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# Anisotropic evolution of a D-brane

**Research Article**

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**Abstract:** The evolution of a probe D-brane in the $p$-brane background is considered. The anisotropic evolution of the world-volume of the D-brane with a given topology of a world-volume in the form of a direct product of a $n$-dimensional flat space and $(3-n)$-dimensional sphere is formulated. In this case the anisotropy is described with the aid of two parameters (Hubble parameters) The special case of this evolution, namely the isotropic evolution corresponds to equality of these two parameters. In the latter case the masses and charges of the background $p$-branes are obtained.

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**Keywords:** D-branes • p-branes • Hubble parameters



## 1. Introduction

The motion of the D-branes in the diversity backgrounds has been considered in several recent papers e.g. [1–7]. The applications of D-branes to the cosmology and gravity are also widely discussed e.g. [3–6]. In these approaches a background is fixed by the solutions of the superstring theory approximated by a ten dimensional supergravity. In type II superstring theory the background fields in the bosonic sector are given by a ten dimensional metric, a dilaton and an antisymmetric field $B$ which come from the NS (Neveu-Schwarz)) sector. The sector RR (Ramond-Ramond) gives antisymmetric fields represented by $p$-forms. In this approximation a Dp-brane is a $p$-dimensional submanifold embedded in the background. Then the motion of the D-brane in the ten dimensional spacetime is determined from the action which consists of the Dirac-Born-Infeld (DBI) term and WZW-like term (Wess-Zumino-Witten). In the string theory the D-brane is the place where open strings terminate. In the world-volume of the D-brane the fields are realized by two mechanisms. The first one is related to the ends of the open strings and the second one is related to the pull-backs of the background fields from NS and RR sectors to the world-volume. The former realization gives the fields which in some cases corresponds to the content of the Standard Model (SM). Hence one can consider the world-volume of the brane as a model of the universe. The second mechanism is related to the realization of gravity in the world-volume. The graviton and other fields from NS and RR sectors are not confined to the world-volume (as the fields of SM) but propagate in the extra dimensions perpendicular to the brane. These extra dimensions can have big sizes e.g. [8–11] or their sizes could be compact and small. In the last case one has to compactify the six spatial dimmensions. The extra dimensions warp on cycles of a compact internal manifold. The topology and

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the geometry of this internal manifold is constrained by the consistency conditions such as a supersymmetry. In the case when fluxes induced by the fields from NS and RR sectors vanish and $N = 2$ supersymmetry is defined in four dimensional Minkowski spacetime, then the internal manifold is Calabi-Yau. Non-vanishing fluxes break supersymmetry and the internal manifold is deformed so the Calabi-Yau manifold is replaced by the generalized complex manifold [12–16].

The D-brane is considered as an embedded submanifold in the ten-dimensional spacetime with nontrivial backgrounds fields. The effective theory is described by the DBI action, which determines the motion in spacetime. From the D-brane point of view this motion is interpreted as the evolution of the world-volume of the brane. In the D-brane models of the universe, all fields and particles of the Standard Model are fixed to the world-volume. Thus evolution of D-brane corresponds to a cosmological evolution for the observer fixed to the world-volume.

The aim of this paper is the determination of the backgrounds and their parameters which are compatible with the observed isotropy of expansion of the universe. This condition is expressed by the Hubble parameters. We consider the cosmological evolution of a probe Dk-brane with the DBI action in the backgrounds of p-branes. The probe Dk-brane means that the backreactions are neglected. We assume that the Dk-brane has the topology of the direct product of two spaces: one is flat and the second has a constant curvature. In this case we will obtain the Hubble parameters for these spaces. These parameters depend on the tangent and the normal directions of the Dk-brane. The condition for the equality of these parameters realize the isotropy of the expansion.

In Sec. 2 we recall the DBI action for a Dk-brane in the backgrounds produced by p-branes and derive its equation of motion in a given embedding. In Sec. 3 we derive a ratio of the Hubble parameters for $k$=3. These parameters are obtained from the metric on the world-volume of D3-brane. This case ($k$=3) can be considered as a toy cosmological model corresponding to 3+1 spacetime. The cosmological models derived from M-theory admit warped factors which depend on time [17]. In support of the type IIB string theory, these factors correspond to different rates of expansion of the tangent directions to a D3-brane. Sec. 4 is devoted to conclusions.

