Title: Proprietary theoretical and methodological computer-oriented system for the development of mathematical competence of students

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PROPRIETARY THEORETICAL AND METHODOLOGICAL COMPUTER-ORIENTED SYSTEM FOR THE DEVELOPMENT OF MATHEMATICAL COMPETENCE OF STUDENTS

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Abstract: *The article is devoted to the use in teaching mathematics of a proprietary, theoretical and methodological computer-oriented system for the development of mathematical competence of students. The relevance of the research is related to the existing contradiction between the importance of acquiring key competencies by students, including mathematical competence, and the actual relatively low level of these competencies. In the European framework, mathematical competence is defined as a combination of knowledge, skills and attitudes appropriate to the situation. Mathematical competence is also set out in the Polish and Czech requirement standards. At the same time e-learning, which today is no longer considered as a fashionable modern trend, but as an effective, modern form, method, technology of teaching - learning proves its utility not only in high school and also at lower levels of education - secondary school, for example, in preparing students for the final secondary school examination in mathematics and in developing students' key competencies, including mathematical competence.*

Keywords: key competences, mathematical competences, e-learning, information and communication technology in education, computer program, distance learning system.

INTRODUCTION

The contemporary education process, which is aimed at, among other things, establishing an interaction enhancing relationships between the participants and ensuring the adaptation of students to modern social and economic conditions, to personal fulfilment and development of the creative potential of a particular person, requires the development of innovative educational technologies, including extensive use of contemporary information and communication technologies for personal and professional development. Currently, the focus of all stakeholders of the educational process is the identity of the learner. More and more emphasis is being laid on the need and necessity of development and improvement of individual creative and intellectual abilities and on, the shaping and strengthening of competences.

Standard professional background, especially in the areas of advanced technology may be not sufficient anymore, which makes it necessary to use new techniques and technologies of teaching, especially in sciences directly related to IT.

Rapid obsolescence of information and knowledge gained by employees, occurring during the very process of training makes it necessary for information and knowledge to be constantly updated and improved.

Research work in the field of efficient use of new information technologies in teaching mathematics in high school was carried out by H. Kakol, A. Nowak, A. Rybak, B. Pabich, E. Smyrnova-Trybulska, J. Vaníček.

Analysis of informatization of education and the related processes was performed by S. Juszczyk, J. Kapounová, K. Kostolanýová, E. Smyrnova-Trybulska, J. Šarmanová.

Prospects for development of the use of contemporary teaching methods, with particular emphasis on remote teaching were presented by S. Brown, B. Johnston, A. Reis, E. Smyrnova-Trybulska, others. Psychological and pedagogical aspects of distance education were discussed by L. Briggs, J. Cowan, S. Juszczyk, D. Peal & B. Wilson, G. Seamens, I. Stuchliková, others. Studies into the development of key competencies were carried out by S. Juszczyk, T. Huk, W. Furmanek, M.M.Sysło, others.

One of the next stages in the development of computer-assisted instruction is to develop a proprietary theoretical and methodological computer-oriented system for the development of mathematical competence of students with the use of selected computer programs and distance education, preparing the learner for the final secondary school examination in mathematics.

In the well-known document, adopted in 2006 by the European Parliament (Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning. (2006/962/EC). The European Parliament and the Council of the European Union, 2006, eight key competences are defined that are needed by every person for self-realization and personal development, for being an active citizen and for achieving full social integration and employment. Competences are defined as a combination of knowledge, skills and attitudes appropriate to the situation. Key competences are those which all individuals need for personal fulfilment and development, active citizenship, social inclusion and employment. The following key competencies have been established:

1. Communication in the mother tongue,
2. Communication in foreign languages,
3. **Mathematical competence and basic competences in science and technology,**
4. Digital competence,
5. Learning to learn,
6. Social and civic competences,
7. Sense of initiative and entrepreneurship,
8. Cultural awareness and expression.

Mathematical competences are ranked third among the key competencies - **"Mathematical competences and basic competences in science and technology".**

To determine the range of mathematical competences, one needs to specify what they are and who they are particularly relevant to in the teaching and learning process.

According to W.Kopaliński, "**competence** is a property, range of powers (...), the scope of one's knowledge, skills, responsibility (...)"(Kopaliński 1967: 201)

On the other hand, a **competent person** is, in other words "authorized, empowered to act, to issue a decision, having the qualifications to express judgments and evaluations (.)" (Skorupka, Auderska, Łempicka 1969).

Mathematical competence is very precisely defined by Mogens Niss (Niss 2001). He has identified **eight elements of mathematical competence.** *He defined* it as "the ability to understand, judge, do, and use mathematics in a

variety of intra- and extra-mathematical contexts. Necessary, but certainly not sufficient, prerequisites for mathematical competence are extensive factual knowledge and technical skills. (...). Mathematical competence includes two overarching sorts of capabilities. The first is to ask and answer questions about, within, and by means of mathematics. The second consists of understanding and using mathematical language and tools." (Niss 2001). *He has identified following eight competencies:*

- Thinking mathematically (mastering mathematical modes of thought);
- Posing and solving mathematical problems;
- Modelling mathematically (i.e., analyzing and building models);
- Reasoning mathematically;
- Representing mathematical entities;
- Handling mathematical symbols and formalisms,
- Communicating in, with, and about mathematics;
- Making use of aids and tools (including information technology),

Mathematical competence, defined in the document **“Key Competences for Lifelong Learning - A European Reference Framework "(MKKE)** are defined as a combination of knowledge, skills and attitudes appropriate to the situation. (European Parliament legislative resolution on the proposal for a recommendation of the European Parliament and of the Council on key competences for lifelong learning)

Knowledge:

- W1. understanding of mathematical terms and concepts;
- W2. well controlled numeracy;
- W3. knowledge of measures and structures;
- W4. knowledge of basic operations and basic mathematical presentations;
- W5. awareness of the questions to which mathematics can offer answers.

Skills:

- U1. apply the key principles and processes of mathematics in everyday situations at home and work (in a mathematical way to reason);
- U2. monitor and evaluate the arguments strings (understand mathematical proof);
- U3. transmit messages using mathematical language;
- U4. use of mathematical text.

Attitudes:

- P1. show respect for the truth;
- P2. strive to search for causes;
- P3. evaluate the validity of inferences and actions.

One of the most popular programs for testing competencies in the classroom is the PISA. It is the Programme for International Student Assessment, conducted under the auspices of the Organization for Economic Cooperation and Development - OECD. The results of PISA tests enable us to compare the achievement of students from different countries. The program is designed to test knowledge and skills of fifteen-year old students. The knowledge and skills are tested for:

- understanding texts (reading comprehension);
- mathematical thinking (mathematics);
- scientific thinking (reasoning in the natural sciences).

In 2003, efforts were focused on testing mathematics.

Table 1.

Percentage distribution of students from Poland and the Czech Republic on different levels of math skills.

