Title: Some aspects of the neutrino theory

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Citation style: Zrałek Marek. (2010). Some aspects of the neutrino theory. "Acta Physica Polonica B" (Vol. 41, no. 7 (2010), s. 1477-1490).

Vol. 41 (2010) *ACTA PHYSICA POLONICA B* No 7

SOME ASPECTS OF THE NEUTRINO THEORY*

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(Received June 06, 2010)

Selected topics in the theory of neutrinos, discussed in last years, are presented. We shortly summarize properties of neutrinos in frame of the original Standard Model (SM) and give the experimental information about their masses and mixing. In the frame of the model with massive neutrinos, the so-called New SM (νSM), two controversial phenomena, the Mössbauer neutrinos problem and the GSI anomaly are explained. Beyond the SM (BSM) we focus on two issues, on the problem of small neutrino masses and large mixing in comparison to the quark sector and on how neutrino oscillation phenomena should be correctly described in the BSM.

PACS numbers: 13.15.+g, 14.60.Lm, 16.60.St, 14.60.Pq

1. Introduction

Neutrinos always have given a new and unexpected information about elementary interactions. By introducing neutrinos in 1930 [1] Pauli has saved the principle of energy and momentum conservation. In 1934 Fermi [2] exploited neutrinos for the formulation of the theory of nuclear beta decay. Again, in 1937 Majorana [3], using neutrinos, has suggested the existence of elementary fermions, which are their own antiparticles. In 1956 neutrinos were used by Lee and Yang [4] to put forward the hypothesis, that the symmetries P and C are not satisfied in nature, and a year later [5], to the experimental verification of this fact. In the sixties neutrinos, together with charged leptons and quarks, gave rise to the formulation of the model of electroweak interactions [6]. In 1973, the measurement of neutral current reactions in the Gargamelle bubble chamber [7] gave the first indication that the neutral gauge boson Z_0 exists. In 1987 neutrinos started to play

* Presented at the Cracow Epiphany Conference on Physics in Underground Laboratories and Its Connection with LHC, Cracow, Poland, January 5–8, 2010.

an active role in astrophysics. Observations of neutrinos from supernova 1987N [8] confirmed existing models of supernova explosion. Two years later, the neutrino re-appeared "on the front pages of newspapers". In the LEP experiments, measurement of the Z_0 decay width [9] has proved, that there are only three different neutrino flavours, and as a consequence, only three families of fermions. In 1998 the phenomenon of neutrino oscillation was discovered [10], showing that neutrinos have mass. This was the first evident indication, that the SM has to be extended. Finally, in 2002 the solar neutrino problem was ultimately resolved [11], proving that the old model of energy production in the Sun (the Bethe model) is correct.

Here we would like to provide information about some recent aspects of neutrino theory which are discussed in literature and, as we think, are important for future experiments (*e.g.* Mössbauer neutrinos), theoretical description of future experimental results (*e.g.* full description of neutrino oscillation) or future extension of the SM (*e.g.* understanding the problem of neutrino mass and flavours).

In the next section we describe neutrino properties in the frame of the SM, and we explain why neutrinos are massless there. In Sec. 3 we shortly summarize recent experimental data concerning neutrino masses and mixing angles. Next, in Sec. 4 we describe present controversial problems concerning Mössbauer neutrinos and fluctuating nuclear decay lifetimes in the GSI experiment. Sec. 5 is devoted to two problems. Firstly, we would like to summarize present efforts to understand small neutrino masses and large mixing angles and next we explain how the oscillation phenomena should be correctly described if neutrinos are produced and detected by interaction beyond the SM. Finally, we make some conclusions.

2. Neutrino properties in the SM

There are four reasons why neutrinos in the SM are massless: *(i)* we do not introduce the right handed fields $\nu_{\rm R}$, *(ii)* only one Higgs doublet is introduced, *(iii)* we require that the theory is renormalizable, and finally, *(iv)* no non-renormalizable effects which are able to give non-vanishing neutrino mass are introduced in the theory. But zero mass neutrinos are not guaranteed by any fundamental theory and resignation from any of the conditions *(i)*–*(iv)* results in a mathematically correct theory. So it is very easy to find a theory with massive neutrinos. The problem is why these masses are so remarkably small.

