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WHITE DWARFS AS A SOURCE OF CONSTRAINTS ON EXOTIC PHYSICS*

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In this paper we briefly review main ideas underlying the constraints on exotic physics coming from Astrophysics already used by the others. Next we present a new bound coming from the White Dwarf cooling. Such stringent bound is possible due to accurate measurements offered by astroseismology. Specifically we consider the G117-B15A pulsating white dwarf (ZZ Ceti star) for which the speed of the period increase has been accurately measured for its fundamental oscillation mode. It has been claimed that this mode detected in G117-B15A is perhaps the most stable oscillation ever recorded in the optical band. Then we review our result concerning the bounds on compactification scale in the theory with large extra dimensions according to Arkani-Hamed, Dimopoulos and Dvali. Because an additional channel of energy loss (Kaluza–Klein gravitons) would speed up the cooling rate, one is able to use the aforementioned stability to derive a bound on compactification scale. We find the lower bound on compactification scale to be $M_s > 14.3$ TeV$/c^2$ which is more stringent than solar or red-giant bounds, as well as the bound coming from LEP. In final section we point out that pulsating hot "pre-White Dwarf" PG 1159-035 (GW Virginis) whose oscillation period increases at the rate of the order of magnitude larger than predicted could be a promising object for further investigations.

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1. Introduction

Last century witnessed a successful application of standard physics in elucidating the properties of celestial bodies which consequently can be used as a source of — sometimes strong — bounds. The idea that astrophysical considerations can constrain "exotic" physics is not a new one. For example it has been implemented to constrain the axion mass [1]. The main idea here is that if "exotic" physics to be tested predicts the existence of weakly interacting particles which can be produced in stellar interiors such weakly interacting particles would serve as an additional source of energy loss in many ways influencing the course of stellar evolution.

There are three main sources of astrophysical bounds invoked in this context. First is the Sun which is a hydrogen burning main sequence star with radiative interior. Back reaction in response to increased energy loss results in rising the internal temperature and shrinking of the radiative interior (for details see [1]). Because enhanced temperature increases the rates of nuclear reactions this would exacerbate solar neutrino problem. Moreover, helioseismology provides an accurate estimate of the internal profile of isothermal sound speed squared [2]. The most accurate value of this quantity, which is proportional to temperature, refers to the radius $r = 0.2\ R_{\odot}$. This provides an effective mean to constrain exotic sources of energy loss.

The second source of bounds comes from red giants — stellar evolutionary phase after the hydrogen has been exhausted and which lasts until the helium ignition takes place during the so called helium flash. Additional cooling would result in one of the following effects. First of all helium flash would not occur if cooling was too effective. If additional cooling is less effective (so that helium flash eventually takes place) the red giant tip on the Hertzsprung–Russel diagram would be located higher above the Horizontal Branch and the time spent on a Horizontal Branch would be shortened. All these effects could be tested on HR diagrams for globular clusters [1].

The last and the most effective of traditional sources of bounds comes from SN1987a — more specifically from the observed duration of neutrino pulse. The pulse would be shortened had the nascent neutron star cooled down more rapidly than standard theory predicted.

In this paper we consider another class of astrophysical objects for which the cooling rate is known from observations, namely the pulsating white dwarfs. We will focus our attention on ZZ Ceti star G117-B15A which was considered previously as a tool of testing fundamental physics (in the context of axion emission) [1, 3] and then we will extend our discussion to the hot pre-white dwarf star GW Vir for which there exist reliable data on the rate of cooling.

2. White dwarf astroseismology as a probe of cooling rate

As we have already pointed out, the main idea of astrophysical constraints on exotic physics is to consider the effects[1] of additional cooling mechanism offered by new kinds of weakly interacting particles predicted by these theories.

White dwarfs constitute another class of objects where astronomical observations provide useful limits on new stellar energy losses. These compact objects are the remnants of stars with initial masses of up to several $M_\odot$ after they ascended the asymptotic giant branch, had lost their envelopes in the planetary nebula stage and developed a degenerate carbon–oxygen core[2]. Their subsequent evolution is governed by cooling, first dominated by neutrino losses throughout their volumes, later by surface photon emission. This suggested a very simple approach first formulated by Mestel [4]. If we assume that surface photon emission (thermal cooling) is the main source of white dwarf's energy one can write:

$$L = -\frac{dU}{dt} = -\frac{\partial U}{\partial T}\frac{dT}{dt} = -C_v M \frac{dT}{dt}\,, \tag{1}$$