Level	**Poland**		**Czech Republic**	
	Number of students	**Percentage**	**Number of students**	**Percentage**
Level 6	108	2%	496	5%
Level 5	432	8%	1289	13%

Level 4	972	18%	2083	21%
Level 3	1350	25%	2381	24%
Level 2	1350	25%	1984	20%
Level 1	810	15%	1190	12%
Below the level of 1	379	7%	496	5%
Total:	5401	100%	9919	100%

Source: PISA 2003 survey in Poland and the Czech Republic.

An analysis of the results of the table yields the conclusion that the students attained better results in the Czech Republic. 1785 (18%) of pupils achieved level 5 and 6 results, while in Poland approximately 540 (10%) of pupils achieved the results of these two levels. 379 (7% of) Polish students and 496 Czech students (5%) achieved below level 1. These students were not able to solve the simplest of problems.

Selected mathematical competences have been included in new Polish and Czech standards of examination requirements for the final secondary school examination in mathematics.

According to **Polish standards of examination requirements in mathematics (**Rozporządzenie Ministra Edukacji Narodowej z dnia 28 sierpnia 2007 r. zmieniające rozporządzenie w sprawie standardów wymagań będących podstawą przeprowadzania sprawdzianów i egzaminów (DzU Nr 157, poz. 1102), the candidate has skills enabling her / him:

- to use and create information;
- to use and interpret the representation;
- to use mathematical modelling;
- to use and development strategies;
- to reason and to present arguments.

1. LATEST DATA (2009) RELATING THE USE OF ICT IN EDUCATION AND E-LEARNING BY POLISH STUDENTS AND TEACHERS OF MATHEMATICS IN SILESIA

The survey was conducted in 2009 by Mgr A. Heba as part of her doctoral thesis research. A total of 500 teachers and 500 secondary school students were interviewed in Poland, in the Silesia province. Questions can be divided

into groups by type of information they provide. Initial questions in the questionnaire provided general information about the respondents, in the case of teachers the information related to: gender, age, type of completed education, work experience, employer, and the way computer skills were acquired. Information about students included: gender, age, type of school attended. Another question concerned the use of contemporary Information and Communication Technology (ICT) in education. Next, respondents were asked what educational multimedia programs were being used in their math classes. Respondents were allowed to select multiple answers. Some results of the surveys were published in several articles of the author (Heba 2009a, 2009b)

1.1 Key findings from the survey

1. Cabri, graphing calculators, Gran, Geogebra are the most popular programs that teachers and students use during math classes.

2. When asked to assess themselves, the teachers and young people thought that they had very well mastered the ins and outs of working with the computer. It can be argued that the students are formally prepared, have an appropriate level of competence in computer skills and use of computer programs and multimedia educational programs in mathematics.

3. However, a large discrepancy occurs between the level of competence in both students and teachers in the computer and the use of computer programs and e-learning.

4. According to the teachers and students, most schools do not use Moodle in teaching.

Comparing both groups of respondents it can be concluded that among the respondents familiar with Moodle, more teachers than students have the skills to use the platform.

This is due to the fact that teachers attend various postgraduate studies and training courses in information technology and use the platform in their work as examiners on regional school-leaving examination boards.

5. Teachers pointed **to the lack of pre-prepared courses in mathematics.**

Due to the absence of ready-made teaching materials on such platforms, the teachers are reluctant to use this tool because it would involve the preparation of lessons and activities, which they consider to be very labour intensive and time consuming.

It is worth noting that the teachers were very keen to use this platform provided that there were pre-prepared courses and classes which they could use during lessons or for homework assignments.

It was pointed out that there was a mismatch between the level of available exercises and math problems on this type of platforms and the level of the school-leaving examination (basic or extended).

2. RESEARCH PROBLEM

After examining the documents, literature, current research we identified a problem concerning the conflict between the importance and priority of of students' possessing key competencies, including mathematical competence and the actual level, which is not that high.

3. GENERAL RESEARCH PURPOSE

After analyzing the literature as well as national and foreign experience in the development of mathematical competence and e-learning, the general objective was formulated: to examine what effect, and to what extent, a proprietary, scientifically sound, methodological theoretical computer-oriented system for the development of mathematical competence of students (e-learning course, a methodological guide for teacher, teaching materials for students, developed goals, form, content, method), using the distance learning platform, based on CLMS Moodle system with the use of selected computer programs, has on students' preparation for the compulsory final secondary school examination in mathematics.

4. RESEARCH TASKS

1. *Development of a proprietary theoretical and methodological computer-oriented system for the development of mathematical competence of students, including:*

- developing educational requirements for the distance teaching of mathematics to students;
- the development of theoretical and methodological requirements for the distance teaching of mathematics to students;
- development of study materials for students;

- develop a guide for teachers;
- development of a distance mathematical course preparing students for the final secondary school examination in mathematics, and developing mathematical competence of students, using selected computer programs;
- development of organizational details of the course curriculum using a distance course.

2. Conducting teaching experiments to verify the effectiveness of the proposed methodology.

5. MAIN HYPOTHESES AND SPECIFIC HYPOTHESES

5.1 Main Hypothesis

H1 – Proprietary, scientifically sound, theoretical-methodological system for computer-oriented development of mathematical competence of students using a distance learning platform and selected computer programs contributes to a higher level of mathematical competence of selected students and improves their results obtained during the compulsory final secondary school examination in mathematics.

5.2. Specific Hypotheses

H1.1 - Proprietary scientifically sound theoretical-methodological computer-oriented system for the development of mathematical competence of students using a distance learning platform and selected computer programs contributes to a higher level of mathematical competence in the use and development of information and their results, obtained during the compulsory final secondary school examination in mathematics in respect of this competence.

H1.2 - Proprietary scientifically sound theoretical-methodological computer-oriented system for the development of mathematical competence of students using a distance learning platform and selected computer programs contributes to a higher methodological level of mathematical competence in the use and interpretation and representation of their results, obtained during the compulsory final secondary school examination in mathematics in respect of this competence.

H1.3 - Proprietary scientifically sound theoretical-methodological computer-oriented system for the development of mathematical competence of students using a distance learning platform and selected computer programs

contributes to a higher level of mathematical competence in the field of mathematical modelling and their results, obtained during the compulsory final secondary school examination in mathematics in respect of this competence.

6. RESEARCH STAGES

The course of the research.

The research Plan contained a combination of quantitative and qualitative methods and included three phases.

6.1. Analytical - ascertaining phase (2008-2009)

An analysis was carried out of research methodological literature concerning teaching methods and psycho-pedagogical literature related to teaching mathematics, including the remote mode as well as available experience in the field of distance education in Poland and abroad.

On 05-09 October 2009, a new extended survey was conducted among mathematics teachers in the Province of Silesia. Overall, 500 teachers were surveyed. The tests were conducted during a conference on the preparation of teachers for the new formula of the compulsory final secondary school examination in mathematics.

In the period 20 October - 20 December 2009 a similar survey was carried out among 500 students in the province of Silesia.

6.2. Search Phase (2009-2011)

The objective during the search phase was to identify ways to improve the curricula and to develop of the concept of remote courses in mathematics in order to improve the quality of education for high school students.

