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FROM KAONS TO NEUTRINOS: QUANTUM MECHANICS OF PARTICLE OSCILLATIONS* **

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The problem of particle oscillation is considered in a pedagogical and comprehensive way. Examples from K, B and neutrino physics are given. Conceptual difficulties of the traditional approach to particle oscillation are discussed. It is shown how the probability current density and the wave packet treatments of particle oscillations resolve some problems. It is also shown that only full field theoretical approach is free from conceptual difficulties. The possibility of oscillation of particles produced together with kaons or neutrinos is considered in full wave packet quantum mechanics language. Precise definition of the oscillation of particles which recoil against mixed states is given. The general amplitude which describes the oscillation of two particles in the final states is found. Using this EPR-type amplitude the problem of oscillation of particles recoiling against kaons or neutrinos is resolved. The relativistic EPR correlations on distances of the order of coherence lengths are considered.

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1. Introduction

The subject is not new. The problem is known since 1955 when Gell-Mann and Pais [1] predicted the existence of two neutral kaons. Earlier, in 1953 in a scheme for classifying the various newly-found particles, Gell-Mann represented the neutral kaon K^0 and its antiparticle $\overline{K}^0$as two distinct

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particles. The decay of both particles into $\pi^+\pi^-$ was observed. If so, how do we know which particle has originated it: the K^0 or the $\overline{K}^0$? The problem has been solved by realizing that what we observe is the mixture of two states, K^0 and $\overline{K}^0$:

$$|K_{\mathrm{S}}\rangle = \frac{1}{\left[2\left(1+|\varepsilon|^2\right)\right]^{1/2}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\overline{K}^0\right\rangle\right], \tag{1}$$

$$|K_{\mathrm{L}}\rangle = \frac{1}{\left[2\left(1+|\varepsilon|^2\right)\right]^{1/2}}\left[(1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\overline{K}^0\right\rangle\right], \tag{2}$$

where ε is a small, complex, later measured parameter responsible for CP symmetry breaking [2]. In this way first time the interference between states of slightly different masses has appeared in quantum mechanics. Inspired by the work of Gell-Mann and Pais, Bruno Pontecorvo turned to consider the possibility of quantum mechanical mixing in another neutral particle — the neutrino. In 1957 he first suggested that a neutrino may oscillate into its antipartner [3]. Oscillation among the different kinds of neutrinos was then proposed by Maki, Nakagawa and Sakata in 1962 [4] and later by many others [5].

The neutral K^0–$\overline{K}^0$ boson system is not the only one where the quantum mechanical mass mixing can be considered. We can expect to observe the same phenomena in other neutral boson systems: D^0–$\overline{D}^0$ and B^0–$\overline{B}^0$. Generally, flavour oscillations of particles can occur when states produced and detected in a given experiment, are superpositions of two or more eigenstates with different masses. The oscillation of K and B meson has been observed experimentally in 1961 [6] and later [7] and has been used to place stringent constraints on physics beyond the Standard Model. If neutrinos are massive and oscillate it is possible to resolve the well-known solar neutrino problem [8]. There are also first experiments in which the neutrino oscillations are observed [9].

The flavour oscillation of particles is a very fascinating demonstration of quantum mechanics in the macroscopic world. It has served as a model for many interesting systems and problems. Various aspects of quantum mechanics, as for example coherence, decoherence, wave packets, measurements, similarity and differences between pure and mixed states, wave function collapse, EPR "paradox" are in action. On the other hand particle mixing is the place where fundamental symmetries and properties of fundamental interactions are studied. Discovering of the CP symmetry violation and the measurement of differences between neutral mesons masses are connected with K, B bosons mixing. Neutrino oscillations have a chance to be the first place where problem of neutrino masses can be resolved.

In this review we will concentrate only on the quantum mechanical description of particle oscillations. Problems connected with testing of the fundamental interactions will not be discussed.

First of all we should mention that interference between states with different masses is not allowed in non-relativistic quantum mechanics. The Galilean invariance forbids a coherent superposition of such states (the so called Bargman superselection rule [10]). Beyond the non-relativistic limit such restrictions do not hold (which clearly follows from experiment). It means that all of our considerations should be done in relativistic quantum mechanics (nevertheless non-relativistic approximations are possible).

First, we would like to describe briefly the traditional approach to the particle oscillation problem. This treatment is simple and elegant but immediately raises a number of conceptual questions. We specify more of them (Chapter 2). Next we show the wave packet treatment, where some of the problems disappear (Chapter 3). The current density approach which is closely connected with the experimental setting, is described in Chapter 4. The problem of constructing the probability current density for a particle with undetermined mass is also considered there.

Next, in Chapter 5, we give some remarks on the field theoretical approach to particle oscillations. Usually (as in the case of neutrino oscillations) the oscillating particle is not directly observed. Only particles accompanying neutrinos, hadrons and charged leptons created in the decay are observed. The proper approach should take all these circumstances into account. The creation of the neutrino in the source, its propagation to the detector and the detection process are treated in the framework of quantum field theory as one large Feynman diagram.

In Chapter 6 we discuss the controversial problem of the oscillation of particles recoiling against kaons or neutrinos from the production process. A detailed approach using wave packets explains the problem of four-momentum nonconservation raised in the literature.

In Chapter 7 we discuss the modern example of an Einstein–Podolsky–Rosen correlation in $K^0 - \overline{K}^0$ and B^0–$\overline{B}^0$ systems. The amplitude approach does not entail the somewhat mysterious "collapse of the wave function" which is usually invoked to describe the EPR effects.

Finally in Chapter 8 we summarize our main conclusions.

2. Problems connected with the traditional approach to the particle oscillation

The usual description of kaon mixing phenomena can be found in many textbooks [11]. Suppose, that we produce K^0 at $t = 0$ by the reaction

$$\pi^- p \to K^0 \Lambda^0 \,. \tag{3}$$

From (1) and (2) the K^0 state at $t=0$ is

$$\left|K^0\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2\,(1+\varepsilon)^2}}\,\left(|K_{\rm S}\rangle + |K_{\rm L}\rangle\right). \tag{4}$$

After time t, as $|K_{\rm S}\rangle$and $|K_{\rm L}\rangle$ states are definite mass eigenstates, we have

$$\left|K^0\,(t)\right\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2\,(1+\varepsilon)^2}}\left({\rm e}^{-i\left(m_{\rm S}-i\frac{\Gamma_{\rm S}}{2}\right)t}\,|K_{\rm S}\rangle + {\rm e}^{-i\left(m_{\rm L}-i\frac{\Gamma_{\rm L}}{2}\right)t}\,|K_{\rm L}\rangle\right), \tag{5}$$

where $m_{\rm L(S)}$ and $\Gamma_{\rm L(S)}$ are masses and inverse mean lifetimes respectively of the long (short)-lived component of K.

The $K^0(\overline{K}^0)$ fraction of the beam after time t is just

$$\begin{aligned} P_{K^0\to K^0(\overline{K}^0)}\,(t) &= \left|\left\langle K^0(\overline{K}^0)\,\middle|\,K^0(t)\right\rangle\right|^2 \\ &= \frac{1}{4}\left[{\rm e}^{-\Gamma_{\rm S}t} + {\rm e}^{-\Gamma_{\rm L}t} \pm 2{\rm e}^{-\frac{1}{2}(\Gamma_{\rm L}+\Gamma_{\rm S})t}\cos\left(\Delta m t\right)\right], \end{aligned} \tag{6}$$

where $\Delta m = m_{\rm L}-m_{\rm S}$. From Eq. (6) we can see that the fraction of $K^0(\overline{K}^0)$ becomes smaller (because of decay) and changes with time with frequency $\omega = \Delta m/2\pi$.

Neutrino oscillations are described in a very similar way [12]. Let us assume that at $t=0$ neutrino with flavour α was born with momentum p perfectly defined (as for example neutrino ν_μ in the pion decay $\pi^+\to\mu^+\nu_\mu$). At this time the neutrino state is described by

$$|\Psi_\alpha(0)\rangle = \sum_a U_{\alpha a}\,|a\rangle\,, \tag{7}$$

where states $|a\rangle$ are energy-momentum eigenstates for neutrinos with mass m_a and $U_{\alpha a}$ are elements of a flavour-mass mixing matrix.

Then

$$H\,|a\rangle = E_a\,|a\rangle\,, \tag{8}$$

where $E_a=\sqrt{p^2+m_a^2}$ with the same momentum p for each neutrino. After time t the state will evolve into

$$|\Psi_\alpha(0)\rangle \to |\Psi_\alpha(t)\rangle = {\rm e}^{-iHt}\,|\Psi_\alpha(0)\rangle = \sum_a U_{\alpha a}{\rm e}^{-iE_a t}\,|a\rangle\,. \tag{9}$$

Then the probability that a neutrino born at $t=0$ with flavour α at time t has flavour β is given by

$$P_{\alpha\to\beta}(t) = |\langle\Psi_\beta(0)|\Psi_\alpha(t)\rangle|^2 = \left|\sum_{a=1}^{n} U^*_{\beta a}{\rm e}^{-iE_a t}U_{\alpha a}\right|^2, \tag{10}$$

where n is the number of interfering light neutrinos[1].

Now usually relativistic approximations are made. As for real, light neutrinos $p \gg m_a$, we have

(i) $E_a \cong p + \frac{m_a^2}{2p}$,

and

(ii) a neutrino born in $x = 0$, at time t will be approximately at position $x \approx t$.