## 2. D-brane evolution

In this section we consider the motion of a Dk-brane. The Dk-brane action is described by the DBI action:

$$S = -T_k \int d^{k+1}\xi e^{-\phi}\sqrt{-\det(\gamma_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + B_{\mu\nu})} + T_k \int \sum_i \widetilde{A}_{(i)} \wedge e^{2\pi\alpha' F+B}, \quad (1)$$

where $\gamma_{\mu\nu}$ is the pull back of the background metric, $B_{\mu\nu}$ is the pull back of the background the NS 2-form, $F_{\mu\nu}$ is the strength of the abelian gauge field on the worldvolume and $\widetilde{A}_{(i)}$ are pull-back of the background $i$-forms $A_{(i)}$ with odd (even) degrees: $i = 1, 3, 5, 7$ ($i = 0, 2, 4, 6, 8$) in the Type IIA (IIB) theory. We consider the background solutions with the symmetry group $\mathbf{R}^1 \times E_{(6-p)} \times SO(p + 3)$, where $E_{(6-p)}$ is the Euclidean group. They are given by [18–24]:

- the metric:

$$ds^2 = -\Delta_+\Delta_-^{-\frac{7-p}{8}} dt^2 + \Delta_+^{-1}\Delta_-^{\frac{(3-p)^2}{2(1+p)}-1} dr^2 + r^2\Delta_-^{\frac{(3-p)^2}{2(1+p)}} d\Omega_{p+2}^2 + \Delta_-^{\frac{1+p}{8}} dX_i dX^i, \quad (2)$$

where:

$$\Delta_\pm (r) = 1 - \left(\frac{r_\pm}{r}\right)^{p+1},$$

$d\Omega_{p+2}^2$ is the metric on a ($p$+2)-dimensional sphere $S^{p+2}$ :

$$d\Omega_{p+2}^2 = h_{rs} d\theta^r d\theta^s,$$

$r, s = 1, ..., p + 2$ and $i = 1, ...6 - p$,

- the gauge strength $F = dA_{(p+1)}$ :

$$F = (p + 1)(r_+r_-)^{(p+1)/2} \varepsilon_{p+2}, \quad (3)$$

$\varepsilon_{p+2}$ is the volume form on $S^{p+2}$,

- the dilaton field:

$$e^{-2\phi} = \Delta_-^a, \quad (4)$$

where $a^2 = (3 - p)^2/4$.

The topological charge $g_{6-p}$ and the mass $m_{6-p}$ of the background are expressed by $r_+$, $r_-$ :

$$g_{6-p} = \frac{vol\left(S^{p+2}\right)}{\sqrt{2}\kappa}(p+1)\left(r_+r_-\right)^{(p+1)/2}, \qquad (5)$$

$$m_{6-p} = \frac{vol\left(S^{p+2}\right)}{2\kappa^2}\left[(p+2)\,r_+^{p+1} - r_-^{p+1}\right]. \qquad (6)$$

The above solution becomes the BPS state for $r_+ = r_- = R$ with the metric:

$$ds^2 = \Delta^{\frac{p+1}{8}}\left(-dt^2 + dX_i dX^i\right) + \Delta^{\frac{p-7}{8}}\left(d\rho^2 + \rho^2 d\Omega^2_{p+2}\right), \qquad (7)$$

where $\rho$ is related to $r$ as follows: $\rho^{p+1} = r^{p+1} - R^{p+1}$ and $\Delta = 1 + (R/\rho)^{p+1}$.

We have considered this background solution since they are general and for $p = 3$ the last mentioned metric has the form used in the warp compactification.

In the general case the Dk-brane and D(6-p)-brane do not intersect if their spatial dimensions obey the relation:

$$6 \geq k + 6 - p.$$

We denote the background coordinates as follows:

$$X^M = (t, X^1, ..., X^{6-p}, r, \varphi^1, ..., \varphi^{p+2}),$$

where $\varphi^1, ..., \varphi^{p+2}$ are coordinates on the sphere $S^{p+2}$, so that $r$ and $\varphi^1, ..., \varphi^{p+2}$ span the transverse directions to the (6-p)-brane. The Dk-brane propagating in this background has $n$-directions parallel to (6-p)-brane and $k - n$ directions perpendicular to (6-p)-brane where the number $n$ is given by [1, 2]:

