CONSTRAINTS ON COSMIC EQUATION OF STATE FROM JOINT ANALYSIS OF STANDARD RULERS AND STANDARD CANDLES*

Beata Malec

Copernicus Center for Interdisciplinary Studies
Gronostajowa 3, 30-387 Kraków, Poland

Marek Biesiada, Aleksandra Piórkowska

Institute of Physics, Department of Astrophysics and Cosmology
University of Silesia, Uniwersytecka 4, 40-007 Katowice, Poland

(Received October 27, 2011)

A key issue of contemporary cosmology is the problem of currently accelerating expansion of the Universe. The nature of this phenomenon is one of the most outstanding problems of physics and astronomy today. Its origin may be attributed to either unknown exotic material component with negative pressure — so-called Dark Energy (DE), to infra red modification of gravity at cosmological scale or requires to relax the assumption of homogeneity of the Universe. The strength of modern cosmology lies in consistency across independent pieces of evidence (like *e.g.* CMB anisotropies, the large-scale distribution of galaxies, the observed abundances of light elements, *etc.*) rather than in single one, crucial experiment. In this spirit we performed a joint analysis of two dark energy models using five different tests. These tests will be called diagnostics and include the data coming from supernovae, Gamma Ray Bursts, CMB acoustic peaks, Baryon Acoustic Oscillations and strong lensing systems. Part of the diagnostics makes use of the angular diameter distance, and part of them uses the luminosity distance splitting these probes into two categories: Standard Rules and Standard Candles. It was shown that combined analysis of them had higher restrictive power in the parameter space. The best fits we obtained for the model parameters in joint analysis turned out to prefer cases effectively equivalent to ΛCDM model. Our findings are in agreement with parallel studies performed by other authors on different sets of diagnostic probes.

DOI:10.5506/APhysPolB.42.2305
PACS numbers: 95.36.+x, 98.80.–k

* Presented by B. Malec at the XXXV International Conference of Theoretical Physics “Matter to the Deepest”, Ustroń, Poland, September 12–18, 2011.

1. Introduction

Discovery of accelerated expansion of the Universe dates back to 1998 when it was inferred from SNIa Hubble diagram by two independent teams [1, 2]. Taking this together with other independent studies like *e.g.* CMBR anisotropies [3] or Baryon Acoustic Oscillations (BAO) imprinted in the large scale structure power spectrum [4], we have now quite convincing picture of our Universe in the large.

The explanation of this phenomenon is far from obvious and honestly speaking we are lacking clear theoretical guidance. Under such circumstances it is quite natural to take up phenomenological approach based on upgrading observational fits of quantities which parametrize the unknown (such like density parameters or coefficients in the cosmic equation of state) and seeking for coherence among alternative tests and techniques.

One can probe DE in a number of ways. Here we focus on geometric probes, which involve distances on cosmological scales. We want to present and combine together five different cosmological probes, including both standard rules and standard candles. These two types of objects are based on two different distance concepts which, although theoretically related to each other, have clearly different systematic uncertainties and different parameter degeneracies. Hence, we expect that their joint analysis can be more restrictive in the parameter space. Standard candles make use of so-called luminosity distance while standard rulers use angular diameter distance.

The main paradigm of modern cosmology is that geometry of the Universe can be described as one of three possible Friedman–Robertson–Walker (FRW) solutions to the Einstein equations representing homogeneous and isotropic space-time. Currently, there exists strong evidence, coming form independent and precise experiments, that the Universe is spatially flat. For example a combined analysis of WMAP5, BAO and supernova data gives $\Omega_{\rm tot} = 1.0050^{+0.0060}_{-0.0061}$. Hence we will assume flat ($k = 0$) FRW model from now on.

2. Standard rulers versus standard candles

2.1. Standard rulers

Strong lensing systems. Phenomenon of strong lensing reveals itself as multiple images of the source. It occurs whenever the source, the lens and observer are so well aligned that the observer–source direction lies inside the so-called Einstein ring of the lens. In cosmological context the source is most often a quasar. Here, we are interested in single galaxies acting as lenses. Radius of the Einstein ring sets the scale of images separation in a system and for the simplest, realistic lens model — Singular Isothermal Sphere (SIS) is given by

$$\theta_{\rm E} = 4\pi \frac{\sigma_{\rm SIS}^2}{c^2} \frac{D_{\rm ls}}{D_{\rm s}} \,. \tag{1}$$

Einstein radius $\theta_{\rm E}$ depends on angular diameter distances ($D_{\rm ls}$ — between lens and source, $D_{\rm s}$ — between observer and lens), which in turn are determined by background cosmology. Then, for testing cosmological models, all that we need is reliable knowledge about $\theta_{\rm E}$ (from image astrometry) and stellar velocity dispersion $\sigma_{\rm SIS}$. In practice, we were using σ_0 as representative to $\sigma_{\rm SIS}$ (from spectroscopy). The arguments in favor of using σ_0 can be found in in [5, 6].