Then from (10) we can find that the probability for our neutrino, born with flavour α, to have new flavour β after travelling a distance x, is

$$P_{\alpha\to\beta}(x) = \sum_{a=1}^{n} |U_{\beta a}|^2 |U_{\alpha a}|^2 + 2\sum_{a>b} \left|U_{\beta a}^* U_{\alpha a} U_{\beta b} U_{\alpha b}^*\right| \cos\left(2\pi \frac{x}{L_{ab}} - \varphi_{ab;\alpha\beta}\right), \tag{11}$$

where L_{ab}, known as *oscillation length* between ν_a and $\nu_b$is defined by

$$L_{ab} = \frac{4\pi p}{m_a^2 - m_b^2}, \tag{12}$$

and

$$\varphi_{ab;\alpha\beta} = \arg\left(U_{\beta a}^* U_{\alpha a} U_{\beta b} U_{\alpha b}^*\right), \tag{13}$$

are phases responsible for CP violation.

From (11) it follows that the oscillation will disappear (the $P_{\alpha\to\beta}(x)$ does not depend on x) if *(i)* all neutrino masses are equal $m_a = m_b$ and/or *(ii)* only diagonal elements of the mixing matrix $U_{\alpha b}$ do not vanish.

The presented arguments seem to be clear and elegant but they are wrong. Many conceptual questions arise when we look at the presentation shown above. A complete treatment of particle oscillation must address the following additional issues.

(1) A necessary condition for particle oscillation to occur is that particle source and detector are localized within the region Δx much smaller than the oscillation length $|L_{ab}|$.

$$|L_{ab}| \gg \Delta x\,. \tag{14}$$

[1] To prove Eq. (10) we have to assume that the scalar product of two eigenmomentum states $\langle b \mid a\rangle = \delta_{ab}$. This means that we must introduce some normalization volume and momentum and energy are quantized.

(2) From Eqs (4) and (7) we see that different mass eigenstates are produced and detected coherently. This is possible only if the momentum (p) and energy (E) of the oscillating particle are spread in such a way that the error in m^2 measurements given by

$$\Delta m^2 = \left[(2E)^2 (\Delta E)^2 + (2p)^2 (\Delta p)^2\right]^{1/2} \tag{15}$$

is larger than $\left|m_a^2 - m_b^2\right| \equiv |\Delta m_{ab}^2|$,

$$\Delta m^2 \geq |\Delta m_{ab}^2|. \tag{16}$$

If this condition is not satisfied and $\left|m_a^2 - m_b^2\right| \geq \Delta m^2$, then also

$$\left|\Delta m_{ab}^2\right| \geq 2p\Delta p. \tag{17}$$

But from the uncertainty relation $\Delta x \geq \frac{1}{\Delta p}$, and Eq. (17) gives $\Delta x \geq \frac{2p}{\left|\Delta m_{ab}^2\right|} = \frac{|L_{ab}|}{2\pi}$, which is in contradiction with Eq. (14).

From both conditions described above we see that the oscillating particle state cannot be described by a plane wave with definite momentum [13] and the wave packet approach must be constructed.

(3) The energy and momentum conservation in processes in which oscillating particles are created (*e.g.* $\pi^- p \to \Lambda K^0$ or $\pi^+ \to \mu^+ \nu_\mu$) implies that different mass eigenstate components have different energy and momentum [14]. Approaches where all oscillating particles have the same momentum and different energies [11,12], or the same energies and different momenta [15–17] are conceptually not correct.

(4) In the traditional approach to find the oscillation probability we calculate the wave functions′ overlap (compare Eqs (6) and (10)). This procedure gives the probability which depends on time. In the real experiment the distance between the source and the detector is known (not the moment in which the measurement is done). To transform $P(t)$ into $P(x)$ the classical formula $x = vt$ is invoked. However to find the probability that the beam of particles produced at $\vec{x} = 0$ will reach a physical detector at a distance $\left|\vec{x}\right|$ the current density $\vec{j}\,(\vec{x}, t)$ should be integrated over the surface of the detector and over the time of observation [18]

$$P\left(\left|\vec{x}\right|, t_1 < t < t_2\right) = \int_{t_1}^{t_2} dt \int_{\partial A} \vec{dS} \cdot \vec{j}\,(\vec{x}, t). \tag{18}$$

In such an approach there is no problem of how to change t into x.

(5) In case of neutrinos additional conceptual problems arise because neutrinos are not "seen" directly. The only things which can be "seen" are hadrons or/and charged leptons in points where neutrinos are produced and detected. So in a realistic description, the external (initial and final) particles should be described by wave packets, and the mass-eigenstate neutrinos should propagate from the production region to a detector [19, 20].

All points which have been mentioned above are not only purely academic. We do not try to derive in a more precise way something which is known from the beginning. We will see that the more precise approach to the particles' oscillation phenomenon gives us new predictions and elucidates mysteries in many points. On the other hand we will see quantum mechanics in action on macroscopic distances.

3. The wave packet treatment of particle oscillation

The wave packet approach to neutrino oscillation was first proposed by Kayser [13] and later considered in more detail in [21, 22]. Nowadays the neutral boson oscillation is also treated in the same way. We will present the formalism for neutrinos, but everything can be repeated also for bosons.

In Eq. (7) states $|a\rangle$ have definite energy and momentum (also the spin direction of neutrinos is defined) so for the sake of precision we should write

$$|a\rangle \equiv |a;p\rangle = \left|a;\vec{p},E,\sigma\right\rangle \,. \tag{19}$$

We can easily construct a state with momentum distributed around a mean value $\vec{p_a}$. Let us assume that, instead of a plane wave, our new state $|a\rangle$ (Eq. (20)) has a Gaussian form, which in the momentum representation, is given by

$$\left\langle a;\vec{p}\,\middle|\,a\right\rangle \equiv \Psi_a\left(\vec{p},\vec{p}_a,\sigma_{pP}\right) = \frac{1}{\left[\sqrt{2\pi}\sigma_{pP}\right]^{3/2}}\exp\left[-\frac{\left(\vec{p}-\vec{p}_a\right)^2}{4\sigma_{pP}^2}\right], \tag{20}$$

where the width σ_{pP} is the same for each massive neutrino in the production (P) process and the same along all three directions. The average momenta $\vec{p_a}$ of the different mass eigenstates are determined by the kinematics of the production process.

In the wave packet approach, the flavour states $|\Psi_\alpha(t)\rangle$ after time t (given by Eq. (9) in the plane wave formalism) are now

$$|\Psi_\alpha(t)\rangle \;=\; \sum_a U_{\alpha a}\mathrm{e}^{-iHt}\int d^3p\left|a;\vec{p}\;\right\rangle\left\langle a;\vec{p}\,\middle|\,a\right\rangle$$

$$= \sum_a U_{\alpha a} \int d^3p\, \Psi_a\left(\vec{p}, \vec{p}_a, \sigma_{pP}\right) \mathrm{e}^{-iE_a(p)} \Big| a; \vec{p}\,\Big\rangle, \tag{21}$$

where $E_a(\vec{p}) = \sqrt{p^2 + m_a^2}$.

The same states in the position representation $\Big|b; \vec{x}\Big\rangle$ are given by

$$\begin{aligned} |\Psi_\alpha(t)\rangle &= \sum_b \int d^3x\, \Big|b; \vec{x}\,\Big\rangle\Big\langle b; \vec{x}\,\Big|\Psi_a(t)\Big\rangle \\ &= \sum_a U_{\alpha a} \int d^3x\, \Psi_a\left(\vec{x}, t; \vec{v}_a, \sigma_{xP}\right) \Big|a; \vec{x}\Big\rangle, \end{aligned} \tag{22}$$

where now the function $\Psi_a\left(\vec{x}, t; \vec{v}_a, \sigma_{xP}\right)$ is defined by

$$\begin{aligned} \Psi_a\left(\vec{x}, t; \vec{v}_a, \sigma_{xP}\right) &= \int d^3p\, \Psi_a\left(\vec{p}, \vec{p}_a, \sigma_{pP}\right) \mathrm{e}^{-iE_a(p)t} \Big\langle a; \vec{x}\,\Big|\, a; \vec{p}\,\Big\rangle \\ &= \frac{1}{(2\pi)^{3/2}} \int d^3p\, \Psi_a\left(\vec{p}, \vec{p}_a, \sigma_{pP}\right) \mathrm{e}^{i\left(\vec{p}\vec{x} - E_a(p)t\right)}. \end{aligned} \tag{23}$$

Since the Gaussian wave packet in momentum space is picked around the average momentum $\vec{p_a}$ we can neglect the spreading of the wave packet and approximate

$$E_a(p) = E_a + \vec{v_a}\left(\vec{p} - \vec{p}_a\right),$$

where

$$E_a = \sqrt{p_a^2 + m_a^2} \text{ and } \vec{v_a} = \left.\frac{\partial E_a}{\partial \vec{p}}\right|_{\vec{p}=\vec{p_a}} = \frac{\vec{p_a}}{E_a}. \tag{24}$$

Then

$$\Psi_a\left(\vec{x}, t; \vec{v}_a, \sigma_{xP}\right) = \frac{1}{\left[\sqrt{2\pi}\sigma_{xP}\right]^{3/2}} \exp\left[i\left(\vec{p_a}\vec{x} - E_a t\right) - \frac{\left(\vec{x} - \vec{v_a}\,t\right)^2}{4\sigma_{xP}^2}\right], \tag{25}$$

with the width σ_{xP} in coordinate space given by

$$\sigma_{xP} = \frac{1}{2\sigma_{pP}}. \tag{26}$$

As earlier in Eq. (10), to find the amplitude of the flavour changing process, we project the states $|\Psi_\alpha(t)\rangle$ on the flavour states $|\Psi_\beta(0)\rangle$

$$A_{\alpha\to\beta}(t) = \langle\Psi_\beta(0)\,|\,\Psi_\alpha(t)\rangle\,. \tag{27}$$

