatment of the collinear phase space regions and for a consistent implementation of the Higgs boson effects. The semi-leptonic channels of reactions (1) and (3) and in particular the reactions $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$ and $e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu\gamma$ have been studied. The latter is well suited for an investigation of effects of the final state photon emission, *e.g.* on the W mass measurement since muons appear well separated from photons in detectors. One could study the quark mass effects due to the different quark flavor channels in $e^+e^- \to \mu^-\bar{\nu}_\mu\gamma$ + hadrons at a high luminosity linear collider, like TESLA. Of particular interest would be a detailed investigation of the single top production channel $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu$. This reaction requires a special treatment because of the pole developed by the t-quark propagator in diagrams (4), (5) of Fig. 1. The pole could be regularized by introducing a constant width of the top which, however, violates the electromagnetic gauge invariance of the bremsstrahlung reaction $e^+e^- \to t\bar{b}\mu^-\bar{\nu}_\mu\gamma$. Therefore, the reliability of results obtained with the constant width prescription has to be carefully studied, before the actual numbers are presented. Having the final state photon resolution in $e^+e^- \to U\bar{D}\mu^-\bar{\nu}_\mu\gamma$ could also make it possible to investigate *e.g.* the quartic γVWW couplings ($V = \gamma, Z$), which are absent on the Born level of $4f$ production.

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