Neutrinos in the SM are introduced in three, well defined flavour states, which were observed experimentally, ν_e neutrino [12], ν_μ neutrino [13], and ν_τ neutrino [14]. As we mentioned before we know from the measurements of the Z_0 decay width, that only three different flavour neutrinos exist in nature [9].

Neutrinos interact with charged gauge bosons $W^{\pm}$:

$$L_{\mathrm{CC}} = \frac{e}{2\sqrt{2}\sin\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) l_\alpha W_\mu^+ + \mathrm{h.c.}\,, \tag{1}$$

and with neutral Z_0:

$$L_{\mathrm{NC}} = \frac{e}{4\sin\theta_W \cos\theta_W} \sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) \nu_\alpha Z_\mu\,. \tag{2}$$

From (1) and (2) we can easily find that the family lepton numbers L_e, L_μ and L_τ are separately conserved and, as a consequence, also the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved. In the SM there is no lepton flavour mixing and no CP symmetry violation in the lepton sector, neutrinos are stable and have no electromagnetic structure (except for the charge radius $\langle r^2 \rangle \neq 0$).

3. What do we know from experiment?

Since the first experiment of Reines and Cowan [12], where the electron antineutrino $\bar{\nu_e}$ was discovered, neutrinos have been observed in many different processes. The cross-sections for (anti)neutrino + electron, (anti)neutrino + nucleon and (anti)neutrinos + nuclei are measured with different neutrino energies and for different final states. All measured cross-sections agree with the SM predictions with massless neutrinos, so only upper limits for the neutrino masses can be found. The best upper limit for the neutrino effective mass (m_i are neutrino masses and U_{ei} are the elements of mixing matrix) [15]:

$$m_\beta = \sqrt{\sum_{i=1,2,3} |U_{ei}|^2 m_i^2}\,, \qquad m_{\min} < m_\beta < m_{\max}\,, \tag{3}$$

has been found in tritium beta decay [16]:

$$m_\beta < 2.2 \text{ eV}\,. \tag{4}$$

From Eq. (3) and (4) we can conclude that $m_{\min} < 2.2\,\mathrm{eV}$. Some information comes also from neutrinoless double beta decays [17], which can occur only if neutrinos are Majorana particles, and if realized, measure the other effective neutrino mass:

$$\langle m_{0\nu} \rangle = |\sum_{i=1,2,3} U_{ei}^2 m_i|\,, \qquad \langle m_{0\nu} \rangle < m_{\max}\,. \tag{5}$$

Recent experimental results from CUORICINO [18] have given

$$\langle m_{0\nu}\rangle < 0.19 \div 0.68\,\text{eV}\,. \tag{6}$$

A recent review of the experimental results on the neutrinoless double beta decay is given in [19].

From neutrino oscillation experiments we also know two differences of neutrino masses squared. The latest global fits [20] give

$$\Delta m^2_{21} \;=\; m_2^2 - m_1^2 = (7.05 - 8.34)\times 10^{-5}\text{eV}^2\,, \tag{7}$$
$$\Delta m^2_{31} \;=\; m_3^2 - m_1^2 = (2.07 - 2.75)\times 10^{-3}\text{eV}^2 \Longrightarrow m_{\max} > 0.045\text{eV}\,. \tag{8}$$

From (8) it follows that the mass of the heaviest neutrino must be $m_{\max} > 0.045\,\text{eV}$.