where U denotes the internal (thermal) energy of the star, T is the central temperature, M is the mass of the white dwarf and C_v is constant volume heat capacity. To solve this equation one needs a relation between L and T. Such a relation can be obtained from the structure equations assuming an analytic power law expression for the opacity (Kramer's law). In the end one can write the following formula relating the luminosity L and central temperature T:

$$\frac{L}{M} = 9.743\; 10^{-21} T^{3.5}\, \frac{L_\odot}{M_\odot}\,. \tag{2}$$

During past few decades, astronomers inferred the cooling speed observationally from the white dwarf luminosity function, *i.e.* the white dwarf number density per brightness interval. The measured luminosity function reveals that there are few bright white dwarfs and many faint ones. The shape of best fitted line is in good agreement with the Mestel's cooling law (at least at the faint end of the curve where photon cooling is a dominant process). This is somewhat surprising but it owes to the fact that several later improvements in the theory of white dwarfs cooling and structure turned out to have cancelling effects when integrated over whole evolutionary time span. The observed luminosity function dips at the bright end due

[1] We actually use the accuracy to which the observations of real objects agree with our standard theoretical picture to derive the bounds.

[2] The mass of the resulting white dwarf star is typically of order of 0.6 $M_\odot$.

to neutrino emission which quickly switches off as the star cools. The luminosity function drops sharply at the faint end. Even the oldest white dwarfs have not yet cooled any further, implying that they were born 8–12 Gyr ago, in good agreement with the estimated age of the galaxy. Therefore, a novel cooling agent cannot be much more effective than the surface photon emission. This was the main conclusion used so far in discussing the astrophysical bounds for exotic physics derivable form the astrophysics of white dwarfs [1].

Now, with an advent of astroseismological techniques we have at hand a new more accurate possibility of tracing actual rate of white dwarf cooling. Namely, in a certain range of surface temperatures white dwarfs are pulsationally unstable and are observed as the so called ZZ Ceti stars. The pulsation period of such stars, which is of the order of a few minutes, depends on the luminosity, and the period decrease gives a direct information about the cooling speed. Let us be a little bit more specific on this issue.

Extensive theoretical studies of non-radial oscillations of white dwarfs had been carried out long time ago [5]. Appropriate theory of stellar oscillations consists in linearising the Poisson equation as well as equations of momentum, energy and mass conservation with respect to small non-radial perturbations [6]. These perturbations (which in ZZ Ceti stars are the so called g-modes) can have oscillatory behaviour $\propto \exp(-i\sigma t)$, where $\sigma^2 \propto -A$. The quantity A is determined by thermodynamical properties of stellar matter: $A = \frac{d\ln\rho}{dr} - \frac{1}{\Gamma_1}\frac{d\ln p}{dr}$, where: ρ denotes the density, r — radial coordinate, p is the pressure and Γ_1 is the first adiabatic index [6]. It is also illustrative to see the connection between oscillations and convective instability. Namely, if an element of a stratified spherically symmetric fluid is displaced from the radius r to $r + \delta r$ then the buoyancy force per unit volume acting upwards on this element is $\rho g A \delta r$. Hence if $A < 0$ the element tends to return to the original position (we say that fluid is stable against convection) and in dissipationless fluid the element oscillates with the so called Brunt–Väisäilä frequency N, where $N^2 = -Ag$. For small $|A|$ frequencies of the g-modes are approximately equal to the Brunt–Väisäilä frequency.

Now, for a zero-temperature degenerate electron gas $A = 0$ meaning that no g-modes are supported. However, if non-zero thermal effects are taken into account one can show [5] that $A \propto T^2$ and consequently $\frac{1}{P} \propto T$ *i.e.* the periods scale like $1/T$ where T is the central temperature. Consequently, the increase of the pulsation period can be calculated from the following formula:

$$\frac{\dot{P}}{P} \propto -\frac{\dot{T}}{T} = -\frac{L}{C_v M_{\rm WD} T}\,. \tag{3}$$

In the second part of the above formula (equality) the aforementioned Mestel cooling law has been applied [4].

The conclusion is that from the rate of period increase one is able to extract the rate of temperature decrease — hence the cooling speed. Now, let us suppose that astroseismological data provided us with the observed speed of pulsation period increase $\dot{P}_{\rm obs}$. Independently to this observable quantity, we have our theoretical expectations[3] concerning an analogous quantity $\dot{P}_0$ without additional cooling. If we confront observations with theory we can expect two situations: a disagreement and an agreement. In the first case disagreement means that $\dot{P}_{\rm obs} > \dot{P}_0$[4] then one can interpret this as an indication of a new channel of energy loss operating with the luminosity $L_{\rm new}$:

$$L_{\rm new} = L_\gamma \left(\frac{\dot{P}_{\rm obs}}{\dot{P}_0} - 1 \right) , \tag{4}$$

where L_γ represents standard photon cooling.