If $|\Psi_\beta(0)\rangle$ is the same as before (Eq. (7)) this means that the momentum of each neutrino v_a is measured precisely. But this is not realistic, so let us assume that also detection process is characterized by the spatial coherence width σ_{pD} connected with the uncertainties in momentum and energy measurements

$$|\Psi_\beta(0)\rangle = \sum_b U_{\beta b} \int d^3p\, \Psi_b\left(\vec{p}, \vec{p}_b, \sigma_{pD}\right) \left| b; \vec{p} \right\rangle . \tag{28}$$

The average values of the momentum $\vec{p_b}$ are the same as in the incoming wave packets Eq. (22). To calculate the spatial decomposition of the detecting flavour state we have to take into account that the detector is placed at a distance L from the origin of the coordinates, so we have

$$\left|\Psi_\beta(\vec{L})\right\rangle = \sum_b \int d^3x_D \left| b, \vec{x_D} \right\rangle \left\langle b, \vec{x_D} \left| \Psi_\beta(0) \right\rangle\right., \tag{29}$$

and after the same approximation as before (Eq. (24)) we obtain

$$\left|\Psi_\beta(\vec{L})\right\rangle = \sum_b U_{\beta b} \int d^3x_D\, \Psi_b\left(\vec{x_D}, 0; \vec{v_b}, \sigma_{xD}\right) \left| b, \vec{x_D} \right\rangle . \tag{30}$$

$\Psi_b\left(\vec{x_D}, 0; \vec{v_b}, \sigma_{xD}\right)$ is given exactly by Eq. (25) after replacements

$$a \to b, \quad \vec{x} \to \vec{x_D}, \quad t \to 0, \quad \sigma_{xP} \to \sigma_{xD}. \tag{31}$$

The amplitude of the flavour changing process is given by the overlap

$$A_{\alpha\to\beta}(\vec{L}, t) = \left\langle \Psi_\beta(\vec{L}) \middle| \Psi_\alpha(t) \right\rangle. \tag{32}$$

We have to remember that the origin of coordinates $\vec{x_D}$ and $\vec{x}$ are not the same, so

$$\left\langle b, \vec{x_D} \middle| a, \vec{x} \right\rangle = \delta_{ab}\delta^{(3)}\left(\vec{x_D} + \vec{L} - \vec{x}\right), \tag{33}$$

and we have

$$A_{\alpha\to\beta}(\vec{L}, t) = \sum_a U^*_{\beta a} U_{\alpha a} \int d^3x\, \Psi^*_a\left(\vec{x} - \vec{L}, 0; \vec{v_a}, \sigma_{xD}\right) \Psi_a\left(\vec{x}, t; \vec{v_a}, \sigma_{xP}\right)$$

$$= \sqrt{\frac{2\sigma_{xP}\sigma_{xD}}{\sigma_x^2}} \sum_a U^*_{\beta a} U_{\alpha a} \exp\left[-i\left(E_a t - \vec{p_a}\vec{L}\right) - \frac{\left(\vec{L} - \vec{v_a}\, t\right)^2}{4\sigma_x^2}\right], \tag{34}$$

where the total production and detection width is now

$$\sigma_x = \sqrt{\sigma_{xP}^2 + \sigma_{xD}^2}\,. \tag{35}$$

Everything we have done up to now can be applied either to neutrino or neutral boson oscillations. The only assumption about the narrow wave packets in the momentum representation (Eq. (24)) can be used in both cases.

Next we calculate the oscillation probability for neutrinos which are relativistic ($p \gg m$) and following Ref. [22] we approximate

$$E_a \cong E + \xi \frac{m_a^2}{2E}\,, \quad p_a \cong E - (1-\xi)\,\frac{m_a^2}{2E}\,, \tag{36}$$

and

$$v_a \cong 1 - \frac{m_a^2}{2E^2}\,.$$

E is the energy determined by kinematics of the production process for a massless neutrino and ξ is a dimensionless quantity of order unity. We will see (next Chapter) how the relativistic approximation for neutrinos causes that the production and detection processes can be factorized out and the standard quantum mechanical approach describes the oscillation phenomenon properly.

In all realistic experiments the distance L is a fixed and known quantity, whereas time t is not measured. The quantity which we measure is the time integral of the probability. Now the time integral can be done, and it is possible to avoid the not properly legitimated (in quantum mechanics) replacement $x = vt$.

The time integral for $\left|A_{\alpha\to\beta}(\vec{L},t)\right|^2$ (Eq. (34)) can be done [22] and after normalization $\left(\sum_\beta P_{\alpha\to\beta}(x) = 1\right)$, instead of Eq. (11) we have

$$\begin{aligned} P_{\alpha\to\beta}(x) &= \int_0^\infty \left|A(\vec{L},t)\right|^2 dt \\ &= \sum_a |U_{\beta b}|^2\,|U_{\alpha a}|^2 + 2\sum_{a>b} \left|U_{\beta b}^* U_{\alpha a} U_{\beta b} U_{\alpha b}^*\right| \cos\left(2\pi \frac{x}{L_{ab}^{\rm osc}} - \varphi_{ab;\alpha\beta}\right) \\ &\quad \times \exp - \left(\frac{x}{L_{ab}^{\rm coh}}\right)^2 \exp - \left(\frac{x}{L_{ab}^{\rm coh}}\right)^2 \exp -2\pi^2\xi^2 \left(\frac{\sigma_x}{L_{ab}^{\rm osc}}\right)^2. \end{aligned} \tag{37}$$

The oscillation lengths are the same as before (Eq. (12)), namely

$$L_{ab}^{\rm osc} = \frac{4\pi E}{\Delta m_{ab}^2}, \quad \Delta m_{ab}^2 = m_a^2 - m_b^2, \tag{38}$$

and $L^{\rm coh}_{ab}$ known as *coherence lengths* [23] are given by

$$L^{\rm coh}_{ab} = \frac{4\sqrt{2}\sigma_x E^2}{|\Delta m^2_{ab}|}. \tag{39}$$

Comparing Eq. (37) to the usual expression for the neutrino oscillation probability we can see that two additional terms appear.

The second factor

$$\exp -2\pi^2\xi^2\left(\frac{\sigma_x}{L^{\rm osc}_{ab}}\right)^2 \tag{40}$$

is equal to unity if $\sigma_x \ll |L^{\rm osc}_{ab}|$. This inequality must be satisfied to observe any oscillation. The presence of the term (40) which goes to zero for $\sigma_x > |L^{\rm osc}_{ab}|$, reflects the requirement which we qualitatively discussed in the previous Chapter: to see the oscillations, the localization of the source and the detector must be much better determined than the oscillation length.

The first factor

$$\exp -\left(\frac{x}{L^{\rm coh}_{ab}}\right)^2 \tag{41}$$

was predicted long ago [23]. It is connected with the fact that two wave packets each with different momentum and energy, have slightly different group velocities. It means that after some time the mass eigenstate wave packets no longer overlap and cannot interfere to produce oscillations. It is very easy to predict the value of coherence length. If both wave packets have width σ_{xP} along the direction of propagation and the difference between group velocities is $|v_a - v_b| = \Delta v$ then we can expect that after travelling a distance L

$$L = \frac{2\sigma_{xP}}{\Delta v}\frac{v_a + v_b}{2} \tag{42}$$

both wave packets cease to overlap each other. This L is just the coherence length.

For relativistic neutrinos, using approximations given by Eq. (36), we reproduce $L^{\rm coh}_{ab}$ from Eq. (39), to a factor of $\sqrt{2}$. We can expect that the coherence length becomes longer if the spreading of the wave packets is taken into account. It is indeed the case as it was proved in Ref. [21].

From Eq. (39) we see that the coherence length $L^{\rm coh}_{ab}$ is proportional to $\sigma_x = \sqrt{\sigma^2_{xP} + \sigma^2_{xD}}$ and not only to σ_{xP} as in Eq. (42). It means that precise measurements of momenta of all particles appearing in the detection process (which implies small σ_{pD}, thus large σ_{xD}) can increase the coherence length [22, 24]. This is a wonderful example of quantum mechanics in action. A measurement can restore the coherence. Two wave packets having negligible overlap in the detector (thus without detector influence they cannot interfere, and the oscillation disappears, $\sigma_x = \sigma_{xP}$), because of precise measurements $(\sigma_{xD} \gg \sigma_{xP})$, may still interfere to give rise to oscillations $(\sigma_x \gg \sigma_{xP})$. This feature of quantum mechanics disagrees with causality. However, it is not the first time when quantum mechanics is at variance with common sense.

But Eq. (37) also restores some common sense. Measurements of momenta and energies of detected particles cannot be too precise if we want to maintain the particle oscillation. As a matter of fact, we have a longer and longer coherence length, but on the other hand increasing σ_x makes the position of the detector to be more and more undefined. If $\sigma_x > |L_{ab}^{\rm osc}|$ the wave packets of neutrinos ν_a and ν_b lose coherence (Eq. (40)), the oscillation between them is wash away. We see, particularly, that the plane wave approach and oscillations are incompatible. For a plane wave $\sigma_x \to \infty$ $(\sigma_p = 0)$ and the factor (40) teaches us that oscillations disappear.

There are also approaches where the neutrino oscillation is treated in a manifestly Lorentz invariant way [25] . The final answer is exactly the same as was presented up to now, but for one difference. In the fully covariant treatment, besides the spatial width σ_x, also temporal width σ_t should appear. It causes only one change in the oscillation probability formula Eq. (37). Instead of the spatial width σ_x, a new effective one, $\overline{\sigma}_{ab}$ appears in the coherence length (39) and in the factor (40)

$$\sigma_x \to \overline{\sigma}_{ab} = \sigma_x + \frac{v_a^2 + v_b^2}{v_a + v_b}\sigma_t\,, \tag{43}$$

where v_a and v_b are group velocities of wave packets.

