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RADIATIVE RETURN METHOD AS A TOOL IN HADRONIC PHYSICS * **

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A short review of both theoretical and experimental aspects of the radiative return method is presented. It is emphasised that the method gives not only possibility of the independent from the scan method measurement of the hadronic cross section, but also can provide information concerning details of the hadronic interactions. New developments in the PHOKHARA event generator are also reviewed. The 3 pion and kaon pair production is implemented within the version 5.0 of the program, together with contributions of the radiative ϕ decays to the 2 pion final states. Missing NLO radiative corrections to the $e^+e^- \to \mu^+\mu^-\gamma$ process will be implemented in the forthcoming version of the generator.

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1. Introduction

Precise hadronic cross section measurements are crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling ($\alpha_{\rm QED}$) from its value at low energy up to M_Z (for recent reviews look [1–3]). When using the scan method one usually needs new experiments to be performed with not negligible costs, while the radiative return method proposed already years

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ago [4] allows for extracting the information on the hadronic cross section basing on the measurements at the existing meson factories, profiting from their huge luminosities and excellent detectors. As it was shown in [5,6] the radiative return method is not only a powerful tool in the measurement of $\sigma(e^+e^- \to$ hadrons), but allows for detailed studies of hadronic interactions. Due to a complicated experimental setup, the use of Monte Carlo (MC) event generators [9–14], which include various radiative corrections [15] is indispensable for both signal and background processes. A more detailed analysis of that subject can be found also in [16, 17].

This paper is aimed as a short review of the results obtained by the radiative return method. It presents also a further potential of the method and shows new developments in the PHOKHARA event generator important for an extraction of the $\sigma(e^+e^- \to$ hadrons) (and other physical quantities) from the measured cross section $\sigma(e^+e^- \to$ hadrons + photons).

2. Hadronic cross section

The extraction of the cross section $\sigma(e^+e^- \to$ hadrons) from the measured cross section $\sigma(e^+e^- \to$ hadrons + photons) relies on the factorisation

$$d\sigma(e^+e^- \to \text{hadrons} + n\gamma) = H \; d\sigma(e^+e^- \to \text{hadrons}) \tag{1}$$

valid at any order for photons emitted from initial leptons, where the function H contains QED radiative corrections. It is known analytically, if no cuts are imposed on photons, at next to leading order (NLO) and has to be provided in form of an event generator of the reaction $e^+e^- \to$ hadrons + photons for a realistic experimental setup. The emission of photons from the final state hadrons has to be controlled as well [11, 17], with an accuracy which allows for an error small enough not to spoil the accuracy of the $\sigma(e^+e^- \to$ hadrons) extraction.

Comparing the scan method and the radiative return method one has to say that they are in many aspects complementary. It is due to the fact that many experimental systematic errors are completely different in both cases. The radiative return method has though the advantage that most of them are the same for all the values of the invariant mass of the hadronic system for which the measurement is performed. That is not true for the scan method, where for each energy one has to perform a separate analysis of many systematical errors (an energy calibration *etc.*). It is also important that using the radiative return method one can use machines, which were built for other purposes (Φ- and B-factories) and one has to 'invest' only in the experimental analysis. In many aspects both methods encounter however the same problems, which have to be solved. The already mentioned final state emission has to be studied carefully in both cases, the photon vacuum polarisation with its fast varying behaviour nearby resonances has to be taken into account, higher orders radiative corrections have to be properly implemented in event generators *etc.* Even if the details are different for scan and the radiative return method the main features remain the same.

The most important hadronic channel, from the point of the hadronic contributions to a_μ, mainly $\pi^+\pi^-$, is an example of that complementarity. Very accurate KLOE measurement [18] provided an important cross check of the CMD-2 data [19] and even if the agreement is not excellent it has allowed to conclude that the disagreement between e^+e^- data and the τ data concerning the pion form factor is not of the experimental origin. Further, mostly theoretical work, will be required to solve that puzzle and one will have to find new sources of the isospin violation effects, which finally will explain that disagreement.