At this stage, an e-learning course was developed, teaching methods were put forward together with guidelines for the course teacher, and the organizational framework of the course was developed, too.

At the next stage the course was approved and implemented and the effectiveness of proposed solutions was verified.

6.3. Formative and generalizing phase (2011)

a) Quantitative section

The e-learning course was run from mid-February 2011 to the end of May 2011.

At the beginning, pre-test surveys were carried out, (pre-check the quantity and quality of information in the area of individual mathematical competences).

After the course, a test was conducted to check the knowledge. The test focused on the same area of knowledge, the same issues, and comprised the same questions.

Next, using statistical methods, verification was carried out of the hypotheses claiming that a newly developed course in mathematics with the use of selected computer programs increases the level of mathematical competence of students.

b) Quality section

After the experiment is conducted, quality tests will be carried out using qualitative research interviews with selected nine students from the experimental group.

7. ON THE RESEARCH CONCEPT

The concept of research and the structure of the system for developing mathematical competences as well as the relationships between system components are shown in Figures 1 and Figures 2

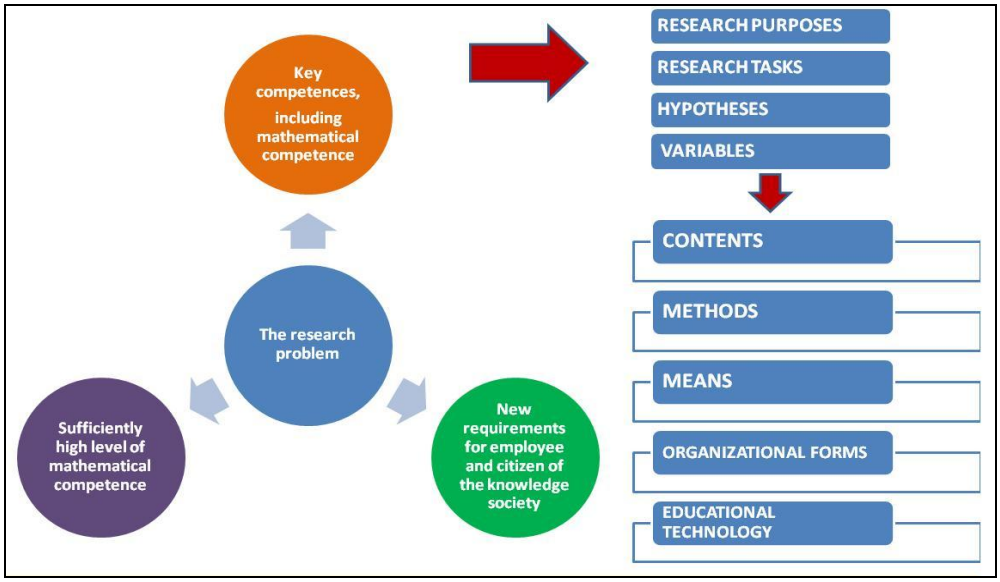

Figure 1. The concept of research.

Source: own

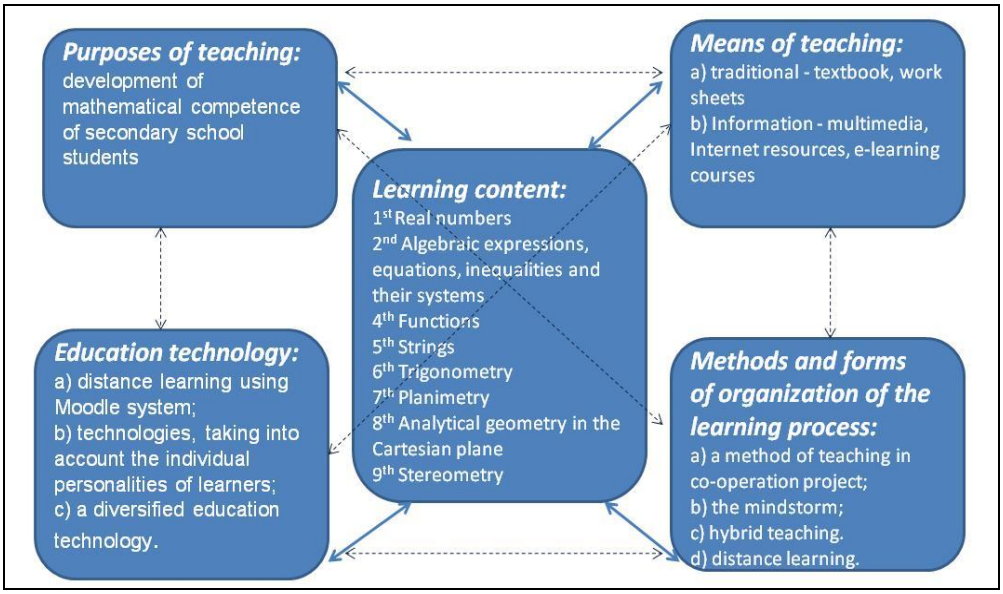

Figure 2. Relationships between components of a system for the development of mathematical competences.

Source: own

8. DESCRIPTION OF A SELECTED MATHEMATICAL COMPETENCE IN USING AND CREATING INFORMATION AND ITS CONNECTION WITH ACTIVITY, TYPES OF EDUCATIONAL MATERIALS, COMPUTER PROGRAMS, THEMATIC SCOPES, AND THEORY OF TEACHING AND MENTAL OPERATIONS

One of the major components of a proprietary, scientifically sound theoretical-methodological computer-oriented system for the development of mathematical competence of students is a **methodological guide for the teacher**, which contains sample lesson plans detailing learning objectives, learning content, teaching technologies, methods and organizational forms of learning process based on the use of the e-learning course, and selected computer programs. The system also provides a detailed description of the developed remote mathematical course preparing for the final secondary school examination in mathematics and developing mathematical competence of students, using some computer programs.

The e-learning course preparing for the final secondary school examination in mathematics was available on the e-learning platform of the Faculty of

Ethnology and Education Sciences University in Cieszyn Silesia in Katowice (http://el2.us.edu.pl/weinoe) (Figure 3, 4, 5, 6). The e-learning course comprised 30 hours of instruction in the traditional mode and 60 in the remote mode.

The course has a modular hierarchical structure and consists of several standard blocks:

- **Introduction to distance course:** Course description, Literature, Glossary, Forum, Registration Survey;
- **Thematic Module:** Pre-Test (diagnostic test); basic teaching materials in the field in question (presentations, text files, videos, etc..). Block of tasks, check, knowledge testing (educational testing). Creative Task Pad, interactive communication block for the lecturer and the students and the students among themselves; Additional resources relating to the learning field in question; knowledge testing (control tests).
- **Module Summary:** Test, Final survey, Reflection survey (evaluation).

I. The topics of the thematic course modules:

1st Real numbers.

2nd Algebraic expressions, equations, inequalities and their systems.

3rd Functions.

4th Strings.

5th The geometry of the Cartesian plane.

6th Planimetry, stereometry.

7th Elements of descriptive statistics. Probability theory and combinatorics.

8th Sets of mathematical problems from the final secondary school examination in mathematics in previous years.