If σ_t is given by the lifetime of particles which produce neutrino, $\sigma_t = \tau$ [25] (*e.g.* pions, kaons or muons) then the second term in (43) is usually much bigger than the first one and the role of factor (40) becomes more important. We should stress however that the independent widths of spatial and temporal characteristics of wave packets cause that freely propagating particles are not necessary on mass-shell. If we insist to have our particle on mass shell (all the time, not for mean values exclusively, which is equivalent with the requirement that our particle's state satisfies the equation of motion) only the momentum (or only the energy) distribution should be applied. Then energy (or momentum) is distributed also but in agreement with the on-shell relation $E = \sqrt{p^2 + m^2}$. Such an approach, which we also used in our presentation above is the standard one.

4. Current density approach to particle oscillations and the problems of states with undefined masses

A typical experiment which tries to observe particle oscillations measures the flux of flavour β type particles in the detector localized at some distance L from the source which produces particles with flavour α. The time of measurements is not known. Usually typical measurements last hours, days or even years (like the observation of solar neutrinos). So the most appropriate way to find the probability (or number of particles) to cross the surface ∂A of the detector (see Fig. 1) is to integrate the probability current density over the surface and integrate the result once more over the duration of measurements

$$P_{\alpha\to\beta}(L) = \int\limits_{t_1}^{t_2} dt \int\limits_{\partial A} \vec{dS} \cdot \vec{j_\beta}\,(\vec{x}, t). \tag{44}$$

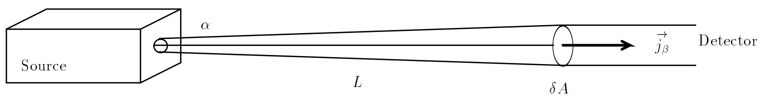

Fig. 1. Particles with the flavour α are produced in the "Source". After travelling the distance L they are detected as particles with flavour β. The probability of such detection is given by the probability current integral over the detector active surface (δA) and over the time of measurements.

This procedure seems to be so easy and natural, that we can ask why people try to use other, more complicated methods. The answer is very simple, there is a problem with the correct definition of the current $j_\beta(\vec{x}, t)$. This current should be defined for particles which we measure, that is $K^0, \overline{K}^0, \nu_e$ or ν_μ. But these particles have undefined masses, and we do not know how to define the probability current for such particles. The problem is more general: how to define properly the creation and annihilation operators for undefined mass states [26,27]? Here we will not discuss all trials [26] to resolve this problem in quantum field theory. We will concentrate only on a simple example of definition of the current j_β for particles with flavour β. For relativistic currents the problem has not been solved. For kaons it was 1
done in Ref. [18]. In the non-relativistic case the free Schrödinger equation for particles with mass m_a can be written in the form

$$i\frac{\partial \Psi_a}{\partial t} = \left(-\frac{\Delta}{2m_a} + m_a\right)\Psi_a, \tag{45}$$

which is the appropriate non-relativistic limit of either Klein–Gordon or Dirac equations ($E \approx \frac{p^2}{2m} + m$). For the Schrödinger equation (Eq. (45)) we know how to define the probability current:

$$\vec{j}_a(\vec{x},t) = \frac{1}{m_a}\mathrm{Im}(\Psi_a^*(\vec{x},t)\,\vec{\nabla}\,\Psi_a(\vec{x},t)) \tag{46}$$

for which the usual continuity equation is satisfied

$$\frac{\partial}{\partial t}(\Psi_a^*\Psi_a) + \mathrm{div}\,\vec{j}_a = 0\,. \tag{47}$$

The problem arises while we try to define the current for the states

$$\Psi_\alpha = c\Psi_a + d\Psi_b \quad \text{and} \quad \Psi_\beta = -d^*\Psi_a + c\Psi_b\,, \quad |c|^2 + |d|^2 = 1 \tag{48}$$

which are an orthogonal mixture of two states with different masses m_a and m_b. We expect that, because of mass mixing, the currents for $\Psi_{\alpha,\beta}$ will not be conserved [18], so let us propose a modified "continuity equation" for $\Psi_{\alpha,\beta}$ states in the form

$$\frac{\partial}{\partial t}|\Psi_{\alpha,\beta}|^2 + \mathrm{div}\,\vec{j}_{\alpha,\beta} = d_{\alpha,\beta}\,. \tag{49}$$

With the following requirements, concerning the new current $\vec{j}_{\alpha,\beta}$ [18], that:

(1) only "velocity" terms with one gradient are included,

(2) for $m_a \to m_b$ the "diffusion" terms $d_{\alpha,\beta}$ should vanish $d_{\alpha,\beta} \to 0$, and

(3) the sum of both flavour currents $\vec{j}_\alpha + \vec{j}_\beta$ is conserved

$$\frac{\partial}{\partial t}(|\Psi_\alpha|^2 + |\Psi_\beta|^2) + \mathrm{div}(\vec{j}_\alpha + \vec{j}_\beta) = 0\,, \tag{50}$$

the currents $\vec{j}_{\alpha,\beta}$ and the diffusion terms $d_{\alpha,\beta}$ can be found.

$$\vec{j}_\alpha = |c|^2\,\vec{j}_a + |d|^2\,\vec{j}_b + \mathrm{Im}\left[cd^*\left(\frac{1}{m_a}\Psi_b^*\mathrm{grad}\Psi_a - \frac{1}{m_b}\Psi_a^*\mathrm{grad}\Psi_b^*\right)\right], \tag{51}$$

and

$$d_\alpha = (m_a - m_b)\,\mathrm{Im}\left[cd^*\left(2\Psi_a\Psi_b - \frac{1}{m_a m_b}(\mathrm{grad}\Psi_a)(\mathrm{grad}\Psi_b^*)\right)\right]. \tag{52}$$

For $\vec{j}_\beta$ and d_β we have

$$\vec{j}_\beta \;=\; \vec{j}_\alpha\,(c \to -d^*, d \to c^*)\,,$$

and

$$d_\beta \;=\; d_\alpha\,(c \to -d^*, d \to c^*)\,. \tag{53}$$

Calculations for $K^0 - \overline{K}^0$mixing probability, using the definition (Eq. (44)) have been done in Ref. [18]. The standard formula (Eq. (6)), with the same oscillation frequency $\omega = \Delta m/2\pi$ was recovered, supporting previous results.

There is, however, one objection concerning this approach. The flavour currents which we use are not conserved. This nonconservation is given by the diffusion term (Eq. (52)) which is proportional to $\Delta m_{ab} = m_a - m_b$. So, we can expect that all our calculations have been done with the some precision. Then, if the result is proportional to Δm_{ab}, our probability flux calculations give the correct answer. The diffusion terms will change the result in higher powers of Δm_{ab}.

5. Treatment of particle oscillation in the framework of quantum field theory

There are several papers where authors investigate the neutrino oscillation problem in the framework of quantum field theory [19,20,24,28–30]. What are the main reasons for those studies?

First until now, to describe particle oscillations, we have used states with undefined masses (Eq. (4) for kaons and Eq. (7) for neutrinos). But, as we have seen in the previous Chapter, there is a problem of proper definition of such states [26]. Only in the extremely relativistic limit the flavour states are defined correctly [27]. As we will see, in quantum field theory the particle oscillation can be treated without resort to weak eigenstates.

Secondly, we have completely neglected the effect of the production and detection processes. It has been shown [27], that the neutrino oscillation probability is independent from the details of the production and detection processes only in the case of extremely relativistic neutrinos.

And finally, in real neutrino oscillation experiments only associated particles, hadrons and charged leptons are observed. Neutrinos are not prepared and not observed directly. One can measure the energy and momentum distributions of other particles which appear in the production and detection processes. As we will see, only the quantum field theoretical approach gives the opportunity to express the neutrino oscillation probability in terms of measured quantities.

Let us now describe briefly how the particle oscillation is treated in field theory. As in Ref. [20,30] we will describe the process of neutrino production (P) and neutrino detection (D) as one Feynman diagram with a virtual neutrino propagating itself on macroscopic distances between the source and the detector. Let us consider the process [20,30]

$$P_I \to P_F + l_\alpha^+ + \nu_\alpha \stackrel{(\nu_\alpha \to \nu_\beta)}{\longrightarrow} \nu_\beta + D_I \to D_F + l_\beta^- \,, \tag{54}$$

where P_I and P_F (D_I and D_F) are the particles in the production (detection) processes.

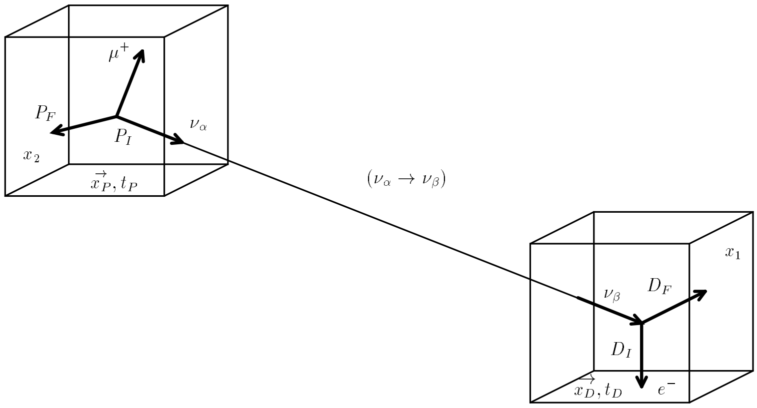

Fig. 2. In the field theory approach to the neutrino oscillation only the source (P_I, P_F, μ^+) and detector (D_I, D_F, e^-) particles are measured. Between the production $(\vec{x}_P, t_P)$ and detection $(\vec{x}_D, t_D)$ points neutrino propagates as virtual particle.