Already now many new valuable physical information was obtained by means of the radiative return method. The BaBar measurement of the $\sigma(e^+e^- \to 2\pi^+2\pi^-, 2K^+2K^-, K^+K^-\pi^+\pi^-)$ [20] are the most accurate to date results, with the $2\pi^+2\pi^-$ mode also extremely important for hadronic contributions to a_μ and $\alpha_{\rm QED}$. The BaBar measurement of the $\sigma(e^+e^- \to \pi^+\pi^-\pi^0)$ [21] has shown that the cross section around the ω'' resonance is actually much bigger as compared to an old measurement by DM2 collaboration [22]. Results coming from BaBar on narrow resonances [20, 21, 23] are also very competitive to the ones coming from the scan method (for a review see [24]). Not to mention the forthcoming results [25,26], which show still growing potential for the radiative return method.

3. Not only the hadronic cross section — looking inside the hadronic interactions

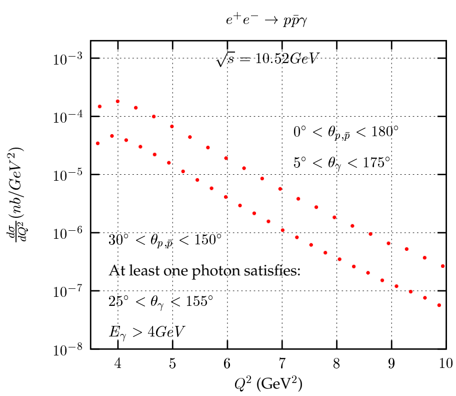
The radiative return method originally proposed for the hadronic cross section measurements [4, 7] can be used to much more detailed studies of the hadronic interaction. The first investigations along these lines were done in [5], where it was shown that it is feasible to measure separately the nucleon form factors in the time-like region at B-factories. That measurements are important for the understanding of the experimental situation in the space-like region (for a review see [27]), where two different type of measurements lead to different results for the ratio of the magnetic and electric proton form factors.

The nucleon electromagnetic current is defined by the form factors as follows:

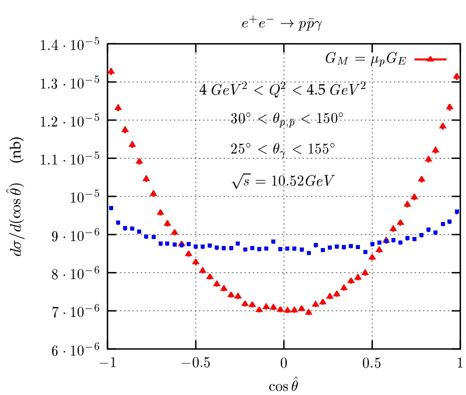
$$J_\mu = -ie\,\bar{u}(q_2)\left(F_1^N(Q^2)\gamma_\mu - \frac{F_2^N(Q^2)}{4m_N}\,[\gamma_\mu, \not{Q}\,]\right)v(q_1)\,, \tag{2}$$

with electric and magnetic form factors $G_M^N = F_1^N + F_2^N$, $G_E^N = F_1^N + \tau F_2^N$.

The statistics is not a problem for a measurement at B-factories with hundreds of fb^{-1} accumulated luminosity as the integrated cross section for the event selection corresponding to lower curve in Fig. 1(a) (angular cuts close to BaBar angular acceptance) is about 59.3 fb for protons and 125 fb for neutrons in the final state. The separation of the electric and magnetic form factors is also possible for quite a big range of the nucleon pair invariant mass. It is particularly easy if one performs analysis in the nucleon pair rest frame as shown in Fig. 1(b), where the proton polar angle distribution is plotted both for a model which predicts the ratio of form factors in agreement with measurements using the Rosenbluth method ($G_M^p = \mu_p G_E^p$, triangles) and a model which predicts the ratio of the form factors with agreement with the measurements using polarisation method (squares). For details concerning both methods see the review article [27] and references therein. For the $4\ \mathrm{GeV}^2 < Q^2 < 4.5\ \mathrm{GeV}^2$ one expects about 2000 events per 100 fb^{-1} accumulated luminosity and clearly two-parameter (G_M^N, G_E^N) fit is possible and its accuracy will be limited mostly by systematic errors.