9th Computer programs used in the e-learning course.

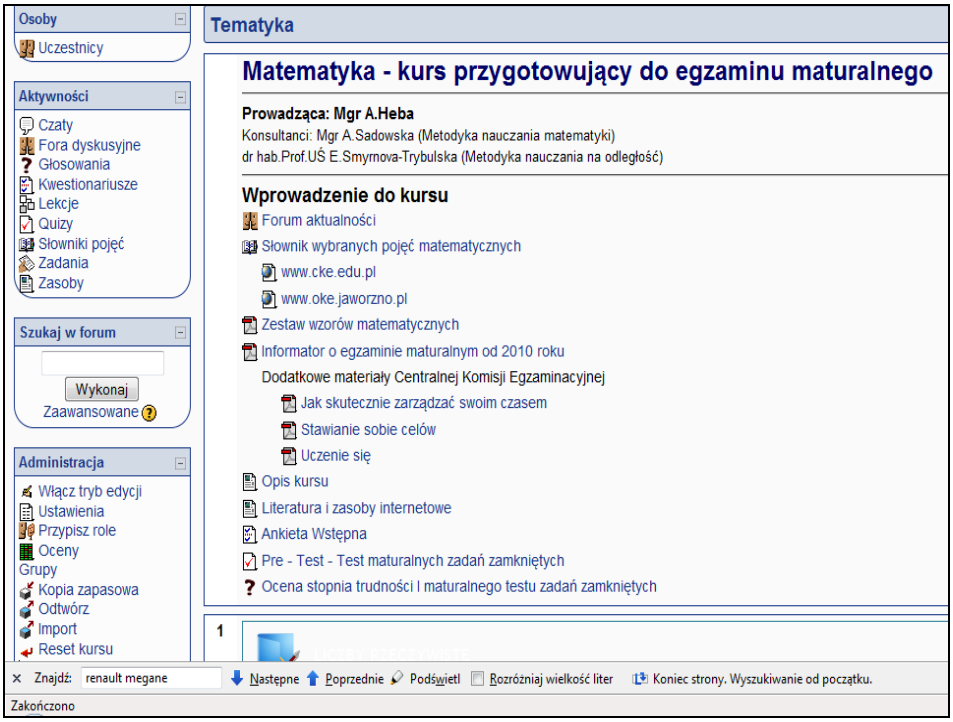

Figure 3. Section of e-learning course. Introduction to the distance course. *Source: http://el2.us.edu.pl/weinoe*

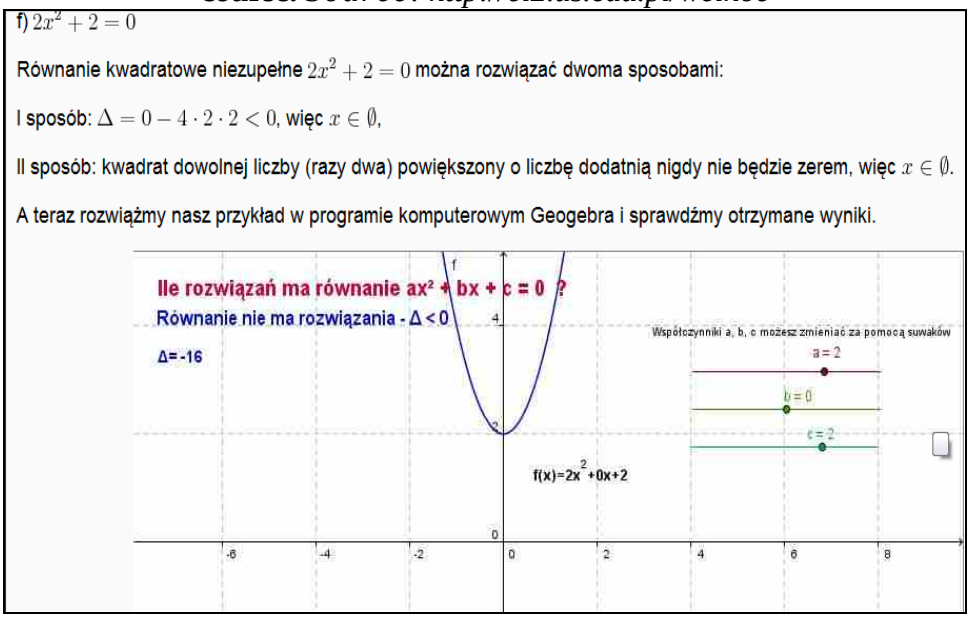

Figure 4. Section of e-learning course. Activity "Lesson". *Source: http://el2.us.edu.pl/weinoe*

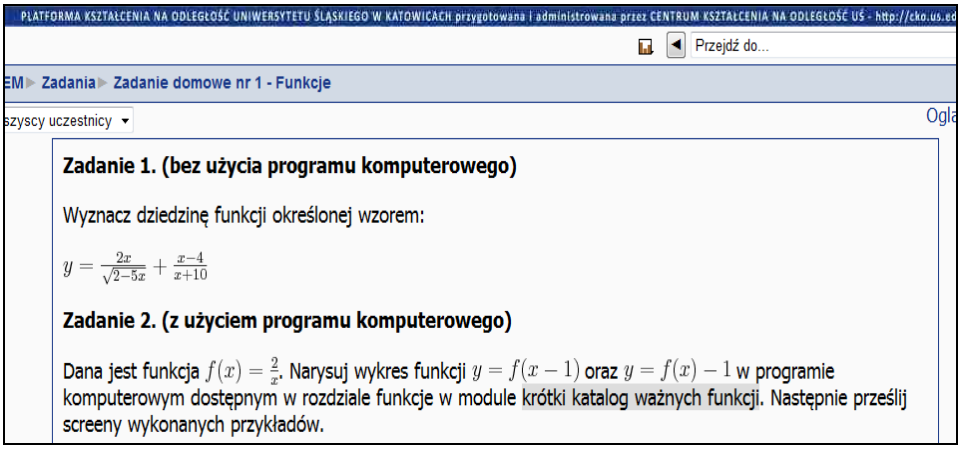

Figure 5. Section of e-learning course. Activity "Assignments".
Source: http://el2.us.edu.pl/weinoe

Figure 6. Section of e-learning course. Activity "Math Crossword".
Source: http://el2.us.edu.pl/weinoe

Annex 1 comprises one of the lesson scripts.

9. DESCRIPTION OF SELECTED MATHEMATICAL COMPETENCE IN USING AND CREATING INFORMATION IN CONTEXT THE RESEARCH

A student who is about to sit the final school-leaving examination in mathematics has skills in:

1) The use and creation of information.

The student interprets a mathematical text and formulates the mathematical results.

The student is able to:

- read the information resulting directly from the content of the problem;
- apply a given formula or specified manner of proceeding;
- perform routine procedure for typical data;
- clearly write up the course and outcome of the calculations and the resulting answer.

Table 2.

Linking competences in the use and creation of information to activity, types of teaching materials, computer programs, thematic scopes, and theories of teaching and mental operations.