The production and detection processes are localized in coordinates $\overrightarrow{x_P}$ ($\overrightarrow{x_D}$) and times $t_P(t_D)$ (see Fig. 2). All initial (P_I, P_F, l_α^+) and final $t(D_I, D_F, l_\beta^-)$ particles are described by wave packets. Their shapes depend on the measurement precision in the production and detection processes. The amplitudes for the full process can be written in the form

$$A_{P\to D} = \left\langle P_F, l_\alpha^+; D_F, l_\beta^- \middle| S \middle| P_I D_I \right\rangle. \tag{55}$$

We can see that there are no neutrinos in the initial and final states. Only particles which really appear in the production and detection "equipments" are observed. Neutrinos with mass m_a propagate virtually between the source and the detector and are described by the Feynman propagators

$$\langle 0| T\left(\nu_a\left(x_1\right)\nu_b\left(x_2\right)\right)|0\rangle = \delta_{ab}\int \frac{d^4k}{(2\pi)^3}\frac{\widehat{k}+m_a}{k^2-m_a^2+i\varepsilon}\mathrm{e}^{-ik(x_1-x_2)}. \tag{56}$$

We will not present the details of all calculations, which are straightforward but tedious. A clear presentation can be found in Refs [20, 28, 30]. We will concentrate only on the discussion of the final results.

First of all, neutrinos are not directly present in Eq. (55), but this is not necessary. The amplitude $A_{P\to D}$ depends on points $\left(\overrightarrow{x_P}, t_P\right)$ and $\left(\overrightarrow{x_D}, t_D\right)$ where neutrinos were born and detected, and this is enough to study oscillations.

Next, the amplitude $A_{P\to D}$ depends also on amplitudes of the production and detection processes and the full structure of $A_{P\to D}$ is the following

$$A_{P\to D} = \sum_a U^*_{\beta a} U_{\alpha a} A_a f_a \left(\vec{x_D} - \vec{x_P}, t_D - t_P \right), \tag{57}$$

where A_a describes the process of neutrino creation and annihilation. The standard oscillation formula is recovered only if A_a can be factorized. This happens, when amplitudes A_a become independent of neutrino masses, $A_a = A$. If all neutrinos are relativistic then $A_a = A\,(m_a \cong 0)$ and the oscillation probability can be defined.

In case of relativistic neutrinos the time integrated neutrino flavour changing probability is given by the similar formula to Eq. (37) with two changes. First, the dumping factor (Eq. (40)) is slightly modified and now equals

$$\exp -2\pi^2 \omega \xi \left(\frac{\sigma_x}{L^{\mathrm{osc}}_{ab}} \right)^2, \tag{58}$$

where ξ is the same quantity as before (Eq. (36)), but ω is the new factor which depends on the production and detection dynamics and can be large (*e.g.* $\omega \cong 10$ is possible [30]). The second modification is a little different definition of the coherence length. Instead of (Eq. (39)) we now have

$$L^{\mathrm{osc}}_{ab} = \sqrt{2\omega} \frac{4E^2 \sigma_x}{|\Delta m^2_{ab}|}, \tag{59}$$

with the same factor ω as in Eq. (58).

The oscillation length L^{osc}_{ab} and the spatial width σ_x are given by the same formulae as before (Eqs (38) and (35)).

There is also an additional very important difference between the present and the former wave packet approaches. Before, σ_{xP} and σ_{xD} (Eq. (35) were two spatial widths of neutrinos specified in some way by the production and detection processes respectively. Now, these quantities are defined by spatial widths of hadrons and leptons which are measured. It turns out (for detail see Ref. [30])

$$\frac{1}{\sigma^2_{xP}} = \frac{1}{\sigma^2_{xP_I}} + \frac{1}{\sigma^2_{xP_F}} + \frac{1}{\sigma^2_{x\alpha}},$$

and

$$\frac{1}{\sigma^2_{xD}} = \frac{1}{\sigma^2_{xD_I}} + \frac{1}{\sigma^2_{xP_F}} + \frac{1}{\sigma^2_{x\beta}}. \tag{60}$$

The widths of observed particles in the production and detection processes define the width of "the neutrino", even if there is no place in this approach for physical neutrinos (only virtual ones appear). In the configuration space (Eq. (60)) the sum of the inverse squares of widths for the observed particles gives the inverse square of the resultant width. Then the smallest ingredient width dominates the values of σ_{xP} or σ_{xD}. It is the opposite for the resultant width σ_x, where $\sigma_x^2 = \sigma_{xP}^2 + \sigma_{xD}^2$. From the definition of the momentum width $\sigma_p = 1/2\sigma_x$, it follows that it is just opposite in momentum space, so then

$$\frac{1}{\sigma_p^2} = \frac{1}{\sigma_{pP}^2} + \frac{1}{\sigma_{pD}^2}, \tag{61}$$

but

$$\sigma_{pP}^2 = \sigma_{pP_I}^2 + \sigma_{pP_F}^2 + \sigma_{p\alpha}^2, \tag{62}$$

and the same for particles in the detection process. We can see explicitly, that precise measurements of momenta of all particles involved in the neutrino detection process (small $\sigma_{pD_I}, \sigma_{pD_F}$ and $\sigma_{p\alpha}$) give a small resultant width σ_{pD}, thus large σ_{xD} and large σ_x. The same subtle thing, which we have discussed before in Chapter 3, that the final measurement is able to recover the interference, in the present interpretation has found a much stronger background.

6. Do particles recoiling against mixed states oscillate?

For many years oscillations of particles like kaons or neutrinos were treated in isolation. The circumstances in which oscillating particles were produced have not been considered. Recently a series of papers has appeared [31,32], in which the kinematics of the production process have been taken into account in detail. Authors claim that, because the produced oscillating particles have neither momentum nor energy defined in explicit way, this fact should have consequences not only for them but also for the recoiling particles.

Let us consider the K^0 production in the reaction

$$\pi^- p \to \Lambda K^0, \tag{63}$$

or the neutrino production in the π^+ decay

$$\pi^+ \to \mu^+ \nu_\mu. \tag{64}$$

If the invariant mass of the initial system is denoted by M ($M^2 = (P_{\pi^-} + P_p)^2$ or $M^2 = m_\pi^2$) then energies (E_i) and momenta (p_i) of outgoing particles depend on masses (m_i) of K^0 or ν_μ. In the CM system there is

$$p_i = \frac{\left[\left(M^2 - m_i^2 - m^2\right)^2 - 4m_i^2 m^2\right]^{1/2}}{2M},$$

and

$$E_i = \frac{M^2 + m_i^2 - m^2}{2M}, \tag{65}$$

where m is the mass of the recoiling particle Λ or μ^+.

From the four-momentum conservation in the production processes (63) or (64), the energy and momentum of Λ or μ^+ are also defined by (65). Authors of Ref. [31] claim that if K^0 or ν_μ oscillate, also the recoiling particles Λ or μ^+ do the same. If it is true, the existence of such phenomena could give a chance for indirect observation of neutrino oscillations which are very difficult to observe in a direct way. However other papers have immediately appeared [33–35] where authors have been strongly against the oscillation of particles produced in association with kaons or neutrinos. We will present here our approach to the problem [36] which also supports the opinion against a visible oscillation of the associated particles. To fix notations, everything will be described for the process (63), but equally well we can show the lack of visible muon oscillations in the pion decay (64).

First of all we would like to specify the kind of oscillation, we can consider for Λ (or μ^+). Two Λ's with different masses do not exist. But even without mass differences the Λ's are produced in association with the long-live K_{L} and the short-live K_{S}. As K_{L} and K_{S} have different masses Λ's will be produced in two orthogonal states with different energy and momentum.

$$|\Lambda_{\mathrm{L}}\rangle = \left|-\vec{p}_{\mathrm{L}}, M - E_{\mathrm{L}}\right\rangle$$

and

$$|\Lambda_{\mathrm{S}}\rangle = \left|-\vec{p}_{\mathrm{S}}, M - E_{\mathrm{S}}\right\rangle, \tag{66}$$

where $\vec{p}_{\mathrm{L(S)}}$ and $E_{\mathrm{L(S)}}$ are the momentum and the energy of the $K_{\mathrm{L}}(K_{\mathrm{S}})$ in total CM frame of the process (63). We do not know in which state $|\Lambda_{\mathrm{L}}\rangle$ or $|\Lambda_{\mathrm{S}}\rangle$ the Λ particles are produced, so let us assume that at $t = 0$ they are produced in some state which is a linear combination of both states (66)

$$|\Lambda(0)\rangle = a\,|\Lambda_{\mathrm{L}}\rangle + b\,|\Lambda_{\mathrm{S}}\rangle, \qquad |a|^2 + |b|^2 = 1. \tag{67}$$

As both ingredient states have different energy they evolve with time in a different way, so there is some chance that after some period of time the state $|\Lambda(0)\rangle$ will oscillate to the orthogonal one

$$|\Lambda'(0)\rangle = -b^*\,|\Lambda_{\mathrm{L}}\rangle + a^*\,|\Lambda_{\mathrm{S}}\rangle. \tag{68}$$

Do we have a chance to recognize both states $|\Lambda(0)\rangle$ and $|\Lambda'(0)\rangle$? In the neutral bosons system, because of the strangeness conservation, K^0 and

$\overline{K}^0$interact strongly in a completely different way and are easily distinguishable. Here we have the same particle Λ with only one decay width Γ_Λ. In spite of that, in principle we can distinguish $|\Lambda(0)\rangle$ from $|\Lambda^{'}(0)\rangle$ but in a much more sophisticated way. Λ's in both states will decay in the same way (mostly to $p\pi^-$). But because two states $|\Lambda_{\rm L(S)}\rangle$ have slightly different momenta (in CM frame) also the angular distribution (*e.g.* for protons) will be slightly different. As two states $|\Lambda(0)\rangle$ and $|\Lambda^{'}(0)\rangle$ are various mixtures of $|\Lambda_{\rm L(S)}\rangle$, the angular distribution of the protons in CM frame which come from $|\Lambda(0)\rangle$ or from $|\Lambda^{'}(0)\rangle$ will be different.