(a)



(b)

Fig. 1. (a) The differential, in Q^2, cross section for the reaction $e^+e^- \to p\bar{p}\gamma$ for two different sets of event selections. (b) The differential, in $\cos\hat{\theta}$ ($\hat{\theta}$ — proton polar angle in the proton pair rest frame), cross section for the reaction $e^+e^- \to \bar{p}p\gamma$ for two theoretical models (see text for details).

Already now BaBar collaboration has preliminary results for the proton electromagnetic form factor measurements [25], which when completed will allow for extensive tests of the theoretical models.

Another example [6], very specific for the radiative return method at DAΦNE energy, is the study of radiative ϕ decays at KLOE (for present status of the experimental situation see [26]). The ϕ decay ($\phi \to f_0(\to \pi^+\pi^-)\gamma$) contributing to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ might in principle cause some problems in the pion form factor extraction from the $\sigma(e^+e^- \to \pi^+\pi^-\gamma)$ measurement. However, as it was shown in [6], the detailed studies of the charge asymmetries allow not only to control that contribution, but also to distinguish between different models of the radiative ϕ decays. That is clearly seen in Fig. 2, where the charge asymmetry for an event selection enhancing the FSR contributions is presented. The differences between predictions coming from different models of the radiative ϕ decays and also from scalar QED (see [6] for details) are sizable in the region of large and small values of the pion pair invariant mass (Q^2) and a measurement can easily distinguish between them leading to tests with unprecedented accuracy.

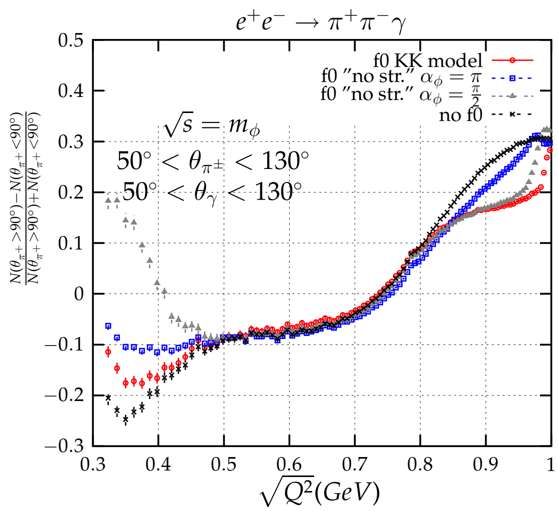


Fig. 2. The pion charge asymmetry for different radiative ϕ decay models [6].

4. New developments in the PHOKHARA event generator

4.1. PHOKHARA5.0: $\pi^+\pi^-\pi^0$ and KK final states

In the newly released version of the PHOKHARA Monte Carlo generator (5.0) three new hadronic channels were added: $\pi^+\pi^-\pi^0$, K^+K^- and $\bar{K}K$. For the K^+K^- both initial and final state photon(s) radiation was taken into account, while for the $\pi^+\pi^-\pi^0$ and the $\bar{K}K$ only initial state photon(s) emission was considered. The kaon hadronic current was adopted from [28], while a detailed analysis of all existing data on the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section was performed in [29]. The constructed model allows not only for an excellent fit to the cross section and a good description of two pion invariant mass distributions (see Fig. 3), but also many three-meson couplings were extracted separately from that fit making possible predictions of various decay rates and cross sections [29].