Activities	Lessons, Tests, Homework, Forum
Types of teaching materials	PPT, PDF, Educational Videos
Computer programs	GRAN-2D, Geogebra, HEXelon Max Calculator
Thematic areas	real numbers, sequences, trigonometry, functions, analytic geometry
Theories of learning	and development of teaching theory, the theory of constructivism, cognitive theory
Mental operations	analysis by synthesis - abstraction - a generalization

Source: Own

10. CONCLUSIONS FROM RESEARCH CONDUCTED

In the course of theoretical and experimental studies the following **results** were obtained:

- A psycho-pedagogical basis was specified for increasing the effectiveness of the process of developing theoretical knowledge and practical skills (competencies) of students from upper high school grades, based on the use of selected computer programs and an e-learning course;
- A number of elements were developed of new information technologies for the teaching of mathematics for use in teaching and for generalization and systematization of knowledge of main aspects of the math curriculum , aimed at raising the level of mathematical competence of students;
- An e-learning course was developed to support learning in the traditional form and to allow for its analysis and correction, along with the implementation of the basic functions of control and coordination of students' cognitive development , taking into account the requirement for them to use relevant components of Moodle system;
- teaching methodology recommendations were developed on the use of selected computer programs and the e-learning course to generalize and systematize important branches of mathematics in order to raise the level of mathematical competence of upper high school grades;
- a pedagogical experiment was conducted and its results were processed which confirm the effectiveness of the proposed educational components of new information and communication technologies: e-learning course, selected computer mathematical programs and handbook for teachers and teaching materials for students.

The research results allow for the following conclusions:

- development of students' competencies can be effectively attained based on the content, which provides the following sequence of mental operations: an analysis by synthesis, abstraction (extracting priority compounds and relationships), semantic generalizations (explanation and how to resolve the overall absorption, ideas,

patterns, principles, characteristics, properties of problems of a given type) and implemented provided that information and communication technologies are systematically used in learning;

- organization of the learning process based on the theory of development learning, post-stage activity formulation theory, using selected computer programs and the e-learning course allows to ensure efficiency as well as the shaping and development of mathematical competence;
- in order to be able to implement the basic control functions including timely adjustments of knowledge, where necessary, and the most effective control in learning activities of students, it is necessary to systematically use information technology including an e-learning course and selected computer programs;
- in order to achieve the most efficient organization of students' cognitive and learning activities it is necessary to effect a rational and focused integration of traditional teaching methods and new information and communication technologies.

The obtained results allow to identify some directions for further research:

- investigating the problem of organization and combination of various forms of scientific and educational activities in the course of teaching the most important branches of mathematics, using computer programs and an e-learning course;
- varying the content of courses (for the ambitious students (optional), compensatory purposes (correction)) in order to provide individualized instruction.

CONCLUSION

The article addresses the use of a proprietary theoretical and methodological computer-oriented system for the development of mathematical competence of high school students. The authors first present the concept of the research, then the relationship between the components of the education system for the development of mathematical competence of students. Next they discuss the concepts of methodology guidance for teachers, present a description of e-learning course and a sample lesson plan. These were used for research carried as part of the dissertation, which is being prepared at the Pedagogical Faculty, University of Ostrava.

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Annex 1

Topic: Solving systems of equations, one of which is at least the second degree

Class: II basic level.

School: secondary.

Time: 45 min

Lesson Objectives:

Formation of selected mathematical competences in the use and interpretation of the representation

General objectives:

– Acquisition of skills for drawing graphs of linear and quadratic functions and solving equations by the graphical method. Familiarization with the computer program GRAN-2D, acquiring skills to use the program to draw graphs of linear and quadratic functions and their analysis.

Specific objectives:

At the level of theoretical knowledge, after the lesson, the student should:

Remember:

- The method of solving systems of equations, of which at least one is a second degree, by the graphical method.
- how to operate computer program GRAN-2D;
- how to operate the e-learning platform based on CLMS Moodle;

Understand:

- The importance of skills in using computer program GRAN-2D;
- The importance of skills in using the e-learning platform based on the CLMS Moodle.

At the level of skills the student should be able to:

In typical situations:

- Analyse possible situations; Determine the number of solutions of equations, of which at least one is a second degree.
- Use the e-learning platform on CLMS Moodle;
- Provide two curves graphically in GRAN-2D;

In practical situations:

- Justify the usefulness of the computer program GRAN-2D in the teaching of mathematics;
- Justify the usefulness of the e-learning platform based on the CLMS Moodle in teaching mathematics.

Methods:

- feeding (lecture, presentation);
- piecemeal research (working with the source material, work in teams of two).
- Functional.

Teaching aids:

- Books:

– M. Karpiński, M. Dobrowolska, M. Braun, J. Lech, Matematyka 1 – Zakres podstawowy, GWO, 2003, ISBN 83-88881-19-1

– D. Masłowska, T. Masłowski, A. Makowski, P. Nodzyński, E. Słomińska, A. Strzelczyk – „Zbiór zadań i testów maturalnych do

obowiązkowej matury z matematyki" Poziom podstawowy Wydawnictwo Aksjomat, Toruń 2009, ISBN 978–83–60689–21–9.

– Cewe, H. Nahorska – „Matura z matematyki od roku 2010 – zbiór zadań maturalnych z zakresu kształcenia podstawowego" Wydawnictwo Podkowa, Gdańsk 2009, ISBN 978–83–88299–77–3.

– P. Wróblewski, L. Dzikowska – Repetytorium "Jak zdać maturę z matematyka" Wydawnictwo LektorKlett Poznań 2009, ISBN 978–83–7608–148–9.

– Kiełbasa – „Matura z matematyki 2010. Zbiór zadań – poziom podstawowy" Wydawnictwo Lubatka, Warszawa 2009, część 1, ISBN 978-83-929478-1-3.

– Zhaldak M., Smyrnova-Trybulska E., Viciuk O.: "Geometria z GRAN-2D", Poradnik metodyczny dla nauczycieli. Wyższa Szkoła Zarządzania i Marketingu w Sosnowcu, Katedra Edukacji Informatycznej, Uniwersytet Śląski, Filia w Cieszynie, - 2005. - 124s.

- **Internet Resources:**
  - Website containing information on GRAN educational program package and demo versions of the programs - .http://www.gran.ata.com.pl/ (access 15.07.2010)
  - Website containing specimen school-leaving examination papers - http://www.operon.pl/matura_z_operonem/ (access 15.07.2010)
  - Website offering interactive exercises, videos, e-lessons, multimedia presentations - http://scholaris.pl/ (access 15.07.2010)
  - Website containing examples of online lessons, tests, crossword puzzles, games - http://www.interklasa.pl/ (access 15.07.2010)
  - Website offering GeoGebra free math software for independent studying and teaching - http://www.geogebra.org/cms/ (access 15.07.2010)
- Computer -19 assuming 1 student per computer station;
- Computer program GRAN-2D;
- E-learning platform based on Moodle CLMS;
- E-learning course;
- Questionnaire for students available on the e-learning platform.