We can see that in principle both states (67) and (68) could be distinguished. But do we have anything which may be distinguished? In other words, if we have flux of Λ's produced with kaons in the reaction (63), will their number in the state $|\Lambda(0)\rangle$ or $|\Lambda^{'}(0)\rangle$ change with distance from the reaction point? For simplicity we will present the answer to this question in the plane wave language. We know that it is not precise, but the value of oscillation length obtained in this way is correct. The full wave packet approach, together with particle correlations (the EPR effect) will be presented in the next Section.

Let us assume that at $t=0$, Λ's are produced in the pure state $|\Lambda(0)\rangle$ (in the reaction (63), the coefficients $a=b=1/\sqrt{2}$, but it is more transparent to leave them undefined). We will consider the production of Λ's and the decay $\Lambda\to p\pi^-$ together. The amplitude for Λ production and decay after a period of time t can be written in a form, where two indistinguishable ways of reaching the final state are added coherently [37]

$$\begin{aligned} A\left(\Lambda\to p\pi^-\right) &= A\left(\Lambda\to\Lambda_{\rm L}\right){\rm e}^{-i\left(m_\Lambda-i\frac{\Gamma}{2}\right)\tau_{\rm L}}A\left(\Lambda_{\rm L}\to p\pi^-\right) \\ &\quad +A\left(\Lambda\to\Lambda_{\rm S}\right){\rm e}^{-i\left(m_\Lambda-i\frac{\Gamma}{2}\right)\tau_{\rm S}}A\left(\Lambda_{\rm S}\to p\pi^-\right), \end{aligned} \tag{69}$$

where $A(\Lambda\to\Lambda_{\rm L,S})$ are amplitudes for Λ production in the states $|\Lambda_{\rm L(S)}\rangle$, $A(\Lambda_{\rm L,S}\to p\pi^-)$ are decay amplitudes from both states, m_Γ and Γ are mass and decay width of the Λ. The different proper times which elapse in the Λ's rest frames during the propagation are the crucial points in our discussion.

If we denote

$$\begin{aligned} \frac{A\left(\Lambda_{\rm S}\to p\pi^-\right)}{A\left(\Lambda_{\rm L}\to p\pi^-\right)} &= \eta_{\rm SL} \\ &= |\eta_{\rm SL}|\,{\rm e}^{i\rho_{\rm LS}}, \quad a=|a|\,{\rm e}^{i\varphi_a}, \quad b=|b|\,{\rm e}^{i\varphi_b}, \end{aligned} \tag{70}$$

the probability for Λ production and decay from the initial state $|\Lambda(0)\rangle$ can be written

$$\begin{aligned} P\left(\Lambda \to p\pi^-\right) &= \left|A\left(\Lambda \to p\pi^-\right)\right|^2 = \left|A\left(\Lambda_{\mathrm{L}} \to p\pi^-\right)\right|^2 \\ &\times\{|a|^2 \mathrm{e}^{-\Gamma\tau_{\mathrm{L}}} + |b|^2 |\eta_{\mathrm{SL}}|^2 \mathrm{e}^{-\Gamma\tau_{\mathrm{S}}} \\ &+2\,|ab\eta_{\mathrm{LS}}|\, \mathrm{e}^{-\frac{1}{2}(\tau_{\mathrm{L}}+\tau_{\mathrm{S}})\Gamma} \cos\left[m_\Lambda\left(\tau_{\mathrm{L}}-\tau_{\mathrm{S}}\right)+\varphi_b+\rho_{\mathrm{LS}}-\varphi_a\right]\}. \end{aligned} \tag{71}$$

The oscillation can possibly arise from the term $m_\Lambda(\tau_{\mathrm{L}} - \tau_{\mathrm{S}})$. If instead of Λ we consider the production and decay of the initial kaon a similar formula would be obtained but with one, as we will see, crucial difference. As masses of $K_{\mathrm{L}} - K_{\mathrm{S}}$ bosons are different, the oscillation factor is equal to $m_{\mathrm{L}}\tau_{\mathrm{L}} - m_{\mathrm{S}}\tau_{\mathrm{S}}$.

How to calculate the proper times? They are measured in different Lorentz frames, in the rest frames of Λ_{L} and Λ_{S}. The basic principle of quantum mechanics — the superposition principle — tells that we can add two states at the same time

$$|\Psi(t)\rangle = |\varphi_1(t)\rangle + |\varphi_2(t)\rangle\,. \tag{72}$$

In the position representation we add wave functions

$$\left\langle \vec{x} \,\middle|\, \Psi(t)\right\rangle = \left\langle \vec{x} \,\middle|\, \varphi_1(t)\right\rangle + \left\langle \vec{x} \,\middle|\, \varphi_2(t)\right\rangle \tag{73}$$

in the same position $\vec{x}$ and at the same time t. It means that the proper times are not suitable variables. We have to add wave function at the same point $\left(\vec{x}, t\right)$ of the same Lorentz system. It is necessary to transform τ_{L} and τ_{S} to the same common frame. The CM frame for the whole $\pi^- p \to \Lambda^0 K^0$ process is the most convenient in this place.

For convenience, we consider only "the one dimensional" problem $\vec{x} = (x, 0, 0)$. Then Lorentz transformations between the rest frames for $\Lambda_{\mathrm{L(S)}}$ and the CM frame are given by

$$\tau_{\mathrm{L}} = \gamma_{\mathrm{L}}\left(t - \beta_{\mathrm{L}} x\right),\; \xi_{\mathrm{L}} = \gamma_{\mathrm{L}}\left(x - \beta_{\mathrm{L}} t\right),$$

and

$$\tau_{\mathrm{S}} = \gamma_{\mathrm{S}}\left(t - \beta_{\mathrm{S}} x\right),\; \xi_{\mathrm{S}} = \gamma_{\mathrm{S}}\left(x - \beta_{\mathrm{S}} t\right),$$

where

$$\gamma_{\mathrm{L,S}} = \frac{E_{\mathrm{L,S}}}{m_\Lambda}\,, \quad \beta_{\mathrm{L,S}} = \frac{p_{\mathrm{L,S}}}{E_{\mathrm{L,S}}}\,. \tag{74}$$

At the beginning $t = 0, x = 0$, and two "ingredients" of the Λ particle, Λ_{L} and Λ_{S} are created at $\tau_{\mathrm{L}} = \tau_{\mathrm{S}} = 0$ and $\xi_{\mathrm{L}} = \xi_{\mathrm{S}} = 0$. But particles in two

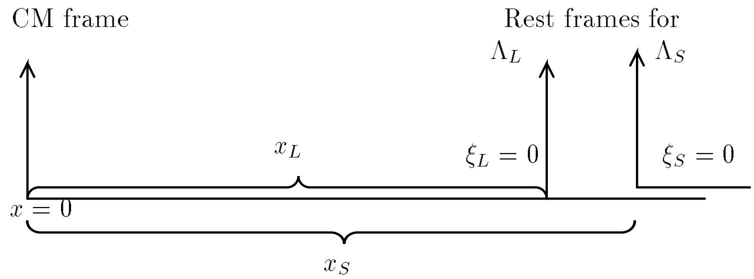

Fig. 3. Relation between CM frame for the production process $\pi^- p \to K^0 \Lambda^0$ and the two rest frames for the Λ_L and Λ_S which move with different speeds. In classical mechanics for $t > 0$ the Λ_L and Λ_S are in different points.

states Λ_L and Λ_S have different speeds and after time t they are in different points in the CM frame (see Fig. 3).

In classical mechanics, for point particles, it is impossible to have a situation that two particles which were born in the same point and at the same time but moving with different speeds would be still in the same, common points at the same time later (Fig. 3). Accordingly to our previous statement (Eq. (73)) such particles will not interfere for any time $t > 0$. But in QM particles are described by wave packets (in the limiting case-plane waves). We do not know at what place the particle was born inside the wave packet and what was the speed of it (see Fig. 4). It is not strange that different parts of two wave packets still interfere. Inside wave packets, energy and momentum are distributed in agreement with QM prescriptions and it is not a surprise that they can be not conserved [32].

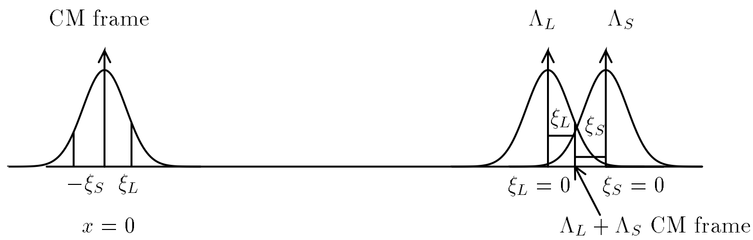

Fig. 4. The same as in Fig. 3, but in quantum mechanics where the Λ_L and Λ_S particles are described the wave packets. Even if the centres of the wave packets at the same time are in different points (like for classical particle in Fig. 3) the two states for Λ_L and Λ_S may interfere. The interference will be possible if the two wave packets overlap.

As we remember, in the wave packet approach, to find the probability as a function of position it is not necessary to assume any relation between t

and x. We simply integrate over t [36]. It is possible, however, to find such a relation between t and x that the oscillation length, which we obtain in this frame, will be (to the first order) the same as in a proper wave packet approach. Such a frame was found [35]. It is the CM frame for $\Lambda_{\rm L}$ and $\Lambda_{\rm S}$, where their momenta are opposite $p^*_{\rm L} = -\, p^*_{\rm S}$ (see Fig. 4). The velocity β of the origin of $\Lambda_{\rm L} - \Lambda_{\rm S}$ center of mass frame in the laboratory system can be easily found from the relation

$$\gamma\left(p_{\rm L} - \beta E_{\rm L}\right) = p^*_{\rm L} = -p^*_{\rm S} = -\gamma\left(p_{\rm S} - \beta E_{\rm S}\right), \tag{75}$$

so

$$\beta = \frac{p_{\rm L} + p_{\rm S}}{E_{\rm L} + E_{\rm S}} = \frac{E_{\rm L} - E_{\rm S}}{p_{\rm L} - p_{\rm S}}\,. \tag{76}$$

Then the movement of the origin of the $\Lambda_{\rm L} - \Lambda_{\rm S}$ CM frame in our laboratory system is described by the obvious relation

$$x = \beta t \tag{77}$$

with β given by Eq. (76).