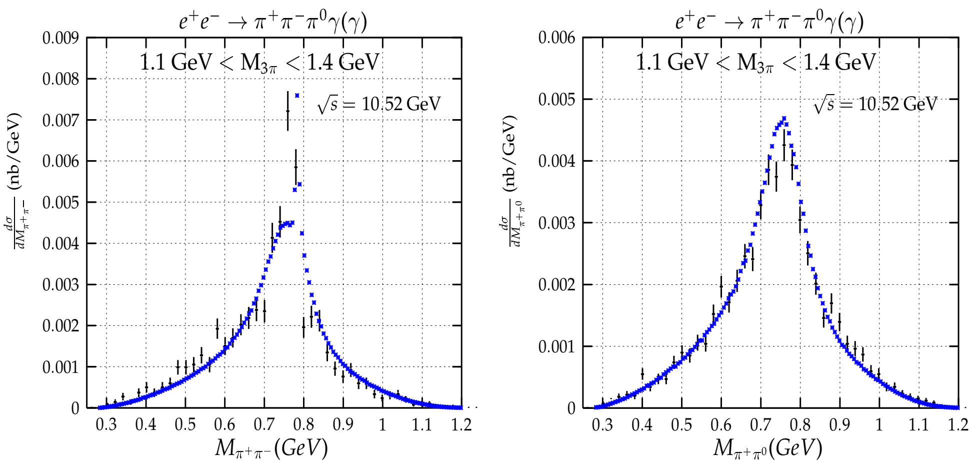


Fig. 3. The two-pion invariant mass distributions: ˙BaBar data [21] (points with error bars) and PHOKHARA5.0 predictions [29] (stars).

4.2. Next to leading order radiative corrections to the reaction $e^+e^- \to \mu^+\mu^-\gamma$

The reaction

$$e^+(p_1)e^-(p_2) \to \mu^+(q_1)\mu^-(q_2)\gamma(k) \tag{3}$$

may serve as a luminosity monitoring process for the radiative return method. If this method is used one measures the ratio

$$\mathcal{R}(s) = \frac{\sigma(e^+e^- \to \text{hadrons} + \gamma)}{\sigma(e^+e^- \to \mu^+\mu^- + \gamma)}\,, \tag{4}$$

and for extraction of the $\sigma(e^+e^- \to \text{hadrons})$ from the data the theoretical knowledge of both processes is needed. Due to a complicated experimental setup that piece of information has to be provided in a form of event generators and the NLO radiative corrections to both processes are indispensable to provide accurate theoretical predictions. Already in [12] a part of the NLO(FSR) radiative corrections was implemented and the missing parts of the generator consist of diagrams shown schematically in Fig. 4. All details of the calculations and implementation in the Monte Carlo event generator PHOKHARA will be presented in separate publications [30] while

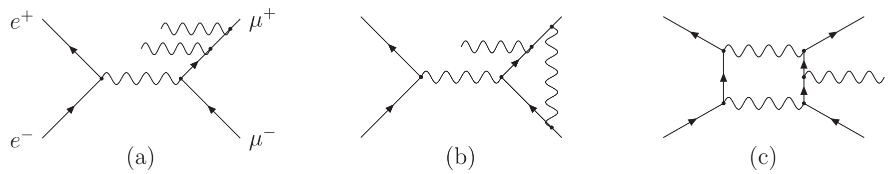


Fig. 4. NLO contributions to the $\sigma(e^+e^-\to\mu^+\mu^-\gamma)$ missing in the PHOKHARA5.0 code.

in this paper few details and the present status of this project is sketched. The contributions from diagrams in Fig. 4(a) are implemented using helicity amplitude method in an analogous way as it was done for two photon ISR contributions [9]. The contributions from Fig. 4(b), even if in principle are analogous to the ISR virtual corrections calculated in [15], have to be treated in a different way as the generator has to work also for DAΦNE energy where the muon mass is not a small parameter. The expansions used for the ISR, which shorten the final result enormously, cannot be applied and even if the whole result is known further work is required to construct formulae for calculation of the radiative corrections fast enough for a Monte Carlo event generator. In both cases the helicity amplitudes can be written as a combination of 14 terms if the gauge invariance and the current conservation is used. The way one writes the result is not unique and we have chosen to write explicitly the part proportional to the Born amplitude plus suitably chosen symmetric and antisymmetric coefficients. The ISR corrections read then

$$\mathcal{M}_{\rm ISR} \sim \bar{u}(q_2)\gamma_\mu v(q_1) \sum_{i=1}^{14} F_i\, \bar{v}(p_1) S_i^\mu u(p_2)\,, \tag{5}$$

where the coefficients F_i contain loop corrections. The FSR corrections have analogous structure.