The second case is more delicate. Had the observations an infinite precision one would say that agreement excluded any new channels of energy loss. In practice we speak about an agreement only in a statistical sense, *i.e.* that the theoretical value $\dot{P}_0$ lies within say 2σ confidence interval of $\dot{P}_{\rm obs}$. Then one can use an upper 2σ value of $\dot{P}_{\rm obs}$ (let's denote it $\dot{P}_{\rm obs,upper}$) to derive a bound:

$$L_{\rm new} < L_\gamma \left(\frac{\dot{P}_{\rm obs,upper}}{\dot{P}_0} - 1 \right) \tag{5}$$

as an estimate for how much of hypothetical new energy loss is allowed by observations combined with the standard theory.

3. G117-B15A as a source of constraints on exotic physics

ZZ Ceti variable star G117-B15A has already been a well recognised object in the context of astroparticle physics [1, 3]. Since its discovery in 1976 [7] G117-B15A has been extensively studied. Regarding its variability the observed periods are 215.2, 271 and 304.4 s together with higher harmonics and linear combinations thereof [8]. Using the data accumulated over the period 1976–1991 Kepler derived an upper limit to the rate of increase of main pulsation period $P = 215.2$ s as $\dot{P} = (12.0 \pm 3.5) \times 10^{-15}$ s s^{-1}. This rate appeared to be about 2–3 times as big as that accommodated within

[3] Based on the standard theory of stellar structure and evolution, but also on the knowledge of actual star's parameters like mass, effective temperature, chemical composition of the core and the envelope, which are determined observationally.

[4] The case $\dot{P}_{\rm obs} < \dot{P}_0$ would mean that there is something fundamentally wrong either with theory or with observations — so in our context this situation is inconclusive.

standard CO dwarf models which predicted $\dot{P} = (2-6) \times 10^{-15}$ s s^{-1}. This motivated Isern *et al.* [3] to conjecture that axion emission could be responsible for the phenomenon. Very recently, with a much longer time interval of acquired data, Kepler *et al.* [20] recalculated the rate of period increase and found significantly lower value of $\dot{P} = (2.3 \pm 1.4) \times 10^{-15}$ s s^{-1}. Hence it has been claimed that the 215.2 s mode of G117-B15A is perhaps the most stable oscillation ever recorded in the optical band (with a stability compared to millisecond pulsars [9]).

This circumstance made it possible to derive bound on axion mass [9] and the compactification scale M_s in the theory with large extra dimensions [10].

3.1. Constraints on large extra-dimensions

The interest in physical theories with extra spatial dimensions has recently experienced considerable revival. In particular, it is has been conjectured [11] that compactification scale could be at the order of a TeV, corresponding to a weak-scale string theory. Such a low compactification scale is attractive from experimental perspective. In this theory, gravity is essentially $n+4$ dimensional whereas all other physical fields are confined to 4-dimensional brane. The relation between the Planck mass in 4 dimensions ($M_{\rm Pl} = 1.2\ 10^{19}$ GeV$/c^2$), the string mass scale in $4+n$ dimensions M_s and the radius R of extradimensional space reads:

$$R^n = \left(\frac{\hbar}{c}\right)^n \frac{M_{\rm Pl}^2}{M_s^{n+2}\Omega_n}\,, \tag{6}$$

where Ω_n is the volume of the unit n-sphere. Present laboratory limits [14] give 1 TeV as lower bound on M_s. Assuming that M_s is of order of TeV (sustaining the hopes of experimental verification of multidimensionality of the world) one can immediately rule out $n = 1$ because in that case one would expect modified gravity at distances $R \approx 10^{15}$ *cm* which is not observed.