Course of lessons

Part I. Introductory:

– (5 minutes) Welcoming and inviting students to work together;

– Writing lesson topic on the blackboard;
– Explanation of the objective of the class;
– Switching on of the computers.

Part II. Fundamental part:

– (5 minutes) Have students recall how to operate the e-learning platform based on CLMS Moodle and the computer program GRAN - 2D; teacher points to the module on the platform where the available information concerning the operation of the program GRAN - 2D for examples using it of solving tasks about quadratic functions by the graphical method (Figure 7, 8).

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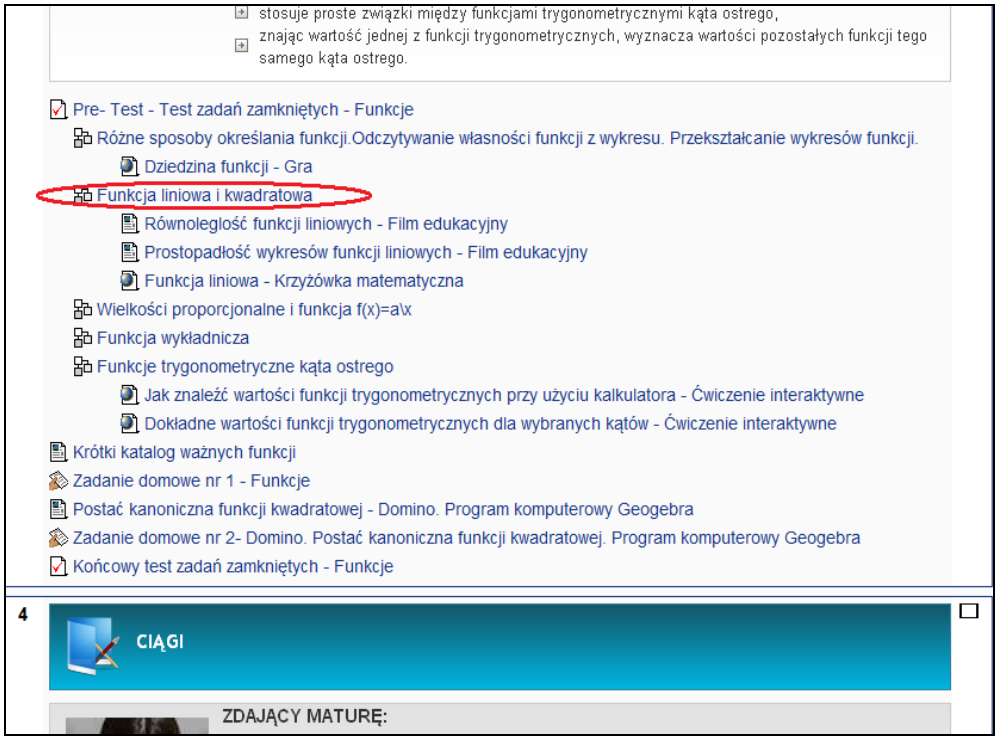


Figure 7. Section of the e-learning course. Lesson activity "Linear and quadratic functions", containing examples of solving tasks by the graphical method, using a GRAN-2D computer program.

Source: http://el2.us.edu.pl/weinoe

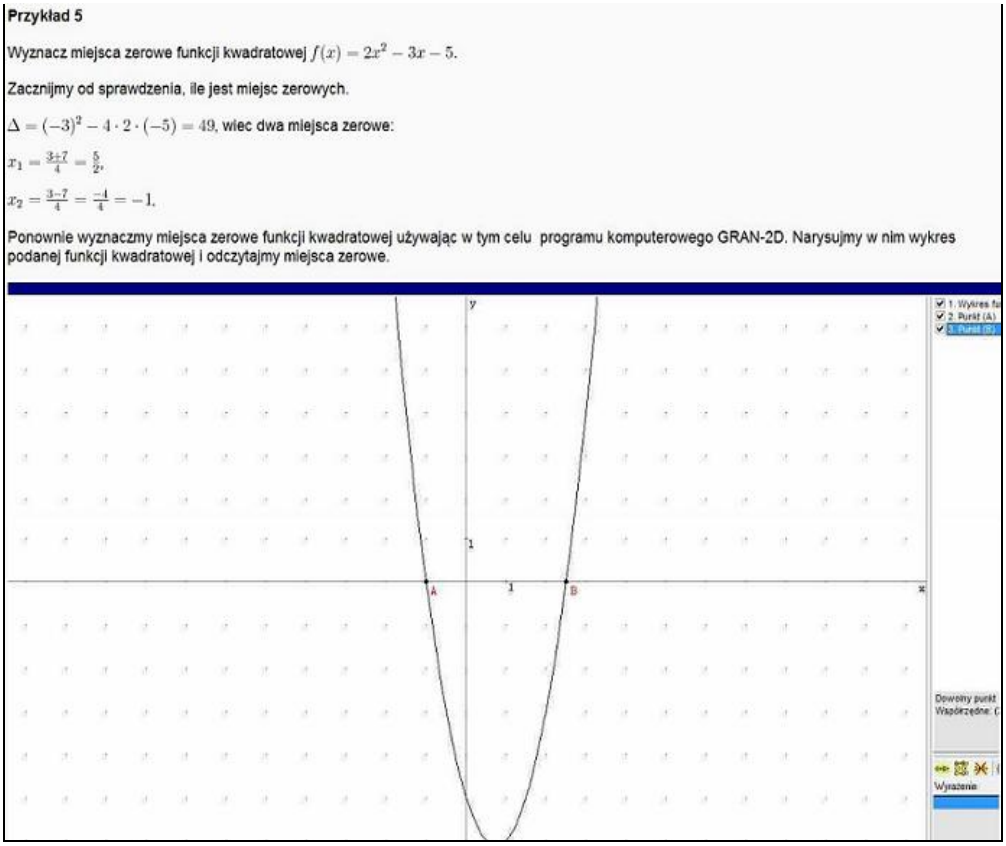


Figure 8. Section of the e-learning course. Activity Lesson about the use of GRAN-2D.