The sample, which was used here, consist of $n = 20$ objects. From the Lens Structure and Dynamics (LSD) survey and the more recent SLACS survey (Sloan Lens ACS Survey[1]) good spectroscopic data for central parts of lens galaxies became available allowing to assess their central stellar velocity dispersions σ_0. We have used essentially the same sample as in [6]. The summary of data can be found in Table 1 of [7].

Due to the fact that cosmological models enter here through a distance ratio and so the Hubble constant gets canceled, our inferences are independent on any assumptions of its value. The method is also not affected by dust absorption or source evolutionary effects. It depends, however, on the reliability of lens modeling (*e.g.* SIS assumption). Cosmological model parameters (coefficients in the equation of state w_i or matter density parameter $\Omega_{\rm m}$ — denoted here for short by $\boldsymbol{p}$) are estimated by minimizing the chi-square

$$\chi^2(\boldsymbol{p}) = \sum_i \frac{\left(\mathcal{D}_i^{\rm obs} - \mathcal{D}_i^{\rm th}(\boldsymbol{p})\right)^2}{\sigma_{\mathcal{D},i}^2} \,, \tag{2}$$

where the sum is over the sample and $\sigma_{\mathcal{D},i}^2$ denotes the variance of $\mathcal{D}^{\rm obs}$. In calculating $\sigma_{\mathcal{D}}$ we assumed that only velocity dispersion errors contribute and the Einstein radii are determined accurately.

CMB shift parameter $\boldsymbol{R}$. Precise measurement of anisotropies in cosmic microwave background (CMB) provides another independent test for the existence of dark energy. Use of the full data set, besides being time consuming and complex, requires detailed assumption about cosmological model, which we want to avoid here. So we took up a common and much simpler approach and use the shift parameter R which neatly summarize CMB data. It is the angular diameter distance to the last scattering surface (at redshift $z_{\rm lss}$) divided by the Hubble horizon size at the decoupling

[1] `http://www.slacs.org/`

epoch. In a standard FLRW cosmology under assumption of spatial flatness it reads: $R(\boldsymbol{p}) = \sqrt{\Omega_{\rm m}} \int_0^{z_{\rm lss}} \frac{dz}{h(z;\boldsymbol{p})}$, where $\Omega_{\rm m}$ is the present day matter density, $h(z)$ is dimensionless expansion rate which depends on cosmological model (through parameters $(\boldsymbol{p})$). It is commonly argued that R parameter allows to constrain the evolution of DE very efficiently, due to its large redshift baseline ($z_{\rm lss} = 1090$). It is also considered as a parameter with least model dependence among others inferred from CMB data, provided that the dark energy density is negligible at recombination, and does not depend on H_0. WMAP 7-year results [3], gave $R(\boldsymbol{p}) = 1.725 \pm 0.018$.

For comparison between theory and observations we will use the chi-square function

$$\chi^2_{\rm CMB}(\boldsymbol{p}) = \frac{[R(\boldsymbol{p}) - 1.725]^2}{0.018^2}\,.$$

Baryon Acoustic Oscillations. BAO originate from perturbations in the early Universe which excite sound waves in the photon-baryon plasma. After recombination they became frozen into the distribution of matter. This fact allow us to define the length of standard ruler, which is the distance sound could travel between Big Bang and recombination. BAO reveal themselves both in the CMB angular power spectrum as the acoustic peaks and in clustering properties of galaxies as well. There exists a preferred scale in spatial distribution of galaxies on which we observe an excess in galaxy numbers *i.e.* as a bump in the two-point correlation function.