At an energy scale much lower than compactification scale, one can construct an effective theory of KK gravitons interacting with the standard model fields [12]. Barger *et al.* [13] have calculated Kaluza–Klein graviton emissivities in five processes interesting from astrophysical perspective: photon-photon annihilation, electron–positron annihilation, Gravi–Compton–Primakoff scattering, Gravi-bremsstrahlung in a static electric field and nucleon–nucleon bremsstrahlung. Kaluza–Klein gravitons can couple to photons which leads to the first process, similarly electron–positron pair can annihilate into Kaluza–Klein gravitons, in the third process scattering of photons by electrons may lead to Kaluza–Klein graviton emission via Compton or Primakoff processes (conversion of photons into Kaluza–Klein gravitons) — respective Feynman diagrams can be found in [13]. First of the last two processes consists in bremsstrahlung emission of Kaluza–Klein gravitons in static electric field of ions, the next one is similar — the bremsstrahlung emission of gravitons by nucleons (in electric field of nucleons). One can expect that first process becomes effective in hot stars where the photon density is large enough, the second one in stars with abundant electron–positron pairs (*i.e.* mainly the protoneutron or young neutron stars) *etc.*

Because white dwarfs are dense and cool one can expect that dominant process of Kaluza–Klein graviton emission is gravi-bremsstrahlung of electrons. The specific (mass) emissivity estimated by Barger *et al.* [13] is

$$\varepsilon = 5.86\ 10^{-75}\frac{T^3 n_e}{\rho M_s^4}\sum_j n_j Z_j^2 \qquad \text{for } n=2\,, \tag{7}$$

$$\varepsilon = 9.74\ 10^{-91}\frac{T^4 n_e}{\rho M_s^5}\sum_j n_j Z_j^2 \qquad \text{for } n=3\,, \tag{8}$$

where T is the temperature of isothermal core, ρ is the density, n_e and n_j are the number densities of electrons and ions, respectively. Total Kaluza–Klein graviton luminosity can be obtained as

$$L_{\mathrm{KK}} = \int\limits_0^{M_{\mathrm{WD}}} \varepsilon\, dm\,. \tag{9}$$

The white dwarf pulsator G117-B15A has a mass of 0.59 $M_\odot$, effective temperature $T_{\mathrm{eff}} = 11620$ K [17] and luminosity $\log(L/L_\odot) = -2.8$ [18] (*i.e.* $L_\gamma = 6.18\times 10^{30}$ erg s^{-1}). Typical model for such CO star predicts the central temperature $T = 1.2\times 10^7$ K [9] and matter density $\rho = 0.97\times 10^6$ g cm^{-3}. We assume that mean molecular weight per electron is $\mu_e \approx 2$. This allows us to estimate electron number density n_e. Following detailed calculations by Salaris [19] chemical composition of the core has been taken as 83% of oxygen by mass and remaining 17% of carbon. Then by virtue of electric neutrality of the star one is able to estimate n_j. Now we can estimate the power radiated away from the white dwarf core in the form of Kaluza–Klein gravitons L_{KK}. In order to derive the lower bound on M_s one can take (in the role of $\dot{P}_{\mathrm{obs,upper}}$ in (5)) an upper 2σ limit of $\dot{P}$ equal to 5.1×10^{-15} s s^{-1} [20]. Following [9] we assume $\dot{P}_0 = 3.9\times 10^{-15}$ s s^{-1}. By virtue of relation (5) one can verify that recently established secular stability of the fundamental oscillation mode of G117-B15A implies $L_{\mathrm{KK}} < 0.308\ L_\gamma$ which translates to:

$$M_s > 14.3\ \ \mathrm{TeV}/c^2 \quad \mathit{for}\ n=2\,.$$

Respective graviton emission rates for $n = 3$ (and greater) theories turn out to be negligible, hence we do not quote the resulting numbers.

It is interesting to compare our result with existing astrophysical constraints on string compactification scale within the framework of the theory proposed by Arkani-Hamed, Dimopoulos and Dvali [11]. Helioseismological and red-giant type considerations were performed by Cassisi *et al.* [15]. They calculated detailed solar models taking into account energy loss in Kaluza–Klein gravitons explicitly in their code and obtained the lower bound for M_s equal to 0.3 TeV/c^2. More stringent limit was derived from simulating the globular cluster Hertzsprung–Russel diagram [15] (implementing Kaluza–Klein graviton emissivity into FRANEC evolutionary code). By virtue of comparing predicted luminosity of RGB tip with observations Cassisi *et al.* obtained a "red-giant" bound for M_s to be 3–4 TeV/c^2. Our estimate of the string energy scale implied by recently reported stability of ZZ Ceti star G117-B15A equal to 14.3 TeV/c^2 is much stronger than above mentioned stellar evolutionary bounds.