In the oscillation experiments, elements of the unitary mixing matrix are parameterized by three angles and one CP-violating phase. Currently not all these parameters are known. Combined solar, atmospheric, reactor and accelerator neutrino data gives [20] (with 3σ interval)

$$\sin^2\theta_{12} \in (0.25 - 0.37)\,,\quad \sin^2\theta_{23} \in (0.36 - 0.67)\,,\quad \sin^2\theta_{13} < 0.056\,. \tag{9}$$

In the SM with massive neutrinos we do not know: *(i)* the nature of neutrinos (are they Dirac or Majorana particles), *(ii)* the spectrum of masses (normal hierarchy, inverted hierarchy, degenerate), *(iii)* the value of θ_{13} mixing angle (is θ_{13} close to zero or rather close to the upper limit) and finally, *(iv)* the CP-violating phases ($\delta_{\rm CP}$ — the only one for Dirac neutrinos, and two additional ones, ϕ_1, ϕ_2 for Majorana neutrinos). It is worth noting the fact that the masses of some neutrinos can be smaller than the experimental uncertainty of the charged lepton masses (for electron $(\Delta m_e)_{\rm exp} = 0.013\,\text{eV}$).

4. Neutrinos in the νSM

We see that experiments tell us that the SM must be extended at least in such a way that neutrinos have mass. There are many beyond the SM (BSM) theories which satisfy this requirement. The simplest and popular scenario is such that the neutrino mass is the only visible symptom of New Physics (NP) at very high energy scale (*e.g.* unification scale $\sim 10^{16}$ GeV), and that all other BSM interactions of quarks and charged leptons are negligible at low, experimentally accessible energies. This model is sometimes called the New SM = νSM. In such a model the NP is "visible" in the neutrino mass Lagrangian and in the neutrino mixing matrix. The mass term and the mixing matrix distinguish Dirac from Majorana neutrinos. In the case of Dirac neutrinos the mass term has the form

$$L_{\rm mass}(D) = \sum_{i=1,2,3} m_i^{\rm D}(\bar{\nu}_{i\rm R}\nu_{i\rm L} + \bar{\nu}_{i\rm L}\nu_{i\rm R})\,. \tag{10}$$

For the Majorana neutrino two kinds of mass Lagrangian are allowed, one built using the left-handed chiral fields

$$L^{\rm L}_{\rm mass}(M) = \sum_{i=1,2,3} m^{\rm M}_{{\rm L}i}(\bar{\nu}^c_{i{\rm R}}\nu_{i{\rm L}} + \bar{\nu}_{i{\rm L}}\nu^c_{i{\rm R}})\,, \qquad \nu^c_{i{\rm R}} = i\gamma^2\nu^*_{i{\rm L}}\,, \tag{11}$$

and the other built using the right-handed fields

$$L^{\rm R}_{\rm mass}(M) = \sum_{i=1,2,3} m^{\rm M}_{{\rm R}i}(\bar{\nu}^c_{i{\rm L}}\nu_{i{\rm R}} + \bar{\nu}_{i{\rm R}}\nu^c_{i{\rm L}})\,, \qquad \nu^c_{i{\rm L}} = i\gamma^2\nu^*_{i{\rm R}}\,. \tag{12}$$

In the νSM the charged (1) and neutral (2) current Lagrangians have new forms

$$L^{\nu{\rm SM}}_{\rm CC} = \frac{e}{2\sqrt{2}\sin\theta_W}\sum_{\alpha,i}\bar{\nu}_i\gamma^\mu(1-\gamma_5)U^*_{\alpha i}l_\alpha W^+_\mu + {\rm h.c.}, \tag{13}$$

and:

$$L^{\nu{\rm SM}}_{\rm NC} = \frac{e}{4\sin\theta_W\cos\theta_W}\sum_{i=1,2,3}\bar{\nu}_i\gamma^\mu(1-\gamma_5)\nu_i Z_\mu\,. \tag{14}$$

As now neutrinos are massive particles, interaction with neutral Higgs particle appears:

$$L_{\rm H} = \frac{e}{2\sin\theta_W}\sum_{i=1,2,3}\left(\frac{m_i}{M_W}\right)\bar{\nu}_i\nu_i H \tag{15}$$

but the ratio $m_i/M_W \ll 1$, and the neutrino coupling with Higgs particles (Eq. (15)) is negligibly small. In such models the lepton flavour numbers are not conserved separately, and neutrinos can oscillate. The total lepton number is (Dirac neutrinos) or is not (Majorana neutrinos) conserved. The CP symmetry is broken if the complex phases in the U mixing matrix, δ, ϕ_1, ϕ_2, are different from $0, \pi/2, \pi$. Neutrinos decay with very large decay lifetime. Dirac neutrinos have non-trivial electromagnetic structure. Both of these latter properties are beyond the possibilities of experimental verification.