Using this classical relation Eq. (77), the proper times $\tau_{\rm L}$and $\tau_{\rm S}$ (from Eq. (74)) are given by

$$\begin{aligned} \tau_{\rm L} &= \gamma_{\rm L}\left(\frac{1}{\beta} - \beta_{\rm L}\right)x, \\ \tau_{\rm S} &= \gamma_{\rm S}\left(\frac{1}{\beta} - \beta_{\rm S}\right)x \end{aligned} \tag{78}$$

and taking Eqs (74) and (76) we have [35]

$$\tau_{\rm L} - \tau_{\rm S} = x\left[\gamma_{\rm L}\left(\frac{1}{\beta} - \beta_{\rm L}\right) - \gamma_{\rm S}\left(\frac{1}{\beta} - \beta_{\rm L}\right)\right] = 0 \tag{79}$$

and the oscillation length (Eq. (71)) is infinitely large. If we calculate, in a analogous frame, the oscillation factor for kaons we obtain [35]

$$\begin{aligned} m_{\rm L}\tau_{\rm L} - m_{\rm S}\tau_{\rm S} &= m_{\rm L}\gamma_{K_{\rm L}}\left(\frac{1}{\beta_K} - \beta_{K_{\rm L}}\right) - m_{\rm S}\gamma_{K_{\rm S}}\left(\frac{1}{\beta_{\rm S}} - \beta_{K_{\rm S}}\right) \\ &= \frac{m_{\rm L}^2 - m_{\rm S}^2}{p_{K_{\rm L}} + p_{K_{\rm S}}} = \frac{\frac{m_{\rm L}+m_{\rm S}}{2}\left(m_{\rm L} - m_{\rm S}\right)}{(p_{K_{\rm L}} + p_{K_{\rm S}})/2} = \frac{m\Delta m}{p} \equiv \frac{\Delta m^2}{2p}, \end{aligned} \tag{80}$$

which reproduces the well known result for the oscillation frequency (Eq. (6))

$$\Delta m t = \Delta m \frac{x}{v} = \frac{m\Delta m}{mv}x = \frac{\Delta m^2}{2p}x\,. \tag{81}$$

The relations (79) and (80) which characterize the oscillation length are obtained in this special Lorentz frame. In other frames, these results are correct only to first order in Δm. Instead of checking the dependence of the oscillation length on the Lorentz frame, in the next Section we will present the more complete wave packet approach. Here we have found that the oscillation length of particles recoiling against mixed states is very large. It means that even if we consider the oscillation of such particles (Λ or μ) separately, without connection to kaons or neutrinos it is impossible to observe such oscillations on any acceptable terrestrial distance. Now we consider the oscillation of both particles (Λ and K or μ and ν) together.

7. Correlations for two oscillating particles, EPR effect

Up to now we have considered the oscillation of one particle without taking into account possible correlations which may appear for two or more particles in the final states from which at least one oscillates in the traditional way. There are many such cases. Some of them, with one oscillating particle, have been discussed above $\left(\pi^- p \to \Lambda^0 K^0 \text{ or } \pi^+ \to \mu^+ \nu_\mu\right)$. There are also interesting processes with two oscillating bosons *e.g.* $\Phi \to K^0 \overline{K}^0$, $\Psi\,(3770) \to D^0 \overline{D}^0$ or $e^+e^- \to \Upsilon\,(4s) \to B^0 \overline{B}^0$.

Let us describe the last of them. At $t = 0$ the state of two bosons is a combination of states with definite masses

$$\left|B^0 \overline{B}^0\right\rangle_{t=0} = \sum_{a,b} \eta_{ab} R_{Ba} R_{\overline{B}b} \left|B_a B_b\right\rangle_{t=0}\,, \tag{82}$$

where R_{Ba} and $R_{\overline{B}b}$ are elements of unitary matrix which describes the mixing. The momentum conservation in the production process gives altogether $n\,(n+1)/2$ independent momenta for all n^2 pairs $B_a B_b$. Sometimes there are additional correlations between various mass states B_a and B_b in Eq. (82). If, for example, the $B^0\overline{B}^0$pairs are produced by the $\Upsilon(4s)$ decay then the state $|B^0\overline{B}^0\rangle$ must be totally antisymmetric [38,39] (since $\Upsilon(4s)$ has intrinsic spin $s = 1$ but B mesons are spinless, the B pair is in a p-wave). The factors η_{ab} in Eq. (71) are responsible for such correlations (see for details Ref. [36]).

Each state $|B_{a(b)}\rangle$ is described by a wave packet which in the momentum representation is given by

$$\left\langle a, \vec{p}\;\middle|\; B_a\right\rangle_{t=0} = \int d^3p\, \Psi_a\left(\vec{p}, \vec{p}_a, \sigma_p\right) \left|a, \vec{p}\right\rangle, \tag{83}$$

where $\Psi_a\left(\vec{p}, \vec{p}_a, \sigma_p\right)$ for simplicity is taken as the Gauss function

$$\Psi_a\left(\vec{p}, \vec{p}_a, \sigma_p\right) = \frac{1}{\left[\sqrt{2\pi}\sigma_p\right]^{3/2}} \exp - \frac{\left(\vec{p} - \vec{p}_a\right)^2}{4\sigma_p^2}, \tag{84}$$

with $\vec{p_a}$ — the average momentum and σ_p — the width of the distribution. After time t (taking into account the particle decay) the state $|B_a\rangle$ will evolve into

$$\begin{aligned} |B_a\rangle_{t=0} &\rightarrow |B_a(t)\rangle = \mathrm{e}^{-iHt} |B_a\rangle_{t=0} \\ &= \mathrm{e}^{-\frac{t}{2\tau_a}} \int d^3p\, \Psi_a\left(\vec{p}, \vec{p}_a, \sigma_p\right) \mathrm{e}^{-iE_a(\vec{p})t} \left|a, \vec{p}\right\rangle, \end{aligned} \tag{85}$$

where $\tau_a = \frac{1}{\Gamma_a}\left(E_a\left(\vec{p}_a\right)/m_a\right)$ is the lifetime of the "a" particle in a chosen Lorentz frame.

The states $|B_a(t)\rangle$ in the position representation will be given by

$$|B_a(t)\rangle = \mathrm{e}^{-\frac{t}{2\tau_a}} \int d^3x\, \Psi_a\left(\vec{x}, t; \vec{v_a}, \sigma_{BP}\right) \left|a, \vec{x}\right\rangle, \tag{86}$$

where $\Psi_a\left(\vec{x}, t; \vec{v_a}, \sigma_{BP}\right)$ is the Fourier transform of the momentum distribution (Eq. (84)) and is given by Eq. (25). Let us assume that two detectors are placed at points $\vec{L}_1$ and $\vec{L}_2$. The detectors will measure the particles with beauty "1" and "2" $(1, 2 = B^0, \overline{B}^0)$ respectively.

The states of the B mesons measured by the two detectors are defined by

$$\left|B_1\left(\vec{L_1}\right)\right\rangle = \sum_c R_{1c} \int d^3x_1\, \Psi_c\left(\vec{x_1} - \vec{L}_1, 0; \vec{v_c}, \sigma_{1D}\right) \left|c, \vec{x_1}\right\rangle, \tag{87}$$

and

$$\left|B_2\left(\vec{L_2}\right)\right\rangle = \sum_d R_{2d} \int d^3x_2\, \Psi_d\left(\vec{x_2} - \vec{L}_2, 0; \vec{v_d}, \sigma_{2D}\right) \left|d, \vec{x_2}\right\rangle. \tag{88}$$

Notations in Eqs (87) and (88) are similar to previously presented in Eq. (30).

We can find the amplitude for two oscillating particles in the same way as before (Chapter 3). Then the amplitude of the process where two particles

B^0 and $\overline{B}^0$ produced at $t=0$ at point $\vec{x}=0$ are detected as particles with beauty "1" ("2") at point $L_1(L_2)$ at $t=t_B\left(t_{\overline{B}}\right)$, is the following

$$\begin{aligned}
&A_{B^0\to B_1,\overline{B}^0\to B_2}(\vec{L_1},t_B;\vec{L_2},t_{\overline{B}})=\left\langle B_1\left(\vec{L_1}\right),B_2\left(\vec{L_2}\right)\middle| B^0\left(t_B\right),\overline{B}^0\left(t_{\overline{B}}\right)\right\rangle\\
&=N\sum_{a,b}R^*_{1a}R_{Ba}R^*_{2b}R_{Bb}\eta_{ab}\exp\left\{-\frac{1}{2}\left(\frac{t_B}{\tau_a}+\frac{t_{\overline{B}}}{\tau_b}\right)\right\}\\
&\times\exp\left\{-i\left(E_at_B-\vec{p_a}\vec{L_1}\right)-i\left(E_bt_{\overline{B}}-\vec{p_b}\vec{L_2}\right)\right\}\\
&\times\exp\left\{-\frac{\left(\vec{L_1}-\vec{v_a}\,t_B\right)^2}{4\sigma_B}-\frac{\left(\vec{L_2}-\vec{v_b}\,t_{\overline{B}}\right)^2}{4\sigma_{\overline{B}}}\right\},
\end{aligned}\tag{89}$$

with the normalization factor

$$N=\left[\frac{4\sigma_{1D}\sigma_{2D}\sigma_{BP}\sigma_{\overline{B}P}}{\sigma_B\sigma_{\overline{B}}}\right]^{1/2},\tag{90}$$

and the effective total widths

$$\sigma_B=\sqrt{\sigma^2_{1D}+\sigma^2_{BP}}\,,\qquad \sigma_{\overline{B}}=\sqrt{\sigma^2_{2D}+\sigma^2_{\overline{B}P}}\,.\tag{91}$$

The amplitude (89) can be used in various situations. If we apply the formula (89) to the description of the EPR effect in the $\Upsilon(4s)\to BB$ decay (Refs [38,39]), the collapse of the BB wave function is included in a natural way. Our approach is an alternative to the amplitude description (Refs [38,39]) and, in case of particle mixing, takes into account the EPR correlations in a much more transparent way (see Ref. [36] for detail).