Source: http://el2.us.edu.pl/weinoe

– (10 minutes) Running GRAN-2D. Once the program is launched, a window opens which shows a Cartesian coordinate system. The teacher introduces students to the operation of the program and discusses individual icons and tabs. Indicates that the *function graph* tool is for drawing a graph of graphs of functions. Asks the students to display *the function graph* dialog window and to set the data range from -10 to 10 (for both axes OX and OY). This functionality inserts both standard functions and parametric as well as polar ones. Here one can enter the model function, define the scope of arguments, colour and line type. The teacher writes on the blackboard model of the linear function y = - 2x + 2 and asks the students to insert this function. Reminds the students that when entering the formula, they should use the symbol "*" as the sign of multiplication. So the formula should look like this: y = - 2 * x + 2 (Figure 9)

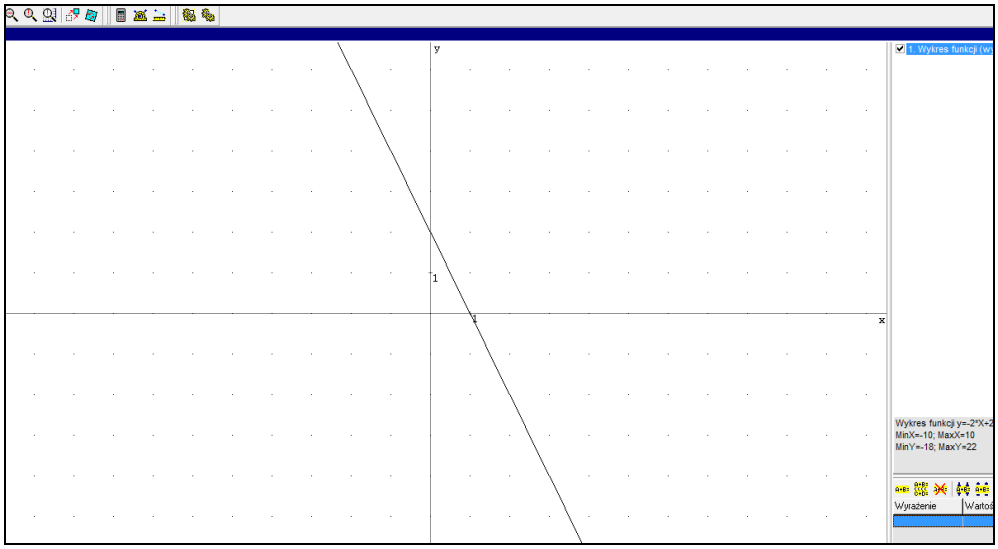


Figure 9. Line y =- 2*x+2, drawn in the GRAN-2D.

The teacher asks the students to insert a quadratic function y = x2 - 3x – 4. Reminds the students that a power of two is written as ^ 2. Students are given some time in which to try out different options for changing line style and colour, etc. (Figure 10)

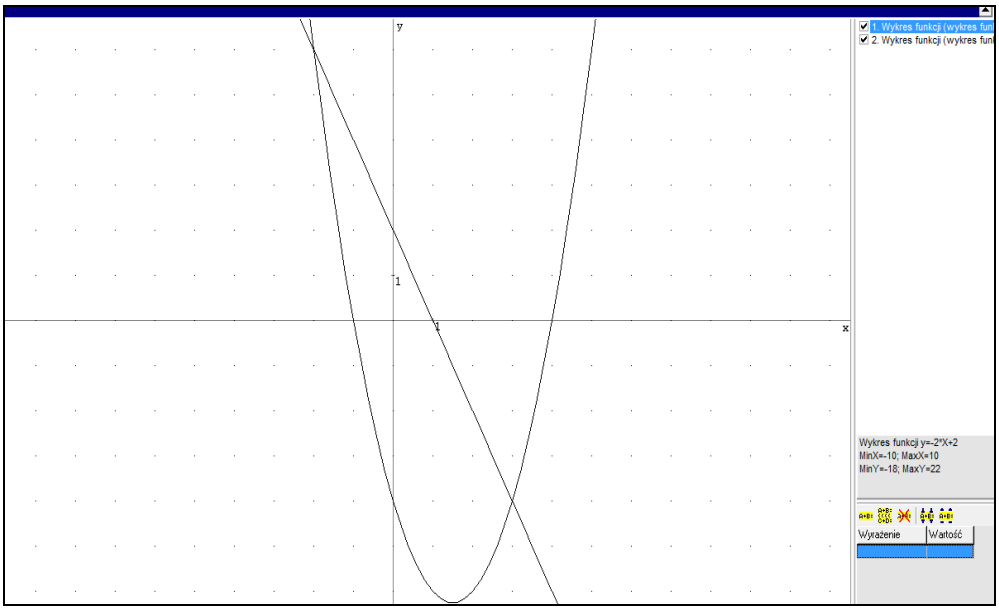


Figure 10. Parabola y = x^2 – 3x – 4, drawn in the GRAN-2D

The teacher reminds the students that the solution is the points of intersection of the two curves. She/he asks how many solutions there are to this system of equations? She/He asks the students to read the points of intersection and write down the solution in their notebooks (-2, 6) or (3, -4). (Figure 11)

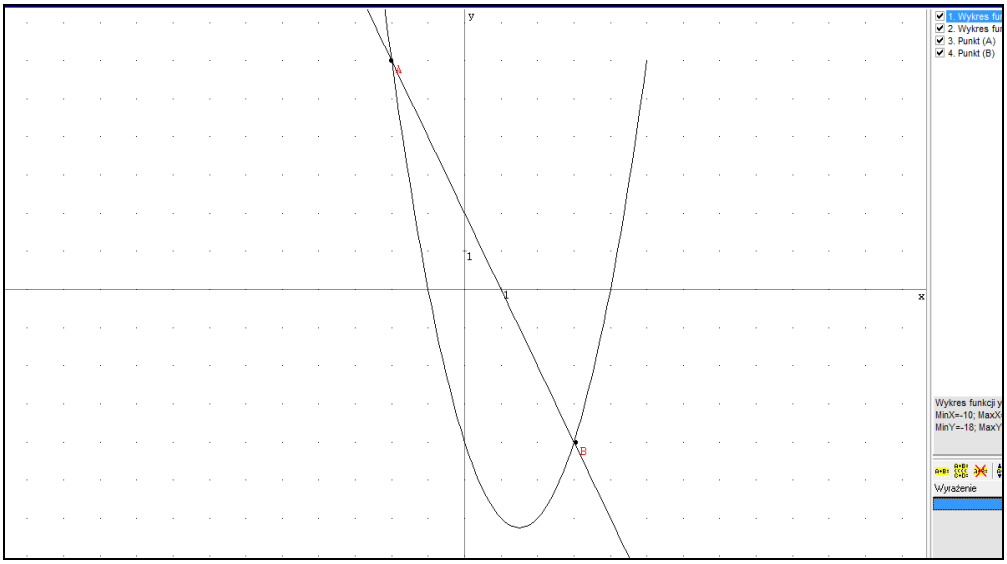


Figure 11. Designation of common points of the line and parabola in the GRAN-2D

The teacher asks how many solutions there can be to a system consisting of a straight line and parabola, and how many solutions there can be to a system consisting of two parabolas? She/he shows the location of the problem on the e-learning platform and asks the students to solve systems of equations included.

– (10 minutes) Students solve the given problem using the computer program GRAN-2D. Students can see examples of unsolved problems in the Lesson activity: “Systems of equations leading to quadratic equations, using GRAN-2D computer program". Then use the module ”Task’’ (Figure 12, 13) in the course send screenshots (screen copy) of the problem as image files to the server.