Neutrinos, which we are dealing with are always relativistic particles $E_i \gg m_i$. In such case it was proved [21] that differences in all observables for Dirac and Majorana neutrinos due to the different mass Lagrangians [(10)–(12)] smoothly disappear for $m_i \to 0$. The other property of massive neutrinos is the oscillation phenomena, proposed many years ago [22] and now commonly observed [10, 11, 23].

During the last years two important properties of oscillation phenomena were discussed, the first connected with the Mössbauer neutrinos and the second with the “oscillating” decays of some nuclei observed in Darmstadt, the so-called GSI anomaly.

4.1. The Mössbauer neutrinos

Soon after the discovery of recoil-free emission and absorption of gamma rays by Mössbauer in 1958 it has been suggested that a similar phenomenon can take place for neutrino emission by an unstable nucleus embedded in a crystal lattice [24]. In last years the idea of recoilless neutrino production and detection was renewed once more, see [25], where the emission process,

$$^{3}\mathrm{H} \longrightarrow ^{3}\mathrm{He} + e^{-}(\mathrm{bound}) + \bar{\nu}_e \,, \tag{16}$$

and the detection process,

$$^{3}\mathrm{He} + e^{-}(\mathrm{bound}) + \bar{\nu}_e \longrightarrow ^{3}\mathrm{He} \tag{17}$$

have been considered. In these recoilless processes (16) neutrinos could be produced with very well defined energy and detected by the process having the resonant nature. Immediately the question arose whether such neutrinos, having definite energy, would oscillate. In the most popular approach we usually assume that oscillation is not a stationary phenomenon and neutrinos energy as well as their momenta are not precisely known [26].

The problem was definitely resolved in [27] using a quantum field theoretical approach. Neither neutrino energy, nor their momenta separately decide about the oscillation process. Neutrino mass states can be added coherently only if their mass difference $\Delta m_{ik} = m_i - m_k$ is smaller than the experimental neutrino mass uncertainty $(\Delta m)_{\mathrm{exp}}$

$$(\Delta m)_{\mathrm{exp}} > \Delta m_{ik} \,. \tag{18}$$

Experimental uncertainties of both energy $(\Delta E)_{\mathrm{exp}}$ and momentum $(\Delta p)_{\mathrm{exp}}$ affect the mass uncertainty

$$(\Delta m)^2_{\mathrm{exp}} = \sqrt{(2E\Delta E)^2_{\mathrm{exp}} + (2p\Delta p)^2_{\mathrm{exp}}} \,. \tag{19}$$

So, even if $(\Delta E)_{\mathrm{exp}} \to 0$ still the mass uncertainty can be large enough, and the oscillation can occur if

$$(\Delta m)^2_{\mathrm{exp}} = \sqrt{2p(\Delta p)_{\mathrm{exp}}} > \Delta m_{ik} \,. \tag{20}$$

For the tritium beta decay neutrino energy and momentum are set by the energies and momenta of all charged particles in the process (16). For this reason, as in any bound state, the neutrino momentum and energy are independent, they are not connected by the on mass shell relation $p = \sqrt{E^2 - m^2}$. Thus we cannot use the normal relation $p\Delta p = E\Delta E$. In conclusion, if Mössbauer neutrinos are produced they can oscillate if their momentum uncertainty satisfies relation (20).