Here, we used distance parameter A which fixes the absolute dimensionless scale of BAO. It is an observable quantity, quite well constrained by the data at redshift $z = 0.35$ as $A = 0.493 \pm 0.017$ [4]. Convenient form for calculating this parameter is given by

$$A(\boldsymbol{p}) = \frac{\sqrt{\Omega_{\rm m}}}{0.35} \left[\frac{0.35}{h(0.35;\boldsymbol{p})} \left(\int\limits_0^{0.35} \frac{dz}{h(z;\boldsymbol{p})} \right)^2 \right]^{1/3}$$

and the corresponding chi-square function is

$$\chi^2_{\rm BAO} = \frac{[A(\boldsymbol{p}) - 0.469]^2}{0.017^2}\,.$$

2.2. Standard candles

Supernovae Ia. For more than a decade now, supernovae Ia are used as standard candles of cosmology. We will use the data set of $n = 557$ supernovae coming from the most recent compilation of SNIa data given in Amanullah *et al.* [8] known as Union2. This data set contains redshifts

z_i and distance moduli μ_i together with their errors σ_i. This leads to the chi-square function

$$\chi^2_{\rm SNIa} = \sum_{i=1}^{N=557} \left[\frac{\mu^{\rm obs}(z_i) - \mu^{\rm th}(z_i;\boldsymbol{p})}{\sigma_i}\right]^2 . \tag{3}$$

The distance modulus: $\mu := m - M = 5\log_{10}(D_{\rm L}(z;\boldsymbol{p})) + 25$ contains (unimportant) constant term which can be enriched by factoring out the Hubble distance scale from the luminosity distance $D_{\rm L}(z;\boldsymbol{p}) = c/H_0\ d_{\rm L}(z;\boldsymbol{p})$. Therefore, instead of minimizing the original chi-square (3) we used an approach equivalent to marginalization over the nuissance parameter.

Gamma Ray Bursts. Being the most luminous astrophysical events observable today they attract recently a lot of attention as another yet cosmological probe. These objects are observed on much higher redshifts than SNeIa, potentially enabling us to probe expansion history much farther, where we expect higher sensitivity to differences among known DE models. The obstacle, however, is that GRBs are not standard candles so it is impossible to build Hubble diagram for them immediately. In order to do so, one needs correlations between distance dependent properties and directly observable features. Schaefer [9] was the first who built GRBs Hubble diagram. He used five two-parameter correlations to estimate distance modulus from each single correlation and then calculated a weighted average. However, in the common calibration procedure particular cosmological model has to enter, causing so-called circularity problem. Here, we used Schaefer's sample recalibrated by Cardone *et al.* [10] in a model independent way, where the authors used supernovae sample to calibrate the nearest GRBs (for details see [10]). For our fitting procedure we used analogous chi-square function as for the supernovae.

3. Cosmological models

While current data are consistent with the concordance model, assuming Λ as a source of DE, a number of other theories are in agreement as well. Here, we focused on two very popular and simple extensions of ΛCDM, namely two qintessential scenarios. One of them is Quintessence with constant equation of state. To this end we promote the coefficient w in DEs equation of state $p = w\rho$ to the role of free parameter. The second model assumes varying equation of state in its most popular parametrization developed by [11], namely $w(z) = w_0 + w_a \frac{z}{1+z}$. In this way, ΛCDM cosmology is included effectively as a special case in both of them. For more details about the models as well as appropriate expansion rates used see [12].

4. Results and conclusion

The results of our joint analysis are displayed in Table I and confidence intervals for respective parameters are shown in Fig. 1. In the class of quintessence models, the recent estimates from [13] pinned down the matter density and equation of state parameter to the range $\Omega_{\rm m} = 0.265 \pm 0.16 \pm 0.025$ and $w = -0.96 \pm 0.06 \pm 0.12$. These results are in perfect agreement with our results shown in Table I. As far as confidence regions (corresponding to 68% and 95% C.L.) in the $(\Omega_{\rm m}, w)$ parameter plane are concerned, one can see the different (almost orthogonal) degeneracy of different techniques resulting in higher restrictive power of combined analysis. Concerning Chevalier–Polarski–Linder parametrization, the joint constraint from WMAP+BAO+H_0+SN provided by [3] gives the bound $w_0 = -0.93 \pm 0.13$, $w_a = -0.41^{+0.72}_{-0.71}$ which is also in agreement with our results.

TABLE I

Fits to cosmological models from: (a) combined standard rulers data (R+BAO+Lenses), (b) from standard candles sample, (c) joint analysis taking into account both classes of objects.