$$n \leq n_0 = \min(k, 6-p). \qquad (8)$$

We will consider free falling Dk-brane in its rest frame with the proper time $\tau$. We assume that $r$ is always transverse to Dk-brane and Dk-brane has the topology of the direct product:

$$V_n \times S^{k-n},$$

where $V_n$ is some n-dimensional flat space and $S^{k-n}$ is the $(k-n)$-dimensional sphere. Thus the embedding field has the form:

$$X^M(\tau) = \left(t(\tau), \xi^1, ..., \xi^n, X^{n+1}, ..., X^{6-p}, r(\tau), \theta^1, ..., ..., \theta^{k-n}, \varphi^{k-n+1}(\tau), ..., \varphi^{p+2}(\tau)\right), \qquad (9)$$

where $\xi^1, ..., \xi^n$ are coordinates on $V_n$ and $\theta^1, ..., \theta^{k-n}$ are coordinates on $S^{k-n}$. The induced metric $\gamma_{\mu\nu}$ on the worldvolume by the embedding (9) has the form:

$$\gamma_{00} = -\Delta_+\Delta_-^{-\frac{7-p}{8}}\dot{t}^2 + \Delta_+^{-1}\Delta_-^{\frac{(3-p)^2}{2(1+p)}-1}\dot{r}^2 + r^2 h_{\widehat{r}\widehat{s}}\dot{\varphi}^{\widehat{r}}\dot{\varphi}^{\widehat{s}}, \qquad (10)$$

$$\gamma_{ab} = \Delta_-^{\frac{1+p}{8}}\delta_{ab}, \qquad (11)$$

$$\gamma_{\widehat{a}\widehat{b}} = r^2 h_{\widehat{a}\widehat{b}}, \qquad (12)$$

where $\widehat{r}, \widehat{s} = k - n + 1, ..., p + 2$, $a, b = 1, ...n$, and $\widehat{a}, \widehat{b} = 1, ..., k - n$. The metrics $h_{\widehat{a}\widehat{b}}$ and $h_{\widehat{r}\widehat{s}}$ are expressed by the metric $h_{rs}$ on the sphere $S^{p+2}$:

$$(h_{rs}) = \begin{pmatrix} \left(h_{\widehat{a}\widehat{b}}\right) & \\ & \left(h_{\widehat{r}\widehat{s}}\right) \end{pmatrix}.$$

The dot over coordinates means the derivative with the respect to the proper time $\tau$. In the case when the background NS form $B$ is zero and the abelian gauge field on the worldvolume vanishes, the WZW term in (1) takes the form:

$$\int \widetilde{A}_{(k+1)},$$

where the form $\widetilde{A}_{(k+1)}$ is given by the pull back of background form $A_{(k+1)}$. In the considered background the only non-vanishing form is $A_{(p+1)} = A_{M_0...M_p} dX^{M_0} \wedge ... dX^{M_p}$ such that $dA_{(p+1)}$ is given by (3). Thus WZW term does not vanish if $k = p$ and the DBI action takes the form:

$$S = -T_k \int d\tau d^n\xi d^{k-n}\theta \left(e^{-\phi}\sqrt{-\det(\gamma_{\mu\nu})} - \delta_{k,p}A\dot{t}\right) \qquad (13)$$

and $A = A_{0...p}$. Since the terms in (13) do not depend on coordinates $\xi$ we get:

$$S = -T_k vol(V_n) \int d\tau d^{k-n}\theta \left(e^{-\phi}\sqrt{-\det(\gamma_{\mu\nu})} - \delta_{k,p}A\dot{t}\right).$$

In the considered background we obtain:

$$e^{-\phi}\sqrt{-\det(\gamma_{\mu\nu})} = \left(\dot{t}^2 - \Delta_+^{-2}\Delta_-^{\frac{1-p}{1+p}}\dot{r}^2 - \Delta_+^{-1}\Delta_-^{\frac{7-p}{8}} r^2 h_{\widehat{r}\widehat{s}}\dot{\varphi}^{\widehat{r}}\dot{\varphi}^{\widehat{s}}\right)^{1/2} \times r^{k-n}\Delta_+^{1/2}\Delta_-^{[5(1-p)+n(1+p)]/16}\sqrt{\det\left(h_{\widehat{a}\widehat{b}}\right)}. \qquad (14)$$