4.2. The GSI anomaly

A GSI experiment in Darmstadt [28] observed time-modulated electron capture β^+ decay of the ions ${}^{140}_{59}\mathrm{Pr}^{58+} \to {}^{140}_{58}\mathrm{Ce}^{58+} + \nu_e$ and ${}^{142}_{61}\mathrm{Pm}^{60+} \to {}^{142}_{60}\mathrm{Nd}^{60+} + \nu_e$. The expected exponential decrease of the parent particles as a function of time has shown additional periodic time modulation

$$\frac{dN(t)}{dt} = N(0)\, e^{-\lambda t} \lambda_{\mathrm{M}}(t)\,, \tag{21}$$

where $\lambda_{\mathrm{M}}(t) = \lambda_{\mathrm{M}}(1 + a\cos(\omega t + \varphi)$ with the period $T = (2pi)/\omega \simeq 7\,\mathrm{s}$ and the amplitude $a \simeq 0.2$.

It has been proposed [28, 29] to interpret the time modulation as the result of the interference between neutrinos with different masses in the final states. At the same time other authors argued [30] that such an explanation of the GSI time anomaly is wrong. We would like to give a simple quantum mechanical argument against neutrino interference in the final state. In order to get time modulation in the final state we would have to add coherently the amplitudes for various neutrino masses. But it is not what we learn from Quantum Mechanics (QM). According to QM, the probability of receiving an eigenvalue from the set $\Delta = (a_1, a_2\,, \dots\, a_N)$ of any physical quantity A is

$$p_\Delta = \mathrm{Tr}\left(\hat{P}_\Delta \hat{\varrho}\right)\,, \tag{22}$$

where $\hat{\varrho}$ is the statistical operator which describes the physical state, and

$$\hat{P}_\Delta = \sum_{i=1,2,..N} \hat{P}_{a_i}\,, \tag{23}$$

where $\hat{P}_{a_i}$ is the projection operator on a state corresponding to the eigenvalue a_i. Thus for the final probability we get

$$p_\Delta = \sum_{i=1,2,..N} \mathrm{Tr}\left(\hat{P}_{a_i} \hat{\varrho}\right) = \sum_{i=1,2\,,\dots\,N} p_{a_i}\,. \tag{24}$$

Such a rule that probabilities (not amplitudes) must be added in the final state works perfectly, and there is no reason to believe that just neutrinos behave in an exceptional way. Adding probabilities for all final neutrinos in the K shell electron capture beta decay in the GSI experiment we cannot expect to get a modulation in time.

5. Neutrinos beyond the SM

New physics may appear also at much smaller energies then the unification scale. There are many hints that really NP operates at a 0 (TeV) scale [31]. Such NP is particularly interesting, because there is a chance to discover it at the LHC, and at the next high energy machines including the future more precise neutrino experiments.

Neutrino mass and mixing can be related to the unification scale as well as to the 0 (TeV) scale. The problem of neutrino oscillations in BSM, presented in Sec. 5.1, refers to NP which operates at future neutrino experiments at the 0 (TeV) scale. If this is the case, it is necessary to take into account new effects, usually neglected in the traditional "neutrino pure state" approach.

5.1. Neutrino mass and mixing

Neutrino masses are much smaller than the masses of charged leptons and quarks. For neutrino mixing angles it is opposite, there are two large mixing angles which contrasts sharply with the smallness of the quark mixing angles. We would like to know why it is so. On the other hand, the problem of particle masses is waiting for a solution. Why do we try to solve separately the problem of neutrino masses and the flavour problem? The ratio of neutrino masses to electron mass $m_\nu/m_e \leq 10^{-6}$ is almost the same as the ratio of electron mass to top quark mass $m_e/m_t \simeq 3\times 10^{-6}$. In spite of that there are several reasons why the smallness of neutrino masses is intriguing. Firstly, the smallness of the neutrino mass remains a question even within one family. Quark mass ratio in the same family is about 10, while for the same lepton generation the mass ratio is smaller than 10^{-6}. Secondly, the problem of neutrino mass may be connected with their nature. The quarks and charged leptons are Dirac particles. Neutrinos are probably Majorana particles. And finally, even if the problem of mass is not resolved, the large difference for lepton masses within a single family can shed light on the necessary extension of the SM. This is probably the main reason why the problem of neutrino mass, usually connected with the flavour problem, has been so intensively studied in recent years [32].