The functions S_i^μ read

$$\begin{aligned}
S_1^\mu &= \frac{1}{2}\left(\frac{2p_1\epsilon^* - \not\epsilon^*\not k}{p_1k}\gamma^\mu - \gamma^\mu\frac{2p_2\epsilon^* - \not\epsilon^*\not k}{p_2k}\right), \quad S_2^\mu = \not k\not\epsilon^*\gamma^\mu\,,\\
S_3^\mu &= \not k\not\epsilon^* p_+^\mu\,, \quad S_4^\mu = \not k\not\epsilon^* p_-^\mu\,, \quad S_5^\mu = \not\epsilon^* p_+^\mu - \not k\epsilon^{*\mu}\,,\\
S_6^\mu &= [\not k\, p_+\epsilon^* - \not\epsilon^* p_+k]\,\gamma^\mu\,, \quad S_7^\mu = [\not k\, p_-\epsilon^* - \not\epsilon^* p_-k]\,\gamma^\mu\,,\\
S_8^\mu &= \not k\,[p_1\epsilon^*\; p_2k - p_2\epsilon^*\; p_1k]\, p_-^\mu\,, \quad S_9^\mu = \not k\,[p_1\epsilon^*\; p_2k - p_2\epsilon^*\; p_1k]\, p_+^\mu\,,\\
S_{10}^\mu &= \not\epsilon^* p_+^\mu - \not k\left[\frac{p_1\epsilon^*}{p_1k}p_2^\mu + \frac{p_2\epsilon^*}{p_2k}p_1^\mu\right], \quad S_{11}^\mu = \not\epsilon^* p_-^\mu + \not k\left[\frac{p_1\epsilon^*}{p_1k}p_2^\mu - \frac{p_2\epsilon^*}{p_2k}p_1^\mu\right],
\end{aligned}$$

$$
\begin{aligned}
S_{12}^{\mu} &= \epsilon^{*\mu} - \frac{p_1\epsilon^*}{p_1 k} p_2^{\mu} - \frac{p_2\epsilon^*}{p_2 k} p_1^{\mu}\,, \qquad S_{13}^{\mu} = [p_1\epsilon^*\; p_2 k - p_2\epsilon^*\; p_1 k]\, p_-^{\mu}\,, \\
S_{14}^{\mu} &= [p_1\epsilon^*\; p_2 k - p_2\epsilon^*\; p_1 k]\, p_+^{\mu}\,,
\end{aligned}
$$

where $p_\pm = p_1 \pm p_2$ and ϵ is the photon polarisation vector.

Within that parameterisation all the coefficients, but the F_1, are free from ultraviolet and infrared singularities, as the S_1 has exactly the spinor structure of the Born (ISR) amplitude. Moreover, most of the coefficients vanish in the massless limit and they are numerically important only for the configurations with the photon collinear to one of the initial leptons.

The diagrams in Fig. 4(c) consist of box and pentabox diagrams. The pentabox tensor integrals (up to the third rank), which has appeared in the calculations, were reduced in D-dimension to standard box diagrams (tensor integrals up to rank two) with method equivalent to the one presented in [31], even if in this particular case the reduction is simple due to the symmetry of the integrals. Further reduction is done using standard Passarino–Veltman reduction to scalar integrals. The reduction of two remaining pentabox scalar integrals to the box scalar integrals has introduced simple denominators $(kp_1 - kp_2)$ and $(kq_1 - kq_2)$, which have zeros in the physical phase space. That problem will be solved by means of the expansion of the resulting around the mentioned zeros. Again here the remaining problem is the size of the result after the tensor reduction and further work is required to produce formulae, which can be used within a Monte Carlo program.

5. Summary

A short review of experimental results obtained by means of the radiative return method is presented, with an extensive discussion of the theoretical basis of the method. The new version of the PHOKHARA Monte Carlo event generator (PHOKHARA5.0) is presented. The status of the work on the NLO radiative corrections to the reaction $e^+e^- \to \mu^+\mu^-\gamma$, a luminosity monitoring process for the radiative return method, is also outlined.

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