The formula (89) can also be used for the $\mu\nu_\mu$ pair "oscillation" from the $\pi\to\mu\nu_\mu$ decay. In this case only one particle, the neutrino, mixes. Then, in the application of Eq. (89) to our present purpose, we have to take one diagonal mixing matrix (*e.g.* $R_{2b}=\delta_{2b},R_{\overline{B}b}=\delta_{\overline{B}b}$) and $\eta_{ab}=\delta_{ab}$. Usually neutrinos are considered as stable or very long living particles. Let us also assume that the "oscillation" of the muons is measured on a distance much shorter than their decay length. Then both factors in Eq. (89), which are responsible for particle decay, may be neglected. In such circumstances, the probability that neutrinos produced as $\nu_\mu=\alpha$ type together with muons at $\vec{x}=0$ are observed as a β-type neutrino at distance L_ν and the muons at distance L_μ, after integrating over times is given by (for details see Ref. [36]),

$$P_{\beta\alpha}\left(L_\mu,L_\nu\right)=\left[\sum_a\frac{U_{\alpha a}}{v_{\mu a}v_{\nu a}}\right]^{-1}\sum_{ab}\sqrt{\frac{4}{\left(v_{\mu a}^2+v_{\mu b}^2\right)\left(v_{\nu a}^2+v_{\nu b}^2\right)}}$$
$$\times U_{\beta b}U_{\alpha b}^*U_{\beta a}^*U_{\alpha a}\exp-2\pi i\left(\frac{L_\mu}{L_{ab}^{\mu\,\mathrm{osc}}}\right)\exp-\left(\frac{L_\mu}{L_{ab}^{\mu\,\mathrm{coh}}}\right)^2\exp-\left(\frac{\sigma_{\mu x}}{L_{ab}^{\mu\,\mathrm{osc}}}\right)^2N_{ab}^\mu$$
$$\times\exp-2\pi i\left(\frac{L_\nu}{L_{ab}^{\nu\,\mathrm{osc}}}\right)\exp-\left(\frac{L_\nu}{L_{ab}^{\nu\,\mathrm{coh}}}\right)^2\exp-\left(\frac{\sigma_{\nu x}}{L_{ab}^{\nu\,\mathrm{osc}}}\right)^2N_{ab}^\nu\,. \tag{92}$$

The oscillation $\left(L_{ab}^{\mu\,\mathrm{osc}}\right)$and coherence $(L_{ab}^{\mu\,\mathrm{coh}})$lengths, and the factor N_{ab}^μ for muons are (for neutrinos the appropriate expressions are similar)

$$L_{ab}^{\mu\,\mathrm{osc}} = 2\pi\left[\left(E_{\mu a}-E_{\mu b}\right)\frac{v_{\mu a}+v_{\mu b}}{v_{\mu a}^2+v_{\mu b}^2}-\left(p_{\mu a}-p_{\mu b}\right)\right]^{-1}, \tag{93}$$

$$L_{ab}^{\mu\,\mathrm{coh}} = 2\sigma_{\mu x}\left(\frac{v_{\mu a}^2+v_{\mu b}^2}{\left(v_{\mu a}-v_{\mu b}\right)^2}\right)^{1/2}, \tag{94}$$

and

$$N_{ab}^\mu=4\pi^2\frac{\left(E_{\mu a}-E_{\mu b}\right)^2\left(v_{\mu a}^2+v_{\mu b}^2\right)}{\left[\left(E_{\mu a}-E_{\mu b}\right)\left(v_{\mu a}+v_{\mu b}\right)-\left(p_{\mu a}-p_{\mu b}\right)\left(v_{\mu a}^2+v_{\mu b}^2\right)\right]^2}, \tag{95}$$

where $E_{\mu a(b)}, p_{\mu a(b)}$ and $v_{\mu a(b)}$ are energy, momentum and velocity of the muon associated with the neutrino $a(b)$.

First of all, we can see from Eq. (92) that the muon oscillation disappears if we do not measure separately the neutrinos with flavour β. In such case the probability given by amplitude (92) is constant in L_μ and L_ν and can be normalized

$$\sum_\beta P_{\beta\alpha}\left(L_\mu,L_\nu\right)=1\,. \tag{96}$$

But even if we measure the β-type neutrino, the muon oscillation will not be seen. We can prove this statement because we know precisely the muon oscillation length (Eq. (93)).

If we denote the difference between the masses of two neutrinos a and b, as

$$m_a-m_b=\Delta m_{ab}, \tag{97}$$

the inverse of the oscillation length may be decomposed in powers of Δm_{ab}. For muons, the term proportional to the first power of Δm_{ab} vanishes, and

$$(2\pi)\left(L_{ab}^{\mu\,\mathrm{osc}}\right)^{-1}=-2\zeta^2\left(1-v_a^2\right)p_a^{-1}\left(\Delta m_{ab}\right)^2+\dots\,, \tag{98}$$

where ζ is the factor in the decomposition

$$p_b = p_a \left(1 + \zeta \left(\frac{\Delta m_{ab}}{p_a} \right) + \rho \left(\frac{\Delta m_{ab}}{p_a} \right)^2 + \dots \right), \tag{99}$$

while for neutrinos there is

$$(2\pi)\,(L_{ab}^{\nu\,\mathrm{osc}})^{-1} = \frac{\Delta m_{ab}^2}{2p_a} + \dots . \tag{100}$$

and we reconstruct the previous formula for mixing particle oscillation length Eqs (12), (38).

From Eq. (98) it follows that $L_{ab}^{\mu\,\mathrm{osc}}$ is very large and for the acceptable neutrino mass difference the muon oscillation length is much bigger than its decay length

$$L_{ab}^{\mu\,\mathrm{osc}} \gg c\tau_\mu \approx 660\mathrm{m}\,. \tag{101}$$

We can see that even if the neutrino and the muon are both measured, the oscillation of muon will not be observed. Taking into account opinions presented in the latest exchange of views [31–35] we agree with the statement that, in practice, oscillations of μ or Λ^0 particle are impossible to observe.

8. Summary

Let us briefly summarize the main results of this review paper

(i) Many conceptual difficulties arise in the plane wave approach to the particle oscillation problem. This approach gives us the shortest way to get the correct expression for the oscillation length, but it fails if we try to describe other aspects of the particle oscillation.

(ii) In real oscillation experiments neither energy nor momentum are the same for all eigenmass particles.

(iii) The wave packet approach

— gives the proper oscillation length L_{ab}^{osc},

— introduces the concept of the coherence length L_{ab}^{coh}, such that for distances greater than L_{ab}^{coh} the particle oscillation disappears,

— in the proper way, takes into account the fact that to observe oscillation, the sizes of particles source and detector must be much smaller than the oscillation length,

— gives a possibility to understand in a simple way the phenomenon, that a precise measurement of the detected particles momenta may restore the coherence between various eigenmass states and, as a consequence, the oscillation between particles,

— temporal and spatial distributions in the wave packet are correlated by the requirement, that particles are on mass shell. Independent distributions for time and space give a wave packet which does not satisfy the free particle wave equation.

(iv) A problem arises with the proper definition of the Fock space for flavour states. As a consequence, the defined probability currents for such states are not conserved. Then the calculated flavour changing probability is correct only to the first power of the mass difference Δm.

(v) The most adequate approach to particle oscillation is given by quantum field theory. It can be seen especially for neutrinos, which are "neither prepared nor observed", and only propagate between sources and detectors. In this approach

— production and detection processes are fully taken into account as in real experiments,

— physical quantities are expressed in terms of measured quantities, like momenta and energies of hadrons or charged leptons in the neutrino creation and detection processes,

— flavour states, which are not well defined, are not necessary to describe the oscillation process. Only the mass eigenstates and the elements of the mixing matrices may be used,

— it is clear in which circumstances the production and detection amplitudes can be factorized out and the separate oscillation probability be defined,

— the oscillation of non-relativistic particles can be described in the proper way.

(vi) We have specified the meaning of oscillation of particles which recoil against mixed states (as Λ in the process $\pi^- p \rightarrow \Lambda^0 K^0$ or μ in the decay $\pi^+ \rightarrow \mu^+ \nu_\mu$). Even if we consider oscillations of such particles separately, without connection to kaons or neutrinos, it is impossible to observe these oscillations on acceptable terrestrial distances.

(vii) We have found the general amplitudes which describe the oscillation of two particles in the final states. These amplitudes can be applied to

— description of the EPR correlations in decays like $\Upsilon(4s) \rightarrow BB$ or $\Phi \rightarrow KK$, including the mysterious collapse of the wave function in a natural way and giving the possibility to discuss the relativistic EPR correlations on distances longer than coherence lengths.

— description of two particles oscillation from which only one has indeterminate mass like ΛK^0 or $\mu\nu_\mu$. Oscillations of particles with known mass (*e.g.* Λ or μ) can be defined only if, in the same time, flavours of the unknown mass particles are measured (K^0 or ν_μ). In this case, however, the oscillation length of particles with determinate mass is very large, much larger than the particle decay length, which makes it impossible to observe their oscillation in practice.

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