In the non-rotating case $\dot{\varphi}^{\widehat{r}} = 0$ the action simplifies and takes the form:

$$S = -T_k vol(V_n) \int d\tau L, \qquad (15)$$

where the Lagrangian $L$ has the form:

$$L = vol\left(S^{k-n}\right)\left(\dot{t}^{2} - \Delta_{+}^{-2}\Delta_{-}^{\frac{1-p}{1+p}}\dot{r}^{2}\right)^{1/2} r^{k-n}\Delta_{+}^{1/2}\Delta_{-}^{[5(1-p)+n(1+p)]/16} - \delta_{k,p}Aw\dot{t} \tag{16}$$

and $w = \int d^{k-n}\theta$. Variation $L$ with respect to $t$ gives:

$$vol\left(S^{k-n}\right)\frac{\dot{t}r^{k-n}\Delta_{+}^{1/2}\Delta_{-}^{[5(1-p)+n(1+p)]/16}}{\sqrt{\dot{t}^{2} - \Delta_{+}^{-2}\Delta_{-}^{\frac{1-p}{1+p}}\dot{r}^{2}}} - \delta_{k,p}Aw = E, \tag{17}$$

where $E$ is a constant of motion. Thus:

$$\left(\frac{dr}{dt}\right)^{2} = \left[1 - \frac{r^{2(k-n)}\Delta_{+}^{1/2}\Delta_{-}^{[5(1-p)+n(1+p)]/16}}{\left(E+\delta_{k,p}Aw\right)^{2}}vol^{2}\left(S^{k-n}\right)\right]\Delta_{+}^{2}\Delta_{-}^{-\frac{1-p}{1+p}}. \tag{18}$$

The proper time $\tau$ of the Dk-brane is expressed by:

$$d\tau^{2} = \gamma_{\mu\nu}d\xi^{\mu}d\xi^{\nu} = g_{MN}\partial_{\mu}X^{M}\partial_{\nu}X^{N}d\xi^{\mu}d\xi^{\nu}.$$

In the rest frame of the Dk-brane and for the consideration of embedding, this proper time has the form:

$$d\tau^{2} = -\left(g_{00} + g_{rr}\dot{r}^{2} + r^{2}h_{\widehat{r}\widehat{s}}\dot{\varphi}^{\widehat{r}}\dot{\varphi}^{\widehat{s}}\right)dt^{2}, \tag{19}$$

where:

$$g_{00} = -\Delta_{+}\Delta_{-}^{-\frac{7-p}{8}}, \qquad g_{rr} = \Delta_{+}^{-1}\Delta_{-}^{\frac{(3-p)^{2}}{2(1+p)}-1}.$$

In the non-rotating case $\dot{\varphi}^{\widehat{r}} = 0$ the derivatives with the respect to the proper time $\tau$ and coordinate time $t$ are related:

$$\left(\frac{dr}{dt}\right)^{2} = \left(\frac{dr}{d\tau}\right)^{2}\left(\frac{d\tau}{dt}\right)^{2},$$

so:

$$\left(\frac{dr}{dt}\right)^{2} = -\frac{g_{00}}{1+g_{rr}\left(\frac{dr}{d\tau}\right)^{2}}\left(\frac{dr}{d\tau}\right)^{2}.$$

From (18) and (19) we obtain the relation between the radial position and the proper time:

$$\left(\frac{dr}{d\tau}\right)^{2} = \frac{\left(E-\delta_{k,p}Aw\right)^{2} - r^{k-n}\Delta_{+}^{1/2}\Delta_{-}^{\beta}vol^{2}\left(S^{k-n}\right)}{\left(E-\delta_{k,p}Aw\right)^{2} - \Delta_{-}^{\gamma} + r^{k-n}\Delta_{+}^{3/2}\Delta_{-}^{\delta}vol^{2}\left(S^{k-n}\right)}\Delta_{+}\Delta_{-}^{\alpha}, \tag{20}$$

where the exponents are:

$$\alpha = \frac{-1+14p-p^{2}}{8(1+p)}, \qquad \beta = \frac{5(1-p)+n(1+p)}{16},$$

$$\gamma = \frac{3\left(p^{2}-10p+9\right)}{8(1+p)}, \qquad \delta = \frac{p^{2}-60p+61+n(1+p)^{2}}{16(1+p)}.$$

The Eq. (20) makes sense if the right side is greater than or equal to zero.