The simplest way to get massive neutrinos is to introduce the right-handed chiral neutrino fields

$$L_{\mathrm{Y}} = -\sum_{\alpha,\beta} f_{\alpha,\beta}\bar{\psi}_{\alpha\mathrm{L}}(-i\sigma_2\varphi^*)\nu_{\beta\mathrm{R}} + \mathrm{h.c.} \tag{25}$$

There is no fundamental reason why we cannot do that, but we do not like this solution. The neutrino mass matrix is proportional to the Yukawa couplings $f_{\alpha,\beta}$ and there is no good reason why these couplings should be so small. Such a solution does not give any indication on how to extend the SM.

The other possibility is to add to the previous model the right-handed mass term

$$L_{\rm RH} = -\frac{1}{2}\sum_{\alpha,\beta} g_{\alpha,\beta}\bar{\nu}^c_{\alpha\rm L}\nu_{\beta\rm R} + \text{h.c.} \tag{26}$$

In such models, by see-saw mechanism we can get three light and three heavy Majorana particles and B–L symmetry is broken. As usually, if two very different scales exist, we meet with the hierarchy problem. If the large scale is of the order of the Planck mass, neutrinos obtain too small masses $m \sim 10^{-5}$ eV.

There are various models which, in a better or worse way, explain the small neutrino masses and the two large mixing angles. There are many ways how the SM is extended. The first option is to continue maintaining the symmetry of the SM and, *(i)* modify the fermion sector, *(ii)* enlarge the Higgs sector, and *(iii)* break spontaneously the B–L symmetry (Majoron(s) appears). The first possibility, as we have discussed previously, is not satisfactory. There are three reasonable ways of the Higgs sector enlargement, where (1) an additional Higgs triplet $\mathit{\Delta}$, (2) a singly charged singlet, h_-, or (3) a doubly charged gauge singlet k_{++} is introduced. These possibilities are very popular. In models with the Higgs triplet the see-saw mechanism is operating. Models with additional singlets, invented by Zee and Babu [33] are very interesting, as NP appears at TeV scale and there is a chance to see some implications at LHC. Neutrino masses are small, as they are generated at either one or two loops. There are also two different realizations of models with Majorons, (1) a gauge singlet and additional right-handed neutrinos are introduced, or (2) only the Higgs sector is extended by adding the Higgs triplet and singly charged scalar.

The second option is to abandon the symmetry group of the SM, and build a model which at the low energy has all features of the SM. Several such models are considered in the literature, *(i)* new gauge group $\mathrm{SU_L}(2)\otimes\mathrm{SU_R}(2)\otimes\mathrm{U}(1)_{\mathrm{B-L}}$ with two Higgs doublets or Higgs doublets and triplets, *(ii)* models of grand unification based on the SU(5), SO(10) or E_6 symmetry groups, *(iii)* supersymmetric models in several versions, the MSSM, the model with broken R-parity and models based on the supersymmetric Left–Right group.

The next problem is connected with the specific structure of the flavour mixing matrix U introduced by Maki Nakagawa and Sakata (see [22]). In order to understand the large values of the mixing angles θ_{12}, θ_{23} and of the much smaller angle θ_{13}, special flavour symmetries are usually imposed in the models.

Despite some successes in understanding the problem of the small neutrino masses and of the large mixing angles, it is difficult to accept that this problem is solved. On the other hand, the experimental knowledge of the masses and mixing matrix is still too poor to select the best theoretical model.

5.2. Neutrino oscillations beyond the SM

There are two different approaches to neutrino oscillation phenomena. The original one has been introduced by Gribov and Pontecorvo [34]. In this theory neutrino oscillation is considered without production and detection processes. The authors assumed that the neutrinos are produced in a pure quantum mechanical α-flavour state, which is a combination of states with a definite neutrino mass

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \,. \tag{27}$$

Then the amplitude of finding a neutrino in the β-flavour state, after passing in vacuum or matter a distance L, is equal to

$$A(L \approx t) = \langle\nu_\beta|e^{-itH}|\nu_\alpha\rangle \,, \tag{28}$$

where H is the Hamiltonian of the free neutrinos in vacuum or interacting inside matter, and as usually we assume that for the relativistic neutrinos $t \approx L$.