Cosmological model	Probe	Best fit	χ^2	χ^2/d.o.f.
Quintessence	Rulers	$\Omega_{\rm m}=0.262 \pm 0.035$ $w=-1.066 \pm 0.188$	63.832	3.19
	Candles	$\Omega_{\rm m}=0.341 \pm 0.053$ $w=-1.198 \pm 0.195$	785.470	1.26
	Joint	$\Omega_{\rm m}=0.278 \pm 0.014$ $w=-1.002 \pm 0.049$	850.725	1.32
CPL	Rulers	$\Omega_{\rm m}=0.224 \pm 0.07435$ $w_0=-1.098 \pm 0.828$ $w_1=-0.0123 \pm 2.036$	60.690	3.19
	Candles	$\Omega_{\rm m}=0.270 \pm 0.065$ $w_0=-1.130 \pm 0.153$ $w_1=0.905 \pm 1.271$	785.965	1.26
	Joint	$\Omega_{\rm m}=0.279 \pm 0.014$ $w_0=-1.042 \pm 0.123$ $w_1=0.218 \pm 0.595$	850.600	1.32
CPL $\Omega_{\rm m} = 0.27$	Rulers	$w_0=-1.583 \pm 0.249$ $w_1=0.218 \pm 0.784$	64.123	3.19
	Candles	$w_0=-1.124 \pm 0.148$ $w_1=0.906 \pm 0.897$	785.960	1.26
	Joint	$w_0=-1.033 \pm 0.096$ $w_1=0.229 \pm 0.417$	851.01	3.19

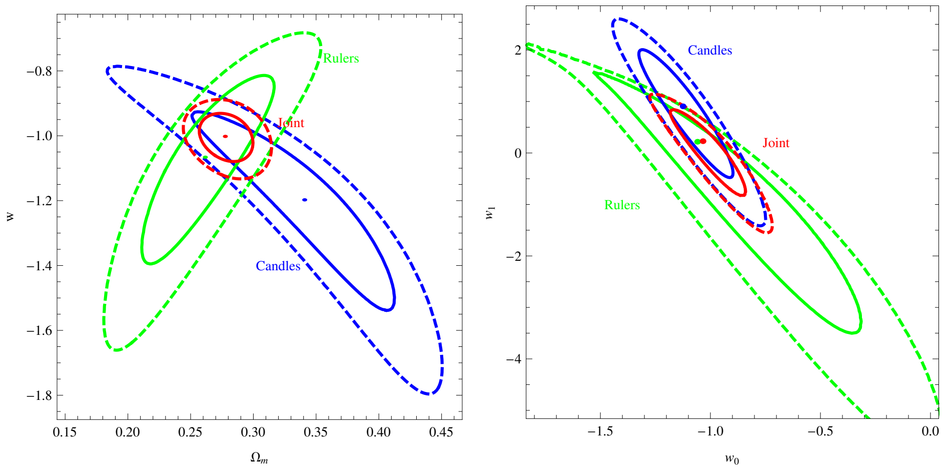

Fig. 1. Best fits (dots) and (68%, 98%) confidence regions for parameters in cosmic equation of state for Quintessence (left panel) and Chevalier–Linder–Polarski parametrization (right panel).

Joint analysis performed in this paper extends the previous one [14] by enrichment of standard candles with GRBs hence reaching deeper in the redshift with these probes. The probes we used came both from standard rulers and standard candles. They invoke different (although theoretically related) concepts of a distance in cosmology, hence they have different parameter degeneracies and different restrictive power in the parameter spaces of cosmological models.

REFERENCES

[1] A.G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).

[2] S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).

[3] E. Komatsu *et al.*, *Astrophys. J. Suppl.* **192**, 18 (2011).

[4] B. Reid *et al.*, *MNRAS* **404**, 60 (2010).

[5] T. Treu *et al.*, *Astrophys. J.* **650**, 1219 (2006).

[6] C. Grillo, M. Lombardi, G. Bertin, *Astron. Astrophys.* **477**, 397 (2008).

[7] M. Biesiada, A. Piórkowska, B. Malec, *MNRAS* **406**, 1055 (2010).

[8] R. Amanullah *et al.*, *Astrophys. J.* **716**, 712 (2010).

[9] B.E. Schaefer, *Astrophys. J.* **660**, 16 (2007).

[10] V.F. Cardone, S. Capozziello, M.G. Dainotti, *MNRAS* **400**, 775 (2009).

[11] M. Chevallier, D. Polarski, *Int. J. Mod. Phys.* **D10**, 213 (2001); E.V. Linder, *Phys. Rev.* **D68**, 083503 (2003).

[12] A. Piórkowska, M. Biesiada, B. Malec, *Acta Phys. Pol. B* **42**, 2297 (2011), this issue.

[13] R. Kessler *et al.*, *Astrophys. J. Suppl.* **185**, 32 (2009).

[14] M. Biesiada, B. Malec, A. Piórkowska, *Res. Astron. Astrophys.* **11**, 641 (2011).