Among existing astrophysical bounds on Kaluza–Klein theories with large extra dimensions only the supernova constraints are more restrictive. They demand $M_s >$30–130 TeV/c^2 [13, 16] and are based on a different mechanism of Kaluza–Klein graviton emission — the nucleon–nucleon bremsstrahlung. On the other hand the lesson learned in testing the physics of axions shown that first straightforward supernova bounds were reduced by an order of magnitude when more accurate nuclear physics was employed [22].

In quite recent paper by Hannesteed and Raffelt [25] it has been argued that EGRET gamma ray flux limit for nearby neutron stars puts even more stringent constraint $M_s > 500$ TeV/c^2 on theories with two large extra dimensions. In so far as these limits could in the end be decisive in the context of Arkani-Hamed, Dimopluolos, Dvali model, this should not weaken our conclusion that astroseismological determination of white dwarf cooling might become a useful tool in broadly understood astroparticle physics.

The bound quoted in this paper is based on white dwarf cooling. The physics underlying this process is very simple hence one can expect that the result is robust (within the framework of the theory proposed by Arkani-Hamed, Dimopoulos and Dvali [11]). Kaluza–Klein graviton emissivity at the relevant densities and temperatures of white dwarfs is dominated by the gravi-bremsstrahlung process taking place in the degenerate and isothermal core. The mass of the core (essentially equal to the mass of the star, since the outer helium layer comprises less than 0.01 $M_{\rm WD}$), its temperature and chemical composition can be reliably estimated by fitting evolutionary models to observational characteristics such like effective temperature or oscillation periods.

4. Perspectives for the future

The cooling rate of B117-G15A pulsating star is constrained by measurements which are performed with great accuracy. The most recent determination of the secular rate of change of the period [20] took into account all the periodicities and the error bars reported therein can be considered as safe. It can be argued [9] that remaining uncertainty – mostly from mode identification procedure and the precise physical characteristics (mass or temperature) is of order of 1×10^{-11} s s^{-1}. Other effects like the contribution of the proper motion [20,21] or the rate of reaction $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ which determines the stratification of the core at final stage of the asymptotic giant branch, both contribute an order of magnitude smaller value to the final uncertainty.

The detailed discussion of using a pulsational code coupled to evolutionary code aimed at constraining an axion mass by observed stability of the fundamental mode of G117-B15A can be found in a recent paper by Córsico *et al.* [9]. One of the conclusions formulated in [9] was that the bounds on axion mass derived from simple estimates like performed in the present paper (in a different context) or in [3] are in good agreement with evolutionary calculations. Although axion emissivity has different temperature dependence than that of Kaluza–Klein gravitons, one can expect the same for gravitons. Hence the B117-G15A pulsator remains an important tool for testing fundamental physics.

Preceding sections illustrate the story how did a certain ZZ Ceti pulsating white dwarf star become the source of very strong astrophysical constraints. A decade ago [3] the same star G117-B15A was considered as suspected for the evidence of extra cooling (supposedly due to axion emission) and since then has been mentioned in this context many times [1]. Improved observational data announced a perfect agreement of the cooling speed of G117-B15A with predictions of standard theory — another triumph of standard physics in understanding the celestial bodies. So we do really have a new astrophysical source of constraints on exotic physics.

Is this the final conclusion? Although it is an important one it is in some sense disappointing since most of scientific efforts are driven by the desire to explain the unknown. In particular it would be good to have a candidate in which a non-standard physics could help in elucidating its peculiar behaviour. In these closing remarks we will point toward such a candidate.

Recent observations of pulsating hot "pre-White Dwarf" PG 1159-035 (GW Virginis), which is a prototype of a new class of pulsating hot White Dwarfs, have indicated that its oscillation period $P = 516$ s [23] increase at the rate

$$\dot{P}_{\rm obs} = (13.07 \pm 0.003) \times 10^{-11} \ \ {\rm s\,s^{-1}}$$

an order of magnitude larger than predicted

$$\dot{P}_0 = 1 \times 10^{-11} \ \ {\rm s\,s^{-1}}\,.$$

Is this an indication of extra cooling due to exotic physics? There are some hints indirectly supporting the view that such possibility could seriously be taken into account. The comparison of Kawaler–Bradley [24] stellar evolutionary models with the observed pulsational spectrum of PG 1159-035 gave a best fit with a mass $0.59 \pm 0.01\ M_{\odot}$ an effective temperature of $\approx 136,000\ K$ and luminosity $\log(L/L_{\odot}) \leq 3$. So the star is really hot and hence one can expect the production of exotic particles much effective than in ZZ Ceti stars. This possibility will be a subject of further investigation.

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