In the coordinate time $t$ the induced metric on the Dk-brane by the embedding (9) has the form:

$$ds^2 = -\left(\Delta_+\Delta_-^{-\frac{7-p}{8}} - \Delta_+^{-1}\Delta_-^{\frac{(3-p)^2}{2(1+p)}-1}\dot{r}^2 - r^2 h_{\hat{r}\hat{s}}\dot{\varphi}^{\hat{r}}\dot{\varphi}^{\hat{s}}\right)dt^2 + \Delta_-^{\frac{1+p}{8}}d\xi_a d\xi^a + r^2 h_{\hat{a}\hat{b}}d\theta^{\hat{a}}d\theta^{\hat{b}}. \tag{21}$$

Using (19) we get:

$$ds^2 = -d\tau^2 + e^{2\lambda}d\xi_a d\xi^a + e^{2\beta}h_{\hat{a}\hat{b}}d\theta^{\hat{a}}d\theta^{\hat{b}}, \tag{22}$$

where $r(\tau)$ is the solution of the (20) and $\exp 2\lambda = \Delta_-^{\frac{1+p}{8}}$, $\exp 2\beta = r^2$. This metric has the form of the Bianchi type I for the homogenous space with two scale factors namely $\Delta_-^{\frac{1+p}{8}}$ and $r^2$.
In the conformal time $x^0$ (which is related to $\tau$ according to $dx^0 = \exp(-\lambda)\,d\tau$) the metric (22) reads:

$$ds^2 = e^{2\lambda}\left(-\left(dx^0\right)^2 + d\xi_a d\xi^a\right) + e^{2\beta}h_{\hat{a}\hat{b}}d\theta^{\hat{a}}d\theta^{\hat{b}}. \tag{23}$$

Then assuming that the evolution of the world-volume seeing by the observer fixed to the brane, is governed by the Einstein gravity, the scale factor $\lambda$ and $\beta$ evolve as follows:

$$n(n-1)\left(\lambda'\right)^2 + 2mn\lambda'\beta' + m(m-1)\left(\beta'\right)^2 + e^{2\lambda-2\beta}\widetilde{R} = 16\pi G\widetilde{T}_{00}, \tag{24}$$

$$\left[2(1-m)\lambda'' - 2n\beta'' + 2n(2-m)\lambda'\beta' + \left(2n+m-m^2-2\right)\left(\lambda'\right)^2 - n(n+1)\left(\beta'\right)^2 - e^{2\lambda-2\beta}\widetilde{R}\right]\delta_{ab} = 16\pi G\widetilde{T}_{ab}, \tag{25}$$

$$\begin{aligned} e^{2\beta-2\lambda}&\left[2(1-n)\beta'' - 2m\lambda'' + (2m-mn-n)\lambda'\beta' - m(m-1)\left(\lambda'\right)^2 - n(n-1)\left(\beta'\right)^2\right]h_{\hat{a}\hat{b}} \\ &+ 2\left(\widetilde{R}_{\hat{a}\hat{b}} - \frac{1}{2}\widetilde{R}h_{\hat{a}\hat{b}}\right) = 16\pi G\widetilde{T}_{\hat{a}\hat{b}}, \end{aligned} \tag{26}$$

where $m = k - n$, the Ricci tensor $\widetilde{R}_{\hat{a}\hat{b}}$ and the scalar curvature $\widetilde{R}$ are obtained from the metric $h_{\hat{a}\hat{b}}$. The prime means derivative with respect to $x^0$ (e.g. : $\lambda' = d\lambda/dx^0$). The energy-momentum tensor $\widetilde{T}_{\mu\nu} = \left(\widetilde{T}_{00}, \widetilde{T}_{ab}, \widetilde{T}_{\hat{a}\hat{b}}\right)$ is given by the matter and the fields in the world-volume of the Dk-brane and is defined with respect to the metric (23):

$$\widetilde{T}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial\mathcal{L}_m}{\partial g^{\mu\nu}}, \tag{27}$$

where $\mathcal{L}_m$ is a Lagrangian density for the matter and the fields in the world-volume. Using relations $\lambda' = \exp(\lambda)\,d\lambda/d\tau$ and $\beta' = \exp(\lambda)\,d\beta/d\tau$ one can rewrite Eq. (24) as follows:

$$n(n-1)H_1^2 + 2mnH_1H_2 + m(m-1)H_2^2 + e^{-2\beta}\widetilde{R} = 16\pi G T_{00}, \tag{28}$$

where $T_{00}$ is the energy density in the metric (22) related to $\widetilde{T}_{00}$ as follows: $\widetilde{T}_{00}e^{-2\lambda} = T_{00}$. The quantities $H_1$ and $H_2$ are Hubble parameters given by: $H_1 = d\lambda/d\tau$ and $H_2 = d\beta/d\tau$. Hence from the Eq. (22) it follows that the factor $\exp(2\lambda) = \Delta_-^{(1+p)/8}$ correspods to the evolution in the flat directions $\xi$ while the second factor $h_{\hat{a}\hat{b}}\exp 2\beta = r^2 h_{\hat{a}\hat{b}}$ concerns the evolution in the curved directions (corresponding to the sphere).

## 3. Hubble parameters

We have related the metric (22) to the two Hubble parameters:

$$H_1 = \frac{1}{\Delta_-^{\frac{1+p}{16}}} \frac{d}{d\tau}\left(\Delta_-^{\frac{1+p}{16}}\right), \tag{29}$$

$$H_2 = \frac{1}{r}\frac{dr}{d\tau}, \tag{30}$$

where it is assumed that there is isotropic evolution which means that $d\left(h_{\hat{a}\hat{b}}\right)/d\tau = 0$. Then the Eq. (29) reads as:

$$H_1 = \frac{(p+1)^2}{16} \cdot \frac{r r_-^{p+1}}{r^{p+1} - r_-^{p+1}} \frac{dr}{d\tau}, \tag{31}$$

where $dr/d\tau$ is given by (20). Thus the ratio of these Hubble parameters is given by the relation:

$$\frac{H_1}{H_2} = \frac{(p+1)^2}{16} \cdot \frac{r^2 r_-^{p+1}}{r^{p+1} - r_-^{p+1}} \equiv \eta(r). \tag{32}$$

It depends on the position $r$ of the Dk-brane and $r$ is given by the solution of the Eq. (20). In the case when $r_+ = r_- = R$ and the metric on the world-volume is given by (22) the Eq. (18) takes the form:

$$\left(\frac{dr}{dt}\right)^2 = \tag{33}$$
$$\left[1 - \frac{r^{2(k-n)}\Delta^{1/2+[5(1-p)+n(1+p)]/16}}{\left(E + \delta_{k,p}Aw\right)^2} vol^2\left(S^{k-n}\right)\right]\Delta^{\frac{1+3p}{1+p}}.$$

The Eq. (33) can be interpreted as a one dimensional particle motion with the zero energy in the potential $-V(r)$:

$$V(r) =$$
$$\left[1 - \frac{r^{2(k-n)}\Delta^{1/2+[5(1-p)+n(1+p)]/16}}{\left(E + \delta_{k,p}Aw\right)^2} vol^2\left(S^{k-n}\right)\right]\Delta^{\frac{1+3p}{1+p}}.$$

Hence $V$ has to be positive and zeros of $V$ gives the turning points. The ratio of the Hubble parameters in this case is:

$$\frac{H_1}{H_2} = \frac{(p+1)^2}{16} \cdot \frac{r^2 R^{p+1}}{r^{p+1} - R^{p+1}}. \tag{34}$$

We restrict ourselves to the case when $k = 3$ which corresponds to the D3-brane. In this case $a, b = 1, \ldots n$, and $\hat{a}, \hat{b} = 1, \ldots, 3 - n$. The relation between the parameters $H_1$ and $H_2$ in other cases is given by the Eq. (28). Thus in the case considered here, we obtain the following equations:

for $n = 0$:

$$H_2^2 + \frac{1}{6}e^{-2\beta}\widetilde{R} = \frac{8\pi G}{3}T_{00}, \tag{35}$$

for $n = 1$

$$H_2^2 + 2H_1H_2 + \frac{1}{2}e^{-2\beta}\widetilde{R} = 8\pi G T_{00}, \tag{36}$$

for $n = 2$

$$H_1^2 + 2H_1H_2 + \frac{1}{2}e^{-2\beta}\widetilde{R} = 8\pi G T_{00}, \tag{37}$$