In the second approach [35] the production and detection processes are included, neutrinos with definite masses are unobserved intermediate states between the source and detection processes and are represented by inner lines in a big Feynman diagram connecting the production and detection points. Such a model can be used in any BSM. The only weakness of this approach is the treatment of the particles propagating on macroscopic distances as non-physical particles, which are not on the mass-shell.

Recently a different approach has been proposed, which also may be used for any model of neutrino interactions, and in which the neutrinos propagate over long distances on mass shell [36]. As a production process, let us consider the three body decay (*e.g.* muon decay or a nuclear beta decay),

$$A \to B + \bar{l}_\alpha + \nu_i(\lambda) \,. \tag{29}$$

Then the state of produced neutrinos, in the rest frame of the decaying particle A, is described by the density matrix which depends on the dynamics for the process (29). In the base where the neutrino mass (m_i) and helicity λ are specified $|\nu_i, \lambda\rangle$, it is described by the well known formula:

$$\varrho^{\alpha}(\lambda, i; \eta, k; E, \theta, \varphi) = \tag{30}$$
$$\frac{1}{N_\alpha} \sum_{\text{spins}} \int \overline{d\text{Lips}}\, A_i^{\alpha}(\lambda_A; \lambda_B, \lambda_l, \lambda; E, \theta, \varphi) \varrho_{\lambda_A, \lambda_{A'}} A_k^{\alpha *}(\lambda_{A'}; \lambda_B, \lambda_l, \eta; E, \theta, \varphi)\,,$$

where the integral $\overline{d\text{Lips}}$ is taken over the part of the phase-space left after excluding the neutrino energy (E) and the momentum direction defined by (θ, φ). The $\varrho_{\lambda_A, \lambda_{A'}}$ is the density matrix which describes the polarization of decaying particle (A) and the factor N_α normalizes the density matrix, so that $\mathrm{Tr}\, \varrho = 1$.

Let us assume that in the detection process the lepton with flavour β is produced in the detector:

$$\nu_i + C \to l_\beta + D\,, \tag{31}$$

then the total cross-section for neutrino detection is calculated in the usual way:

$$\begin{aligned} \sigma_{\alpha \to \beta}(E, L) &= \frac{1}{64\pi^2 s} \frac{p_f}{p_i} \frac{1}{2s_C + 1} \\ &\times \sum_{\text{spins, masses}} \int d\Omega\, f_i^{\beta}(\lambda) \varrho^{\alpha}(L; i, \lambda; k, \eta)\ f_k^{\beta *}(\eta)\,, \end{aligned} \tag{32}$$

where the $f_i^{\beta}(\lambda)$ are spin amplitudes for the detection process (31) of neutrino with mass m_i and helicity λ. The $\varrho^{\alpha}(L; i, \lambda; k, \eta)$ is the density matrix after neutrino propagation, calculated from

$$\varrho^{\alpha}(E, L) = e^{-iHL} \varrho^{\alpha}(E, L = 0) e^{iHL}\,. \tag{33}$$

In this approach, depending on the neutrino interaction in the production process, the initial neutrino state can be pure, as in the νSM, or mixed. The final formula for the detection rate does or does not factorize to neutrino oscillation probability times detection cross-section (for details see [36]).

6. Conclusions

Several selected topics in the theory of neutrinos have been described. First, a short presentation of neutrino properties in the frame of the original Standard Model (SM), together with the experimental informations about their masses and mixing was given. In the frame of the model with massive neutrinos, the so-called NewSM (νSM), two controversial phenomena have been presented: the Mössbauer neutrinos problem and the GSI anomaly. In theories beyond the SM, the problem of small neutrino masses and large mixing has been summarised. A few comments about the theories of neutrino oscillations have been made.

This work has been supported by the Polish Ministry of Science and Higher Education under the grant No. N N202 064936 and by the European Community Marie-Curie Research Training Network under contract MRTN-CT-2006-035505.

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