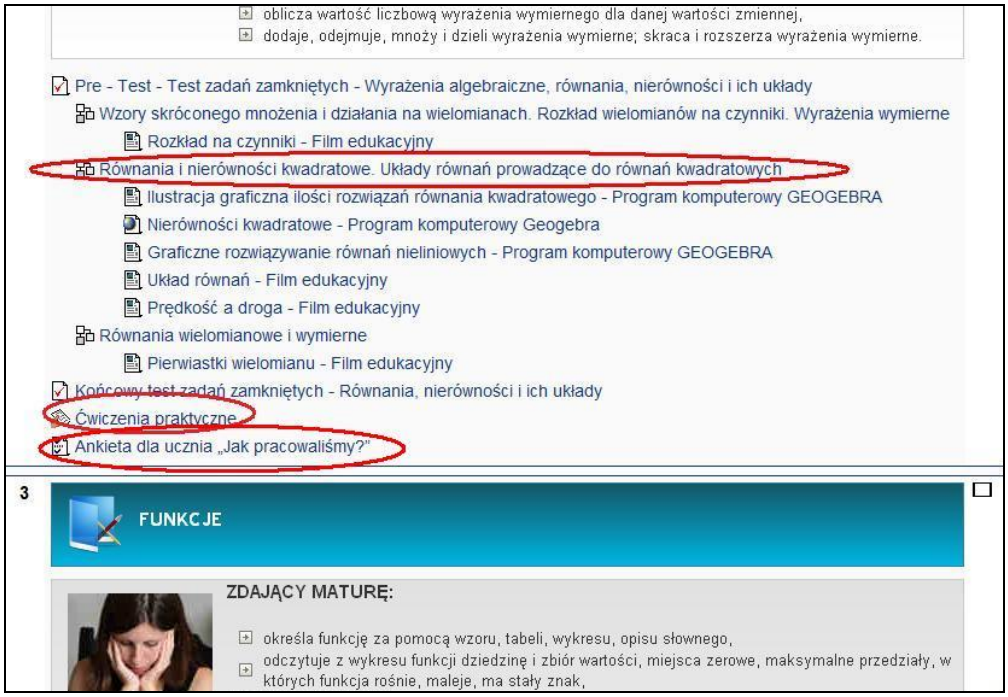


Figure 12. Section of the e-learning course. Activities: Lesson "Systems of equations leading to quadratic equations, using GRAN-2D computer program", Task, Questionnaire.

Source: http://el2.us.edu.pl/weinoe

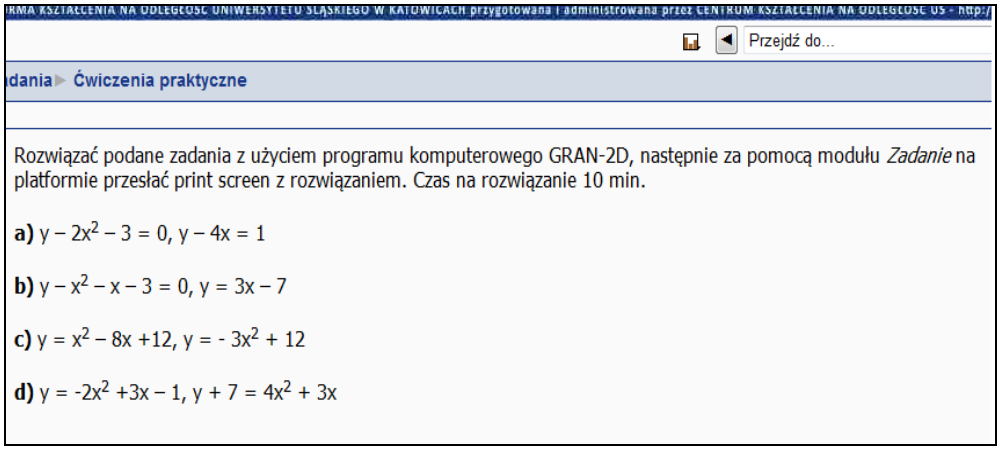


Figure 13. Section of the distance courses. "Task" activity.
Sources: http://el2.us.edu.pl/weinoe

Part III. Completion and recapitulation of activities:

- (5 minutes) Summary of lessons.
- Students turn off the computers.
- Thanking students for participating in the lesson and working together. Invitation to use the computer program GRAN-2D and e-learning platform based on the CLMS Moodle.

Practical exercises

– Solve the given problem using computer program GRAN-2D, then use the module ”Task“ in the course task to send a screenshot (screen copy) the solved problem as an image file to the server so that the teacher can inspect it. Time available: 10 min.

(The teacher shall give one problem to each group)

a) $y - 2x^2 - 3 = 0$, $y - 4x = 1$

b) $y - x^2 - x - 3 = 0$, $y = 3x - 7$

c) $y = x^2 - 8x + 12$, $y = -3x^2 + 12$

d) $y = -2x^2 + 3x - 1$, $y + 7 = 4x^2 + 3x$

Summary of items from the lesson:

– Talking about ways to solve a graphical system of two equations, one of which is at least the second degree, and analysis of possible outcomes (alternatives) solutions,

– Discussion on the effectiveness of e-learning platform Moodle and the computer program GRAN-2D,

– Students’ completing questionnaires "How have we worked?" available on the e-learning platform (Figure 12, 14.).

Survey for the students "How have we worked?"

1. Was the method of conducting lessons interesting for you?
2. Was GRAN - 2D helpful when discussing the topic?
3. Did you take an active part in lessons?
4. Do you remember a lot of information from the lesson?
5. Would you like this method to be used in math lessons?

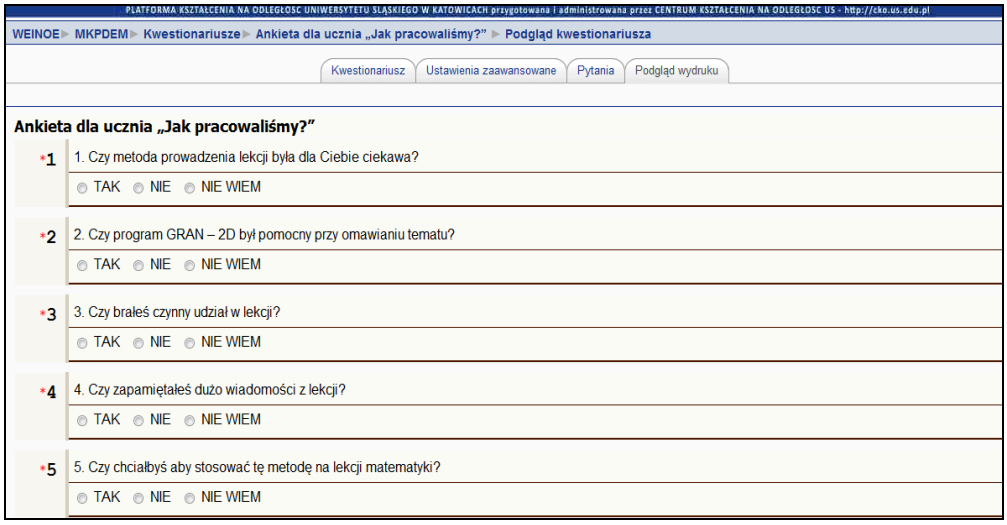
6. 

Figure 14. Section of the e-learning course. Activity “Questionnaire – survey for the student summarizing the lesson.

Source: http://el2.us.edu.pl/weinoe