for $n = 3$

$$H_1^2 = \frac{8\pi G}{3}T_{00}. \tag{38}$$

The Eqs. (35) and (38) are equations only for $\beta$ and $\lambda$, respectively. These equations govern the isotropic evolution of the world-volume of the D3-brane. While the Eqs. (36) and (37) give relation between $H_1$ and $H_2$. One can see that the cases $n = 1$ and $n = 2$ are symmetric, so we focus on the case $n = 1$. Using the Eq. (32), Eq. (36) takes the form:

$$H_2^2(1 + 2\eta) + \frac{1}{2}e^{-2\beta}\widetilde{R} = 8\pi G T_{00}. \tag{39}$$

All the terms in the above equation are functions of $r$ which evolve with respect to the coordinate time $t$ according to Eq. (33) which, in our case ($k = 3$), takes the form:

$$\left(\frac{dr}{dt}\right)^2 = \tag{40}$$
$$\left[1 - \frac{r^{2(3-n)}\Delta^{1/2+[5(1-p)+n(1+p)]/16}}{\left(E + \delta_{3,p}Aw\right)^2} vol^2\left(S^{3-n}\right)\right]\Delta^{\frac{1+3p}{1+p}}.$$

In order to derive the change (34) in the time $t$ we would like to solve the Eq. (40). These solutions, among others, depends on the dimension $p$ of the background branes. Thus we have to consider each dimension $p$ separately. Although in the type IIB the dimensions $p$ are odd we also consider the background produced by D-particle, thus $p = 0, 1, 3$ and 5. The solutions of (40) for different $p$ are given below where the number of the non-compact dimensions $n$ is given by the condition (8).

For $p = 0$ (D-particle) the Eq. (40) gives the following result:

$$\int \frac{\sqrt{r}dr}{\sqrt{r-R}\sqrt{1 - r^{2(\alpha-\beta_0)}(r-R)^{2\beta_0}\sigma_n^2}} = t + t_0, \tag{41}$$

where

$$\sigma_n^2 = \left(\frac{vol\left(S^{3-n}\right)}{E}\right)^2$$

and $\alpha = 3 - n$, $2\beta_0 = (13 + n)/16$. The number $n$ of the flat dimensions is:

$$n = 0, 1, 2, 3.$$

The cases $n = 0$ and $n = 3$ correspond to the only one Hubble parameter.
For $p = 1$ (D-string) we get:

$$\int \frac{r^2 dr}{\left(r^2 - R^2\right)\sqrt{1 - r^{2(\alpha-\beta_1)}\left(r^2 - R^2\right)^{2\beta_1}\sigma_n^2}} = t + t_0, \quad (42)$$

where $2\beta_1 = (9 + 2n)/16$.
For $p = 3$:

$$\int \frac{r^5 dr}{\left(r^4 - R^4\right)^{5/4}\sqrt{1 - r^{2(\alpha-\beta_3)}\frac{\left(r^4-R^4\right)^{2\beta_3}}{(E+Aw)^2} vol^2\left(S^{3-n}\right)}} = t + t_0, \quad (43)$$

where $2\beta_3 = (2n - 1)/8$.
In the both the above cases the number $n$ of the flat dimensions is equal to:

$$n = 0, 1, 2, 3.$$

The first and the last cases correspond to the only one Hubble parameter.
For $p = 5$:

$$\int \frac{r^8 dr}{\left(r^6 - R^6\right)^{4/3}\sqrt{1 - r^{2(\alpha-\beta_5)}\left(r^6 - R^6\right)^{2\beta_5}\sigma_n^2}} = t + t_0, \quad (44)$$

where $2\beta_5 = 3(n - 2)/8$. The number $n$ of the flat dimensions is equal to: $n = 0, 1$.
The above integrals are complicated. One can evaluat them in the limit when the parameter $E$ goes to infinity (it means that $\sigma_n \to 0$). In this case all the above integrals have simple asymptotes: $r \sim t$. It means that the D3-brane and background $p$-branes do not form a bounded system. Thus one can notice from (34) that:

$$\frac{H_1}{H_2} = \eta \underset{r\to\infty}{\longrightarrow} \begin{cases} \infty, & p = 0, \\ R^2/4, & p = 1, \\ 0, & p > 1, \end{cases} \quad (45)$$

and $\eta$ is singular for all $p$ in $r = R$. As mentioned above, the considered background solutions are right for $r > R$ and Eq. (45) is valid for big $r$. Thus the Eq. (39) for big $r$ and the background produced by D1-branes (D-strings) takes the form:

$$H_2^2\left(1 + \frac{1}{2}R^2\right) + \frac{1}{2}e^{-2\beta}\widetilde{R} = 8\pi G T_{00}. \quad (46)$$

This equation becomes the ordinary Friedmann equation with the constant space curvature $6\widetilde{R}$ if $R = 2$ which means that $H_1 = H_2$. The condition $R = 2$ puts a constraint on a topological charge $g_5$ and a mass $m_5$ of a dual D5-branes to the background D1-branes, because $R$ is related to the charge and the mass by (5) and (6). Thus these relations for $r_+ = r_- = R$ are:

$$g_5 = \frac{3vol\left(S^3\right)}{\sqrt{2}\kappa}R^3, \quad (47)$$

$$m_5 = \frac{3vol\left(S^3\right)}{2\kappa^2}R^3. \quad (48)$$

Thus we obtain the following values of $g_5$ and $m_5$:

$$g_5 = 24\sqrt{2}\pi^2/\kappa, \quad (49)$$

$$m_5 = 24\pi^2/\kappa^2. \quad (50)$$

In the background produced by D1-branes the condition of the isotropic expansion leads to the (49) and (50).
For $p = 0$ one can see from (45) that the expansion of the flat dimensions is much faster than non-flat dimensions. The second posibility for $p = 0$ is following: $H_2 = 0$ which means that the non-flat space is static. For $p > 1$ the non-flat dimensions expand faster than flat or $H_1 = 0$ which means that the flat space is static.

## 4. Conclusions

In this paper we have obtained the Hubble parameters for Dk-brane embedded in the backgrounds produced by the black p-branes. These parameters are related to the topology of the Dk-brane: the Dk-brane is represented as the Cartesian product of the $n$-dimensional flat space and some $(n-k)$-dimensional space space with a constant curvature (in our case this space is a sphere). In the general case these parameters have different values. It means that the evolution from the point of view of an observer fixed to the Dk-brane in the flat directions is different. The ratio of these parameters has been obtained in explicit form for big values of $r$. This ratio is equal to one only in one case for $p = 1$ (D-strings) and for the special value of $R = 2$.

It means that in the asymptotic region ($r \to \infty$) and for $R = 2$, the expansion is the same in all directions in the world-volume of the D3-brane. In this case the topological charge and the mass are given by Eqs. (49)–(50). The above results are valid if D3-brane and the background branes do not form a bounded system. This is true for sufficiently big parameter $E$. In the general case the ratio $\eta$ (Eq. (32)) depends on the position of the D3-brane.

The considered model is an example of a toy cosmological model. The observed isotropic expansion of our universe is realized in this model as a condition on the equality of the Hubble parameters. This condition puts a constraint on the allowed masses and charges of the background D5-branes which are dual to D1-branes. The values of the mass and the charge were obtained under the assumption that the D3-brane has the topology of the direct product of the $n$-dimensional flat space and $(3 - n)$-dimensional sphere. It will be interesting to relate the concomitant results to the ideas presented in [25].

According to the mirage cosmology [3, 4], the evolution of the world-volume is driven not only by the energy density in the world-volume but also by the interactions with the bulk which consist of the fields and the branes coming from string theory. The observer fixed to the world-volume does not see the additional dimensions in which the fields and the other branes are propagated, and interprets these interactions as ones with the real fields in the world-volume. The geometry (gravity) of the world-volume is given by the induced metric and its dynamics is described by the Friedmann-like equations with the energy density modified by the contribution coming from the bulk. In this work, the energy density ($\widetilde{T}_{00}$) obtained from the matter lagrangian density (Eq. (27)) is modified by the factor $\exp(-2\lambda)$ which depends on the position of the D3-brane in the bulk. Thus in the Eqs. (35)–(38) the energy density $T_{00}$ depends not only on the bare $\widetilde{T}_{00}$ but also on the position of the D3-brane. From the world-volume perspective this phenomenon is equivalent to introducing some fictitious matter fields with the energy density given by $T_{00}$. In this way the consequent results are related to the mirage cosmology.

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